CHAPTER VI

MHD COUETTE FLOW WITH HEAT TRANSFER IN PRESENCE OF
CONSTANT HEAT SOURCE

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6.1 INTRODUCTION

MHD is the science of motion of an electrically conducting fluid in presence of magnetic field. In MHD, an induced magnetic field appears perturbing the original magnetic field and also perturbing the original motion an induced electric field appears. These two are the basic effects of MHD. The dynamo and motor is a classical example of MHD principle. Engineers apply MHD principle in fusion reactors, dispersion of metals, metallurgy, design of MHD pumps, MHD generator and MHD flow meter etc. Geophysics encounters MHD characteristics in the interaction of conducting fluid and magnetic field. MHD convection problems are also very significant in the fields of stellar and planetary magnetospheres, aeronautics and electrical engineering. The principle of MHD also finds its application in medicine and Biology. The MHD principle is also utilized in stabilizing a flow against the transition from laminar to turbulent flow.

MHD in its present form is due to the pioneer contribution of several notable authors like Alfven (1942), Cowling (1957), Shercliff (1965), Ferraro and Plumpton (1966) and Crammer and Pai (1978). The phenomenon of heat transfer plays a significant role in many industrial and environmental problems. Industrial problems include those of production and conversion of energy and electrical power generation, minimization of the rates of heat transfer for maintaining the integrity of materials in high temperature environments, working and design of propulsion systems and cryogenics. Environmental problems include those of local and global climatology. Studies on thermal effects in buildings and other structures require applications of heat transfer principles. The study of heat transfer related problems concerning the flows of electrically conducting fluids through channels becomes very interesting and fruitful from physical point of view when a magnetic field is applied to the flow.
The first experimental and theoretical work on MHD channel flow was undertaken by Hartmann (1937a) and Hartmann and Lazarus (1937b). Following these works, several other investigators have carried out model studies on MHD channel flow. Some of them are Chang and Yen (1961), Sutton and Sherman (1965), Soundalgekar and Bhatt (1984), Lahjomri et al. (2002), Badosa and Borkakati (2003), Makinde and Mhone (2005), Ganesh and Krishnambal (2006), Chaudhary et al. (2006) and Anwar et al. (2009). Ahmed and Kalita (2010) have studied the problem of two dimensional steady MHD forced convective Poiseuille flow and heat transfer between two porous plates with constant pressure gradient and a heat source, taking into account the induced magnetic field. Recently Ahmed (2012) have investigated the effect of thermal diffusion and constant heat source in a Hartmann flow.

In this present work the main objective is to investigate the effect of the applied magnetic field on the velocity field, temperature field, skin friction and Nusselt number at the plates, induced magnetic field, current density and the induced electric field. It is also proposed to study the effects of dissipative heat and Prandtl number on the heat transport characteristics.

6.2 MATHEMATICAL FORMULATION

The equations governing the steady motion of an incompressible viscous electrically conducting fluid in presence of a magnetic field and a constant heat source are:

Continuity equation: \( \nabla \cdot \vec{q} = 0 \) (6.2.1)

Gauss's law of magnetism: \( \nabla \cdot \vec{H} = 0 \) (6.2.2)

Magnetic induction equation: \( \nabla \times (\vec{q} \times \vec{H}) + \eta \nabla^2 \vec{H} = 0 \) (6.2.3)
where, $\eta = \frac{1}{4\pi\mu_0\sigma}$

Ampere’s law: $\vec{V} \times \vec{H} = 4\pi \vec{J}$ \hspace{1cm} (6.2.4)

Ohm’s law: $\vec{J} = \sigma \left[ \vec{E} + \vec{q} \times \vec{B} \right]$ \hspace{1cm} (6.2.5)

Momentum equation: $\rho \left( \vec{q} \vec{V} \right) \vec{q} = -\vec{V}p + \vec{J} \times \vec{B} + \mu \nabla^2 \vec{q} + \rho \vec{g}$ \hspace{1cm} (6.2.6)

Energy equation: $\rho C_p \left( \vec{q} \vec{V} \right) \vec{q} = k \nabla^2 T + \frac{\vec{V}^2}{\sigma} + \phi + Q$ \hspace{1cm} (6.2.7)

We now consider a steady laminar flow of an incompressible viscous electrically conducting fluid through a channel bounded by two infinite horizontal isothermal parallel plates separated by a distance $L$ in presence of applied magnetic field $\vec{H}_0$ and a constant heat source. The plate at the distance $L$ is moving with uniform velocity.

