Chapter 6

\textbf{B}_c \textbf{ meson as a heavy-light meson in the QCD potential model}

6.1 Introduction

The \( B_c \) meson is a particularly interesting hadron, since it is the lowest bound state of two heavy \((b, c)\) quarks with different flavors. Because of the fact that the \( B_c \) meson carries the flavor explicitly, there is no gluon or photon annihilation via strong interaction or electromagnetic interaction but decay only via weak interaction. Since both \( b \) and \( c \) quarks forming the \( B_c \) meson are heavy, the \( B_c \) meson can decay through the \( b \rightarrow q \) \((q = c, u)\) transition with \( c \) quark being a spectator as well as through the \( c \rightarrow q \) \((q = s, d)\) transition with \( b \) quark being a spectator. The former transitions correspond to the semileptonic decays to \( \eta_c \) and D mesons, while the latter transitions correspond to the decays to \( B_s \) and \( B \) mesons.

The CDF Collaboration reported the discovery of the \( B_c \) ground state in \( p\bar{p} \) collisions already more than fourteen years ago [213]. More experimental data on masses and decays of the \( B_c \) meson are expected to come in near future from the Tevatron at Fermilab and the Large Hadron Collider (LHCb) at CERN. The estimates of the \( B_c \) decay rates indicate that the \( c \) quark transitions give the dominant contribution while the \( b \) quark transitions and
weak annihilation contribute less. However, from the experimental point of view the $B_c$ decays to charmonium are easier to identify. Indeed, CDF and D0 observed the $B_c$ meson and measured its mass analyzing its semileptonic decays of $B_c \rightarrow J/\psi l\nu$.

There are many theoretical approaches to study the exclusive semileptonic decay of $B_c$ meson. The paper by Bjorken in 1986, on the decays of long lived $B_c$ meson is considered to be the pioneering work for $B_c$ meson [214]. A lot of efforts was then directed to study this specific meson on the basis of modern understanding of QCD dynamics of heavy flavours in the framework of different approaches. Some of these approaches are: QCD sum rules [215, 216, 217], the relativistic quark model [218, 219, 220], the quasi-potential approach to the relativistic quark model [176, 221, 222], the non-relativistic approach of the Bethe-Salpeter(BS) equation [223], based on the BS equation, the relativistic quark model [224, 225], the QCD relativistic potential model [226], the relativistic quark-meson model [227], the nonrelativistic quark model [228], the covariant light-front quark model [229] and the constituent quark model [230, 231, 232, 149] using BSW(Bauer, Stech, and Wirbel) model [233] and ISGW (Isgur, Scora, Grinstein, and Wise) model [234].

The consequence of heavy quark spin symmetry is that the number of form factors which parametrize the matrix elements is reduced and simplifies the semileptonic transitions. However, spin symmetry does not fix the normalisation of the form factors at any point of the phase space. The normalisation of the form factors near the zero recoil point must be computed by some nonperturbative approach [235]. So far, Jenkins et al., in Ref.[236] estimated the universal form factors of semileptonic decays of $B_c$ meson using non-relativistic meson wavefunctions and in Ref.[237], it is computed by employing the ISGW model at the zero-recoil point. In this chapter, we extend the QCD potential model and check its sensitivity in studying the universal form factor Isgur-Wise function for $B_c$ meson with two different approaches: a) linear part of the Cornell potential as perturbation with Coulombic part as parent and b) Coulombic part as perturbation with linear part as parent.
6.2 Formalism

