ADDENDA
Open flavour charmed mesons in a quantum chromodynamics potential model

KRISHNA KINGKAR PATHAK1,* and D K CHOU DHURY2

1Department of Physics, Arya Vidyapeeth College, Guwahati 781 016, India
2Department of Physics, Gauhati University, Guwahati 781 014, India
and Physics Academy of Northeast, Guwahati 781 014, India
*Corresponding author. E-mail: kkingkar@gmail.com

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Abstract. We modify the mesonic wave function by using a short distance scale $r_0$ in analogy with hydrogen atom and estimate the values of masses and decay constants of the open flavour charm mesons $D$, $D_s$ and $B_c$ within the framework of a QCD potential model. We also calculate leptonic decay widths of these mesons to study branching ratios and lifetime. The results are in good agreement with experimental and other theoretical values.

Keywords. Heavy-light mesons; masses; decay constants; branching ratio.


1. Introduction

Recently, we have reported a regularization procedure to avoid the singularity by introducing a flavour-dependent short distance scale at the origin to study the oscillation frequency of $B$ and $B$ mesons in a QCD potential model [1]. The purpose of this paper is to use the potential model to calculate the masses and decay constants of open flavour charmed mesons $D$, $D_s$ and $B_c$ and then to find the decay width and branching ratio of the same.

If the CKM element is well known from other measurements, then $f_D$ can be measured well. If, on the other hand, the CKM element is not known or poorly measured, having theoretical input on $f_D$ can allow a determination of the CKM element. These decay constants can be accessed both experimentally and through lattice quantum chromodynamics (lQCD) simulations. While for $f_\pi$, $f_K$, $f_D$, experimental measurements agree well with lattice QCD calculations, a discrepancy is seen for the value of $f_{D_s}$: The 2008 PDG average for $f_{D_s}$ is $273 \pm 10$ MeV [2], about $3\sigma$ larger than the most precise $N_f = 2 + 1$ lQCD result from the HPQCD/UKQCD collaboration [3], $241 \pm 3$ MeV. On the other hand, experiments and lQCD calculations agree very well with each other on the value

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The Oscillation Frequency of $B$ and $\bar{B}$ Mesons in a QCD Potential Model with Relativistic Effects

Krishna Kingkar Pathak1**, D. K. Choudhury2

1Department of Physics, Arya Vidyapeeth College, Guwahati-781016, India
2Department of Physics, Gauhati University, Guwahati-781014, India

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Wavefunction at the origin, with the incorporation of a relativistic effect, leads to singularity in a specific potential model. To regularize the wavefunction, we introduced a short distance scale and used it to estimate the mass and decay constants of $B_d$ and $B_s$ mesons within the QCD potential model. These values were then used to compute the oscillation frequency, $\Delta m_{B_d}$, of $B_d$ and $B_s$ mesons. The values were found to be in good agreement with experimental and other theoretical values.


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Investigations of weak decays of mesons composed of a heavy quark and an antiquark give us an important insight into heavy-quark dynamics. Research into the mixing and decay constants of $B$ meson also provides us with useful information about the dynamics of quarks and gluons at the hadronic scale. The weak eigenstates of neutral mesons are different from their mass eigenstates. This leads to the phenomenon of mixing, whereby neutral mesons oscillate between their matter and antimatter states. This was first observed in the Kaon sector, and subsequently in $B_d$ and $B_s$ mesons. The mass difference $\Delta m_B$ is a measure of the frequency of the change from a $B$ into a $\bar{B}$, and is called the oscillation frequency. The decay constants of heavy mesons, one of the input parameters for oscillation frequency, are crucial for interpreting data on particle-antiparticle mixing in the neutral $B$ meson system, and for anticipating and interpreting new signatures for CP violation.

If the CKM element is well known from other measurements, then the pseudoscalar decay constant $f_p$ can be well measured. If, on the other hand, the CKM element is less well or poorly measured, having theoretical input on $f_p$ can allow a determination of the CKM element. A measurement of the decay constant $f_B$ is difficult, since $B^+ \rightarrow l^+\bar{\nu}_l$ is cabibosuppressed in the standard model. Hence $f_{B_d}$ has to be provided from theory.

In this Letter, we calculate the pseudoscalar masses $M_{B_s}$ and pseudoscalar decay constants $f_{B_s}$ to compute the oscillation frequency $\Delta m_{B_s}$, $q = d, s$ within the framework of a potential model. To incorporate the relativistic effect, the necessity of a short distance scale, in analogy to QED, is also pointed out.