6.2.1 The wavefunctions in the model

With Cornell potential one obtains the advantage of choosing the Coulombic part as perturbation with linear part as parent as well as linear part as perturbation with Coulombic part as parent. It is expected that a critical role is played by scale the $r_0$, where the potential $V(r) = 0$. Aitchison and Dudek in Ref.[239] put an argument that if the size of a state (meson here) measured by $\langle r \rangle < r_0$, then the Coulomb part as the "parent" will perform better and if $\langle r \rangle > r_0$, the linear part as "parent" will perform better. Aitchison's work also showed that the results with Coulombic part as parent (VIPT) for bottomonium spectra are well explained where as charmonium states are well explained with linear part as parent. Moreover in Ref. [240], we have analysed that the critical distance $r_0$ is not a constant and can be enhanced by reducing $b$ and $c$ or by increasing $\alpha_s$. Thus for a fixed value of $b$ and $c$, $\alpha_s$ plays an important role in choosing the perturbative term. However in this manuscript we allow the same range of $\alpha_s$ obtained from the theoretical bounds of slope and curvature of I-W function and check the applicability of the model wavefunctions in the two approaches for the semileptonic decay of $B_c$ meson into $c\bar{c}$ ($\eta_c, J/\psi$) states.

The wavefunction computed by Dalgarno method, with Coulombic part $-\frac{4\alpha_s}{3r} + c$ of the potential as perturbation and linear part $br$ as parent has been reported in Ref.[241] and the alternate approach of choosing the linear part $br + c$ as perturbation has been discussed in the previous chapters.

The main equations of the wavefunction with Coulombic part as perturbation and linear part as parent are discussed below.
6.2.2 Wavefunction with Coulombic part as perturbation

The wavefunction with linear part as parent becomes an Airy function, which in fact makes the total wavefunction a complicated one since Airy function is a diverging function. Thus, the total wave function corrected up to first order and considering up to order $r^3$ are given by [241, 242]

$$\psi_{\text{coul}}(r) = \psi^{(0)}(r) + \psi^{(1)}(r)$$

$$= \frac{N_1}{2\sqrt{\pi}} \left[ \frac{\text{Ai}((2\mu b)^{\frac{3}{2}} + \rho_{01})}{r} - \frac{4\alpha_s}{3} \left( \frac{a_0}{r} + a_1 + a_2 r \right) \right]$$

where $N_1$ is the normalisation constant for the total wave function $\psi_{\text{coul}}(r)$ where subscript “coul” means Coulombic potential as perturbation and $\rho_{0n}$ are the zeros of the Airy function which is given by [239, 243]:

$$\rho_{0n} = -\left[ \frac{3\pi (4n - 1)}{8} \right]^{\frac{3}{4}}$$

$$a_0 = \frac{0.8808 (b\mu)^{\frac{3}{4}}}{(E - c)} - \frac{\alpha_2}{\mu (E - c)} + \frac{4W^1 \times 0.21005}{3\alpha_s (E - c)}$$

$$a_1 = \frac{ba_0}{(E - c)} + \frac{4 \times W^1 \times 0.8808 \times (b\mu)^{\frac{3}{4}}}{3\alpha_s (E - c)} - \frac{0.6535 \times (b\mu)^{\frac{3}{4}}}{(E - c)}$$

$$a_2 = \frac{4\mu W^1 \times 0.1183}{3\alpha_s}$$

$$W^1 = \int_0^{+\infty} r^2 H' |\psi^{(0)}(r)|^2 dr$$

where $H' = -\frac{4\alpha_s}{3r} + c$ is the perturbed Hamiltonian and

$$E = -\left( \frac{b^2}{2\mu} \right)^{\frac{1}{2}} \rho_{0n}.$$ 

In the Ref. [241], the value of $c = 1 \text{ Gev}$ was taken where as here in this chapter, we choose
6.2. Formalism

c = -0.4 GeV in consistency with the chapters 4 and 5.

6.2.3 The strong coupling constant $\alpha_s$ in the Model

We use the same prescription of $\alpha_s$ as is used in our previous chapters (Eqn.3.18). In chapter 5, we discussed that the physically plausible range of effective $\Lambda_{QCD}$ can be deduced from the allowed range of the slope and curvature of the I-W function and considering the theoretical bounds on slope $3/4 < \rho^2 < 1.51$ [202, 203] and curvature $C > \frac{202}{4}$ [203] of the I-W function, we obtained an allowed range of $\Lambda_{QCD}$ in the model as $382 MeV \leq \Lambda_{QCD} \leq 430 MeV$ for $B$ meson. We extend this theoretical bounds of $B$ meson to $B_c$ meson in its semileptonic decay of charmonium states ($B_c \rightarrow c\bar{c}(\ell^+\nu_\ell)$) and compute the slope and curvature of the Isgur Wise function.

6.2.4 Form factors and Decay rates of $B_c \rightarrow c\bar{c}(\ell^+\nu_\ell)$ transitions

In the semileptonic transitions of $B_c \rightarrow c\bar{c}(\ell^+\nu_\ell)$ states, the hadronic part of the matrix element is contributed by the vector ($V^\mu = \bar{c}\gamma^\mu b$) or axial vector ($A^\mu = \bar{c}\gamma^\mu\gamma_5 b$) current between $B_c$ and $c\bar{c}$ states. For transition between two pseudoscalar mesons ($B_c \rightarrow \eta_c$), axial current $A^\mu$ vanishes and vector current $V^\mu$ only contributes. This hadronic current, $V^\mu$ between the two $J^P = 0^-$ mesons is expressed in terms of two form factors $f_+(q^2)$ as [179]

$$
\langle \eta_c(p')|V^\mu|B_c(p)\rangle = f_+(q^2)(p + p')^\mu + f_-(q^2)(p - p')^\mu
$$

(6.9)

where $q$ is the four momentum transfer which varies within the range $m_T^2 \leq q^2 \leq (m_{B_c} - m_{\eta_c})^2 = q_{\max}^2$ and $f_+(q^2)$ and $f_-(q^2)$ are two weak transition form factors.

For the transition of pseudoscalar to vector mesons ($B_c \rightarrow J/\psi(p', e)$) both the vector
and axial vector current contributes and we get four independent form factors as,

\[ \langle J/\psi(p', e) | \bar{c} \gamma^\mu b | B_c(p) \rangle = 2ie^{\mu \nu \rho \beta} \frac{e_{\nu} \gamma_{\rho} p_{\beta}}{M_{B_c} + M_{J/\psi}} V(q^2) \]  

(6.10)

\[ \langle J/\psi(p', e) | \bar{c} \gamma^\mu \gamma_5 b | B_c(p) \rangle = (M_{B_c} + M_{J/\psi}) \left[ e^\mu - \frac{e \cdot q q^\mu}{q^2} \right] A_1(q^2) - e \cdot q \left[ \frac{(p + p')^\mu}{M_{B_c} + M_{J/\psi}} - \frac{(M_{B_c} - M_{J/\psi}) q^\mu}{q^2} \right] A_2(q^2) + 2M_{J/\psi} \frac{e \cdot q q^\mu}{q^2} A_0(q^2) \]  

(6.11)

In the present study we treat \( B_c \) system as a heavy-light one in analogy to \( D \) system as the ratio of the constituent quark masses in the \( B_c \) meson is very close to that of \( D \) meson and extend HQET for the study of \( B_c \) meson also. On the basis of HQET, the most general form of the transition discussed by Eqns. 6.9 and 6.10 can be expressed as [179, 242],

\[ \frac{1}{\sqrt{M_{B_c} M_{\eta_c}}} \langle \eta_c(v') | V^\mu | B_c(v) \rangle = (v + v')^\mu \xi(\omega) \]  

(6.12)

\[ \frac{1}{\sqrt{M_{B_c} M_{J/\psi}}} \langle J/\psi(v', \epsilon_5) | V^\mu | B_c(v) \rangle = ie^{\mu \nu \rho \beta} \epsilon_{\nu} \gamma_{\rho} v_{\beta} \xi(\omega) \]  

(6.13)

\[ \frac{1}{\sqrt{M_{B_c} M_{J/\psi}}} \langle J/\psi(v', \epsilon_5) | A^\mu | B_c(v) \rangle = [(1 + \omega) e^\mu - (\epsilon \cdot v) v^\mu] \xi(\omega), \]  

(6.14)

where \( v \) and \( v' \) is the four velocity of \( B_c \) meson before and after the transition in the rest frame of the initial meson and \( \xi(\omega) \) is the universal form factor known as Isgur Wise function.