For the light heavy flavor bound system of $q\bar{Q}$ or $\bar{q}Q$, the hamiltonian can be written as

$$H = -\frac{\nabla^2}{2\mu} + V(r),$$

where $V(r)$ is the spin-independent quark-antiquark potential.

$$V(r) = V_{\text{coul}}(r) + V_{\text{conf}}(r),$$

where $V_{\text{coul}}$ represents the coulombic part of the potential and $V_{\text{conf}}$ represents the confining potential. The vector and scalar confining potentials in the non-relativistic limit reduce to

$$V_{\text{conf}}^v(r) = (1 - \epsilon_1)(br + C),$$

$$V_{\text{conf}}^s(r) = \epsilon_1 (br + C),$$

reproducing

$$V_{\text{conf}}(r) = V_{\text{conf}}^v(r) + V_{\text{conf}}^s(r) = br + C,$$

$$V_{\text{coul}}(r) = -\alpha_c/r,$$

where $\alpha_c = 3\alpha_s$ with $\alpha_s$ being the strong running coupling constant, $\epsilon_1$ the mixing coefficient, and $b$ and $C$ the potential parameters as used in our previous work.

Considering the linear part of the potential as perturbation, the coulombic part as the parent, and then using the dalgarno method, the wavefunction in the ground state is obtained, i.e.

$$\psi_{\text{rel+conf}}(r) = \frac{N'}{\sqrt{\pi a_0^3}} e^{-r/a_0} \left(C' - \frac{\mu ba_0^2}{2} \right)^{-1/2},$$

$$N' = 2^{1/2} \cdot \left( (2\epsilon(3 - 2\epsilon)C')^2 - \frac{1}{4} \mu ba_0^2 \Gamma(5 - 2\epsilon)C' + \frac{1}{64} \mu^2 b_0^2 \Gamma(7 - 2\epsilon) \right)^{-1/2},$$

$$C' = 1 + c A_0 \sqrt{\pi a_0^3},$$

$$\mu = \frac{m_u m_s}{m_d + m_s},$$

$$a_0 = \left( \frac{4}{3} \mu \epsilon \right)^{-1},$$

$$\epsilon = 1 - \sqrt{1 - \left( \frac{4}{3} \mu \epsilon \right)^2}.$$
LEPTONIC DECAY OF HEAVY–LIGHT MESONS
IN A QCD POTENTIAL MODEL

KRISHNA KINGKAR PATHAK
Department of Physics, Arya Vidyapeeth College, Guwahati 781016, India
kkingkar@gmail.com

D. K. CHOU DHURY
Department of Physics, Gauhati University, Guwahati 781014, India

N. S. BORDOLOI
Department of Physics, Cotton College, Guwahati 781001, India

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We study the masses and decay constants of heavy–light flavor mesons \(D, \ D_s, \ B\) and \(B_s\) in a QCD potential model. The mesonic wave function is used to compute the masses of \(D\) and \(B\) mesons in the ground state and the wave function is transformed to momentum space to estimate the pseudoscalar decay constants of these mesons. The leptonic decay widths and branching ratio of these mesons for different leptonic channels are also computed to compare with the experimental values. The results are found to be compatible with available data.

Keywords: Heavy–light mesons; masses; decay constants; branching ratio.

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1. Introduction

Heavy hadron spectroscopy has played a major role in the foundation of QCD. In the last few years, however, it has sparked a renewal of interest due to the numerous data available from the \(B\) factories, CLEO, LHCb, the Tevatron and by the progress made in the theoretical methods. The remarkable progress at the experimental side for the study of hadrons has opened up new challenges in the theoretical understanding of light–heavy flavor hadrons.

The study of the wave functions of heavy-flavored mesons like \(B\) and \(D\) are important both analytically and numerically for studying the properties of strong interaction between heavy and light quarks as well as for investigating the
On the bounds of CKM matrix element $V_{cb}$ in a potential model

D K. Choudhury

Department of Physics, Gauhati University, Guwahati-781014, India
Physics Academy of North-East, Guwahati-781014, India

Krishna K. Pathak*

Department of Physics, Arya Vidyapeeth College, Guwahati-781016, India
kkingkar@gmail.com

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Considering the allowed range of slope $p^2$ and curvature $C$ of Isgur-Wise function for $B$ meson, we obtain an allowed range of strong coupling constant $\alpha_s$ as well as the QCD scale parameter $\Lambda$ in a specific prescription. The allowed range of $\Lambda$ is then used to obtain the theoretical bounds on $V_{cb}$ in a potential model. The recent available data of $V_{cb}$ are found to lie within this computed range $0.0375 < |V_{cb}| < 0.0410$.

Keywords: Mesons; $I$-$W$ function; CKM elements; branching ratio.

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1. Introduction

In a recent communication,\(^1\) we have reported the results of pseudoscalar decay constants of heavy-light mesons in a potential model with linear part of the potential as perturbation. The technique used was the quantum mechanical perturbation theory with plausible relativistic correction.