For small, nonzero recoil, Isgur-Wise function can be written by the formula (Eqn. 5.5 of chapter 5):

\[ \xi(v, v') = \xi(Y) = 1 - \rho^2 (Y - 1) + C (Y - 1)^2 + \ldots \]  

(6.15)
where $Y$ is given by,

$$ Y = v \cdot v' = \left[ m_{B_c}^2 + m_{c\bar{c}}^2 - q^2 \right] \frac{2m_{B_c}m_{c\bar{c}}}{2m_{B_c}m_{c\bar{c}}} . \quad (6.16) $$

For heavy-light mesons, I-W function can also be expressed by another formula [244, 245]:

$$ \xi (Y) = \int_0^{+\infty} 4\pi r^2 |\psi (r)|^2 \cos prdr \quad (6.17) $$

where

$$ p^2 = 2\mu^2 (Y - 1) . \quad (6.18) $$

In Eqn.6.17, we employ the two wavefunctions (Eqn.2.16 and Eqn.6.1) to compute the slope and curvature of the Isgur-Wise function and present the results in Table.6.1. The input parameters used in the numerical calculations are the same as is used in our previous chapters. For the masses of $B_c$, $\eta_c$ and $J/\psi$, we use the experimental masses from PDG2012 [2].

Table 6.1: The slope $\rho^2$ and curvature $C$ of the I-W function with linear part as perturbation and Coulombic part as perturbation.

<table>
<thead>
<tr>
<th>$\Lambda$</th>
<th>$\rho^2$</th>
<th>$C$</th>
<th>$\rho^2$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>382 MeV</td>
<td>9.59</td>
<td>117.783</td>
<td>3.78</td>
<td>0.057</td>
</tr>
<tr>
<td>430 MeV</td>
<td>5.45</td>
<td>31.39</td>
<td>3.83</td>
<td>0.051</td>
</tr>
</tbody>
</table>

In Ref.[235], the slope and curvature of the universal form factor for $B_c$ meson is computed in the framework of QCD relativistic potential model and is shown in Table.6.2.
Table 6.2: Parameters of the form factors for the channel of $B_c \rightarrow \eta_c(J/\psi)$ with $\Lambda = 397\,MeV$ (from ref.[235]).

<table>
<thead>
<tr>
<th>Channel</th>
<th>$F(1)$</th>
<th>$\rho^2$</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c \rightarrow \eta_c(J/\psi)$</td>
<td>0.94</td>
<td>2.9</td>
<td>3</td>
</tr>
</tbody>
</table>

The result of Table.6.1 is found to be closer to that of Ref.[235] in one of the approach of our model with Coulombic part as perturbation. Interestingly, the scale $\Lambda = 397\,MeV$ (used in Ref.[235]) lies within our range of $382\,MeV \leq \Lambda_{QCD} \leq 430\,MeV$. In Fig.6.1 and Fig.6.2, we show the variation of Isgur-Wise function with its four velocity transfer ($Y=v.v'$) in the two different approaches.