Exclusive semileptonic decays of hadrons containing a bottom quark provide a path to measure the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements $V_{cb}$, an important parameter to test the Standard Model. It is well known in the literature that in case of heavy to heavy transitions like $b \rightarrow c$ decays, all heavy quark bilinear current matrix elements are described in terms of only one form factor, which is called the Isgur–Wise (IW) function in leading order. The IW function, particularly its slope ($\xi'(1)$) at the zero recoil point is important since it allows a model

*Corresponding author.
Comments on the perturbation of Cornell potential in a QCD potential model

D K Choudhury¹ and Krishna Kingkar Pathak²

¹Physics Department, Gauhati University, Guwahati 781014, India.
²Physics Department, Arya Vidyapeeth College, Guwahati 781016, India.
E-mail: kkingkar@gmail.com

Abstract. We find in the analysis that the linear part of the Cornell potential can be treated as perturbation for a set of larger values of \( \alpha_s \) in the range 0.4 < \( \alpha_s \) < 0.75 with a constant shift within the range of -0.4 GeV < \( c \) < -1 GeV. Moreover with the same range of constant shift in the Potential, we expect better results with Coulombic part as perturbation for \( \alpha_s \) < 0.4.

1. Introduction

In the potential models, the effective potential between a quark and antiquark can be taken as the Coulomb-plus-linear potential,

\[
V(r) = -\frac{4\alpha_s}{r} + br + c.
\]

(1)

This potential has received a great deal of attention in particle physics, more precisely in the context of meson spectroscopy where it is used to describe systems of quark and antiquark bound states. However, it has been found to be questionable about the numbers of free parameters (\( \alpha_s, b, c \)) and numbers of findings in any potential model. The success of a phenomenological model depends on reducing the free model parameters to obtain more precise values with proper arguments and analysis.

In this letter, we put forward the comments on linear part of the Potential as perturbation with Coulombic part as Parent [1, 2] as well as Coulombic part as perturbation with linear as parent [3] in a potential model and attempt to put some constraints on the model parameters.

2. The method of perturbation

It is well known that one cannot solve the Schrödinger equation in quantum mechanics with the QCD potential (equation (1)) except for some simple models. Perturbation theory has been helpful since the earliest applications of quantum mechanics in this regard. In fact, perturbation theory is probably one of the approximate methods that most appeals to intuition [4].

The advantage of taking Cornell Potential for study is that it leads naturally to two choices of “parent” Hamiltonian, one based on the Coulomb part and the other on the linear term, which can be usefully compared. It is expected that a critical role is played by \( r_0 \) where the Potential \( V(r) = 0 \). Aitchison and Dudek in Reference [5] put an argument that if the size of a state measured by \( \langle r \rangle < r_0 \), then the Coulomb part as the “Parent” will perform better.
ISGUR–WISE FUNCTION IN A QCD POTENTIAL MODEL WITH COULOMBIC POTENTIAL AS PERTURBATION

BHASKAR JYOTI HAZARIKA*, KRISHNA KINGKAR PATHAK† and D. K. CHOUDHURY‡

*Department of Physics, Pandu College, Guwahati-781012, India
†Department of Physics, Arya Vidyapeeth College, Guwahati-781016, India
‡Department of Physics, Gauhati University, Guwahati-781014, India

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We study heavy light mesons in a QCD inspired quark model with the Cornell potential \(-\frac{4g_s}{3r} + br + c\) Here we consider the linear term \(br\) as the parent and \(-\frac{4g_s}{3r} + c\), i.e. the Coulombic part as the perturbation. The linear parent leads to Airy function as the unperturbed wave function. We then use the Dalgarno method of perturbation theory to obtain the total wave function corrected up to first order with Coulombic piece as the perturbation. With these wave functions, we study the Isgur–Wise function and calculate its slope and curvature.

Keywords: Dalgarno method; Isgur–Wise function; slope and curvature.


1. Introduction

Considerable efforts have been made in understanding the physics of hadrons containing at least one heavy quark since long.\(^1\)–\(^9\) It is well known that the heavy quark symmetry in the heavy quark limit leads to a single form factor called the Isgur–Wise (I-W) function which can describe the heavy quark bilinear current matrix elements of weak decay. The basic ingredient of the I-W function is the hadronic wave function, the determination of which becomes such a crucial factor. The potential models for this purpose is quite helpful as they contain more input parameters and hence has its firm basis.

Under such circumstances the I-W function has been investigated\(^3\)–\(^9\) with considerable success of valid degrees in different models. In the potential models, "Cornell potential" is found to be more useful than the others. It leaves two options of choosing the parent (1) the Coulombic part \(-\frac{4g_s}{3r}\) and (2) the linear potential part \(br\).