Figure 6.1: Variation of I-W function with $Y$ for different scales of $\Lambda$ with linear part as perturbation.
6.2. Formalism

Applying HQET, the most general form of the transition discussed by Eqns.6.9 and 6.10 can be expressed in terms of Isgur Wise function as [179]

\[ f_4(q^2) = \xi(Y) \frac{m_{B_c} \pm m_c}{2 \sqrt{m_{B_c} m_c}} \tag{6.19} \]

and

\[ V(q^2) = A_2(q^2) = A_0(q^2) = \left[ 1 - \frac{q^2}{(M_{B_c} + M_{J/\psi})^2} \right]^{-1} A_1(q^2) = \frac{(M_{B_c} + M_{J/\psi})^2}{4M_{B_c}M_{J/\psi}} \xi(Y) \tag{6.20} \]

Here, we have applied the HQET to relate the form factors of the semileptonic transitions of \( B_c \to c\bar{c} \) states with the Isgur-Wise function in Eqn.6.19 and Eqn.6.20. These equations are based on the heavy flavour symmetry and is broken in the case of mesons containing two heavy quarks[236]. Spin symmetry breaking effects can occur when the c-quarks recoil momentum is larger than \( m_c \). However, we expect that the equations are applicable to other
kinematic point since the recoil momentum of $c\bar{c}$ state is small ($y_{\text{max}} - 1 = 0.26$) due to its heavy mass [246]. In Ref.[235], Pietro Colangelo and Fulvia De Fazio showed that the normalization of the form factor $\Delta$ describing the transition $B_c \rightarrow J/\psi \ell^+ \nu_\ell$ is close to 1 ($\simeq 0.94$) at the zero-recoil point, as being the overlap of wave-functions, although it is not constrained by symmetry arguments.

The differential semileptonic decay rates can be expressed in terms of these form factors by

(a) $B_c \rightarrow Pev$ decay ($P = \eta_c$)

$$\frac{d\Gamma}{dq^2}(B_c \rightarrow Pev) = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} |f_+(q^2)|^2.$$  \hspace{1cm} (6.21)

(b) $B_c \rightarrow Vev$ decay ($V = J/\psi$) The decay rate in transversely(T) and longitudinally(L) polarized vector mesons are defined by [247]

$$\frac{d\Gamma_T}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{96\pi^3} \frac{q^2}{M_B^2} |H_0(q^2)|^2,$$

$$\frac{d\Gamma_L}{dq^2} = \frac{d\Gamma_+}{dq^2} + \frac{d\Gamma_-}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{96\pi^3} \frac{q^2}{M_B^2} (|H_+(q^2)|^2 + |H_-(q^2)|^2).$$  \hspace{1cm} (6.23)

where helicity amplitudes are given by the following expressions

$$H_+(q^2) = \frac{2M_{B_c} \Delta}{M_{B_c} + M_V} \left[ V(q^2) \mp \frac{(M_{B_c} + M_V)^2}{2M_{B_c} \Delta} A_1(q^2) \right].$$  \hspace{1cm} (6.24)

$$H_0(q^2) = \frac{1}{2M_V \sqrt{q^2}} \left[ (M_{B_c} + M_V)(M_{B_c}^2 - M_V^2 - q^2)A_1(q^2) - \frac{4M_{B_c}^2 \Delta^2}{M_{B_c} + M_V} A_2(q^2) \right].$$  \hspace{1cm} (6.25)

Thus the total semileptonic decay rate is given by

$$\frac{d\Gamma}{dq^2}(B_c \rightarrow Vev) = \frac{G_F^2 |V_{cb}|^2}{96\pi^3} \frac{q^2}{M_{B_c}^2} \left( |H_+(q^2)|^2 + |H_-(q^2)|^2 + |H_0(q^2)|^2 \right).$$  \hspace{1cm} (6.26)
where $G_F$ is the Fermi constant, $V_{cb}$ is CKM matrix element,

\[
\Delta \equiv |\Delta| = \sqrt{\frac{(M_{B_c}^2 + M_{PV}^2 - q^2)^2}{4M_{B_c}^2} - M_{PV}^2}.
\]

Integrating over $q^2$ of these formulas (Eqn.6.21 and Eqn.6.26), we compute the total decay rate of the corresponding semileptonic decay and present the results in Table.6.4. In Fig.6.3, Fig.6.5 and Fig.6.6, we plot the differential semileptonic decay rates $dT/dq^2$ for semileptonic decays $B_c \rightarrow \eta_c ev$ and $B_c \rightarrow J/\psi ev$ within the two approaches of our model.

Figure 6.3: Differential decay rates $(1/V_{cb})dT/dq^2$ of $B_c \rightarrow \eta_c ev$ (in GeV$^{-1}$) with linear part as perturbation. The red and blue curves correspond to $\Lambda = 382$ MeV and 430 MeV respectively.
Figure 6.4: Differential decay rates \( \frac{1}{|V_{cb}|^2} \frac{d\Gamma}{dq^2} \) of \( B_c \rightarrow \eta_c e^- \nu_e (\text{in GeV}^{-1}) \) with coulombic part as perturbation. The red and blue curves correspond to \( \Lambda = 382 \text{ MeV} \) and \( 430 \text{ MeV} \) respectively.
Figure 6.5: Differential decay rates \( \frac{1}{|V_{cb}|^2} \frac{d\Gamma}{dq^2} \) of \( B_c \rightarrow J/\psi ev(\text{in GeV}^{-1}) \) with linear part as perturbation. The red and blue curves correspond to \( \Lambda = 382 \text{ MeV} \) and \( 430 \text{ MeV} \) respectively.
Figure 6.6: Differential decay rates \( (1/|V_{cb}|^2)d\Gamma/dq^2 \) of \( B_c \rightarrow J/\psi\nu\bar{\nu} \) (in \( GeV^{-1} \)) with Coulombic part as perturbation. The red and blue curves correspond to \( \Lambda = 382 \ MeV \) and \( 430 \ MeV \) respectively.

Table 6.3: Decay width for \( B_c \rightarrow \bar{c}\bar{c}(\ell^+\nu_\ell) \) decay. In the braces "linear" means the result with linear part as perturbation and "coul" means Coulombic part as perturbation.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Decay width(( \Gamma )) ( \times 10^{-15} \ GeV )</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Lambda = 382 \ MeV ) ( \Lambda = 430 \ MeV )</td>
<td></td>
</tr>
<tr>
<td>( B_c \rightarrow \eta_c(\ell^+\nu_\ell) )</td>
<td>415 (linear) 1.8 (coul)</td>
<td>32 (linear) 1.7 (coul)</td>
</tr>
<tr>
<td>( B_c \rightarrow J/\psi(\ell^+\nu_\ell) )</td>
<td>424 (linear) 15 (coul)</td>
<td>51 (linear) 14 (coul)</td>
</tr>
</tbody>
</table>
Table 6.4: Branching ratio for $B_c \rightarrow cc(\ell^+\nu_\ell)$ decay. In the braces "linear" means the result with linear part as perturbation and "coul" means Coulombic part as perturbation.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Branching ratio $(BR) \times 10^{-2}$ $\Lambda = 382MeV$</th>
<th>$\Lambda = 430MeV$</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c \rightarrow \eta_c(\ell^+\nu_\ell)$</td>
<td>28 (linear) 0.12 (coul)</td>
<td>2.3 (linear) 0.11 (coul)</td>
<td>0.81 [220] 0.42 [221] 0.76 [225] 0.15 [226] 0.51 [227]</td>
</tr>
<tr>
<td>$B_c \rightarrow J/\psi(\ell^+\nu_\ell)$</td>
<td>29 (linear) 1.0 (coul)</td>
<td>3.5 (linear) 0.98 (coul)</td>
<td>2.07 [220] 1.23 [221] 2.01 [225] 1.47 [226] 1.44 [227]</td>
</tr>
</tbody>
</table>