\(^1\) Corresponding author
SEMILEPTONIC DECAY OF $B_c$ MESON INTO $c\bar{c}$ STATES IN A QCD POTENTIAL MODEL

KRISHNA KINGKAR PATHAK  
Department of Physics, Arya Vidyapeeth College,  
Guwahati-781016, India  
kkmgkar@gmail.com

D K CHOUDHURY  
Department of Physics, Gauhati University,  
Guwahati-781014, India

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The slope and curvature of Isgur-Wise function for $B_c$ meson is computed in a QCD potential model, in two different approaches of choosing the perturbative term of the Cornell potential. Based on heavy quark effective theory the exclusive semileptonic decay rates of $B_c$ meson into the $c\bar{c}(\eta_c, J/\psi)$ states are exploited. Spin symmetry breaking effects are ignored up to a particular point and the form factors are connected with Isgur-Wise function for other kinematic point since the recoil momentum of $c\bar{c}$ from $B_c$ is small due to its heavy mass.

Keywords: Dalgaro method, Isgur-Wise function; form factors; decay width.


1. Introduction

The $B_c$ meson is a particularly interesting hadron, since it is the lowest bound state of two heavy $(b, c)$ quarks with different flavors. Because of the fact that the $B_c$ meson carries the flavor explicitly, there is no gluon or photon annihilation via strong interaction or electromagnetic interaction but decay only via weak interaction. Since both $b$ and $c$ quarks forming the $B_c$ meson are heavy, the $B_c$ meson can decay through the $b \rightarrow q \ (q = c, u)$ transition with $c$ quark being a spectator as well as through the $c \rightarrow q \ (q = s, d)$ transition with $b$ quark being a spectator. The former transitions correspond to the semileptonic decays to $\eta_c$ and $D$ mesons, while the latter transitions correspond to the decays to $B_s$ and $B$ mesons. The CDF Collaboration reported the discovery of the $B_c$ ground state in $p\bar{p}$ collisions.
Semileptonic Decay of $B_c$ Meson into $S$ Wave Charmonium in a QCD Potential Model with Coulombic Part as Perturbation

Krishna Kingkar Pathak¹, Dilip Kumar Choudhury²
¹Department of Physics, Arya Vidyapeeth College, Guwahati, India
²Department of Physics, Gauhati University, Guwahati, India
Email: kkingkar@gmail.com

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ABSTRACT
We present the semileptonic decay of $B_c$ meson in a QCD potential model with the coulombic part of the Cornell potential $\frac{-4\alpha_s}{3r} + br + c$ as perturbation. Computing the slope and curvature of Isgur-Wise function in this approach, we study the pseudoscalar and vector form factors for the transition of $B_c$ meson to its $S$ wave charmonium $cc$ states. Numerical estimates of widths for the transitions of $B_c \rightarrow J/\psi (\eta_c) \ell \nu$ are presented.

Keywords: Dalgarno Method; Isgur-Wise Function; Form Factors; Decay Width

1. Introduction
The investigation of weak decays of mesons composed of a heavy quark and antiquark gives a very important insight in the heavy quark dynamics. The exclusive semileptonic decay processes of heavy mesons generated a great excitement not only in extracting the most accurate values of Cabbibo-Kobayashi Maskawa (CKM) matrix elements but also in testing diverse theoretical approaches to describe the internal structure of hadrons. The great virtue of semileptonic decay processes is that the effects of the strong interaction can be separated from the effects of the weak interaction into a set of Lorentz-invariant form factors, i.e., the essential informations of the strongly interacting quark/gluon structure inside hadrons. Thus, the theoretical problem associated with analyzing semileptonic decay processes is essentially that of calculating the weak form factors.

The decay properties of the $B_c$ meson are of special interest, since it is the only heavy meson consisting of two heavy quarks with different flavor. The decay into $J/\psi \pi$ channel is the most studied semileptonic decay of $B_c$ meson. The investigation of semileptonic decay of $B_c$ meson is of particular interest due to its high branching ratio and its sensitivity to the quarkonium states.

The decay properties of $B_c$ meson were studied experimentally by the CDF Collaboration at Fermilab and the D0 Collaboration at the Tevatron. The measured branching fractions for the decay $B_c \rightarrow J/\psi \ell \nu$ are 

$$\mathcal{B}(B_c \rightarrow J/\psi \ell \nu) = (3.8 \pm 0.6)\times10^{-4}$$

There are many theoretical approaches to the calculation of exclusive $B_c$ semileptonic decay modes. Some of them are: QCD sum rules [2-4], the relativistic quark model [5-7] based on an effective Lagrangian describing the coupling of hadrons to their constituent quarks, the quasipotential approach to the relativistic quark model [8-10], the instantaneos nonrelativistic approach to the Bethe-Salpeter (BS) equation [11], the relativistic quark model based on the BS equation [12,13], the QCD relativistic potential model [14], the relativistic quark-meson model [15], the nonrelativistic quark model [16], the covariant light-front quark model [17], and the constituent quark model [18-21] using BSW (Bauer, Stech, and Wir--)