The computed decay rates and branching ratios for the semileptonic decay of $B_c \rightarrow cc(\ell^+\nu_\ell)$ shows that the results overshoot in case of linear part as perturbation and falls short with Coulombic part as perturbation. With Coulombic part as perturbation, the decay rate and branching ratio for $B_c \rightarrow J/\psi(\ell^+\nu_\ell)$ semileptonic decay give comparable results with that of Ref.[221] for both $\Lambda = 382MeV$ and $\Lambda = 430MeV$. However, with $\Lambda = 382MeV$ the numerical result is more comparable to that of Ref.[221] and we consider this small difference of decay rate for $\Lambda = 382MeV$ and $\Lambda = 430MeV$ significantly. This is because the smaller value of the QCD scale $\Lambda$ in Eqn.3.18 provides a smaller value in $\alpha_s$ and hence weakens the Coulombic part of the potential to treat the latter as perturbation. Thus the results with $\Lambda = 382MeV$ for Coulombic part as perturbation is considered to be more comparable. This fact is even more clear when we check the status of perturbation in Table.6.5, where we show the dominance of parent term over the perturbation by comparing the numerical values for I-W function for the total wave function and parent term only. The result shows that, with linear part as perturbation the condition of $\xi_{total}(Y) > \xi_{parent}(Y)$ is sustained for a narrow range of $Y$ $(1 \leq Y \leq 1.06)$ where as with Coulombic part as perturbation the range of $Y$ is quite large $Y$ $(1 \leq Y \leq 1.22)$.
Table 6.5: Variation of I-W function with total wavefunction and parent wavefunction only for $\Lambda = 382 \, MeV$.

<table>
<thead>
<tr>
<th>$Y$</th>
<th>Linear part as perturbation</th>
<th>Coulombic part as perturbation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_{\text{total}}(Y)$</td>
<td>$\xi_{\text{parent}}(Y)$</td>
<td>$\xi_{\text{total}}(Y)$</td>
</tr>
<tr>
<td>1.01</td>
<td>0.916</td>
<td>0.509</td>
</tr>
<tr>
<td>1.06</td>
<td>0.848</td>
<td>0.772</td>
</tr>
<tr>
<td>1.08</td>
<td>0.9865</td>
<td>2.145</td>
</tr>
<tr>
<td>1.20</td>
<td>3.79</td>
<td>25.06</td>
</tr>
<tr>
<td>1.24</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

6.3 Results and Discussion

In this chapter, we have computed the slope and curvature of Isgur Wise function for $B_c$ meson, considering the Coulombic part of the Cornell potential as perturbation in one approach and linear part as perturbation in the other. The numerical calculations are done for two different scales of QCD $\Lambda = 382 \, MeV$ and $\Lambda = 430 \, MeV$. The values of slope and curvature of the I-W function seems to be acceptable with Coulombic part as perturbation where as with linear part as perturbation, the result overshoot the possible values. The former is also closer to the result of Ref.[235] obtained in a QCD relativistic potential model [250, 251]. For a slight lower value of $\Lambda_{QCD} < 382 \, MeV (\Lambda \approx 280 \, MeV)$, one can obtain the values of slope $\rho^2$ at par with the Ref.[235]($\rho^2 = 2.9$), which lies outside the range.

Moreover, it conforms to the expectation of perturbation since with higher value of $\Lambda$, the coupling constant $\alpha_s$ increases making the Coulombic term of the potential more dominant to treat the linear part as perturbation. Similarly, with lower value of $\Lambda$ (hence lower value of $\alpha_s$), the Coulombic part of the potential becomes less dominant so as to be considered as perturbation.

To conclude, within the prescription of the strong coupling constant of Ref.[201, 137], the predicted behaviour of $B_c$ meson is closer to typical heavy-light mesons like $B$ and $D$. 
6.3. Results and Discussion

if the linear part of the potential is more dominant than the Coulombic part. It will be interesting to see if this feature is scheme invariant.

In a sense, the present work is complimentary to the work on leptonic decays (chapter 4), where $\Lambda_{QCD} = 200 \text{ MeV}$ was chosen with linear part as perturbation within the same prescription of running coupling constant [201, 137]. The apparent change of $\Lambda_{QCD}$ (and hence equivalent strong coupling constant) in the present case is attributed to the decrease of available momentum transfer in semileptonic decays compared to leptonic ones.