We introduce a coordinate system $(\vec{x}, \vec{y}, \vec{z})$ with $X$-axis along the direction of the flow, $Y$-axis normal to the plates and $Z$-axis along the width of the plates. Let $\vec{q} = (\vec{u}, 0, 0)$ be the fluid velocity and $\vec{H} = (\vec{H}_1, \vec{H}_0, 0)$ be the magnetic field at the point $(\vec{x}, \vec{y}, \vec{z})$ in the fluid as shown in the figure 6.1.
which shows that \( u \) is independent of \( x \). In the present problem \( u \) does not depend on \( z \) and hence \( u = u(y) \) (6.2.9)

The Gauss’s law of magnetism yields

\[
\left(\frac{\partial}{\partial x}\right) \frac{V}{L} = 0, \quad \text{which shows that} \quad H_j = H_j(y) \quad (6.2.10)
\]

We assume \( p = p_0(x) + p_1(y) \) (6.2.11)

Under the assumption (6.2.11), the equation (6.2.6) can be broken down to the following equations:

\[
\frac{\mu_0 H_0}{4\pi} \frac{dH_1}{dy} + \frac{\mu}{d^2u}{d^2y}^2 = \frac{dp_0}{dx} - \bar{P} = \text{a constant} \quad (6.2.12)
\]

and

\[
\frac{d\bar{p}_1}{dy} + \rho g + \frac{\mu_0}{4\pi} H_1 \frac{dH_1}{dy} = 0 \quad (6.2.13)
\]
The equation (6.2.3) reduces to
\[
\frac{d^2 H}{dy^2} + 4\pi\sigma \mu_0 H_0 \frac{d\bar{u}}{dy} = 0
\] (6.2.14)

The energy equation (6.2.7) yields
\[
\frac{k}{\rho C_p} \frac{d^2 T}{dy^2} + \frac{\mu}{\rho C_p} \left( \frac{d\bar{u}}{dy} \right)^2 + \frac{\sigma}{\rho C_p} \left( E + \mu_0 \bar{u} H_0 \right)^2 + \frac{Q}{\rho C_p} = 0
\] (6.2.15)

The equations (6.2.4) and (6.2.5) give
\[
\bar{J} = \sigma [E + \mu_0 \bar{u} H_0] = -\frac{1}{4\pi} \frac{d\bar{H}_1}{dy}
\] (6.2.16)

where, \( \bar{J} = |J| \) and \( \bar{E} = |E| \)

We recall that there is no externally applied current, for which
\[
\int_0^L J \, dy = 0
\] (6.2.17)

The relevant boundary conditions are:
\[
\bar{y} = 0: \quad \bar{u} = 0, \; \bar{T} = \bar{T}_0, \; \bar{H}_1 = 0, \; \bar{p}_1 = 0
\] (6.2.18)

\[
\bar{y} = L: \quad \bar{u} = \bar{U}, \; \bar{T} = \bar{T}_1, \; \bar{H}_1 = \bar{I}
\] (6.2.19)

We introduce the following non-dimensional quantities:
\[
M = \mu_0 H_0 L \sqrt{\frac{\sigma}{\mu}}, \quad y = \frac{\bar{y}}{L}, \quad x = \frac{\bar{x}}{L}, \quad z = \frac{\bar{z}}{L}, \quad u = \frac{4\pi \sqrt{\mu \sigma} \bar{u}}{H_0}, \quad h = \frac{\bar{H}_1}{H_0}, \quad E = \frac{4\pi \bar{E} L \sigma}{H_0}, \quad Pr = \frac{\mu C_p}{k},
\]
\[
P = \frac{4\pi L^2 P \sqrt{\sigma}}{H_0^2 \sqrt{\mu}}, \quad g = \frac{4\pi L \rho g}{\mu_0 H_0^2}, \quad J = \frac{4\pi L \bar{J}}{H_0}, \quad p_0 = \frac{4\pi L \sqrt{\sigma} \bar{p}_0}{H_0 \sqrt{\mu}}, \quad p_1 = \frac{4\pi \bar{p}_1}{\mu_0 H_0^2}, \quad U = \frac{4\pi \sqrt{\mu \sigma} \bar{U}}{H_0},
\]
\[
I = \frac{\bar{I}}{H_0}, \quad \theta = \frac{\bar{T} - \bar{T}_0}{\bar{T}_1 - \bar{T}_0}, \quad Ec = \frac{H_0^2}{16\pi^2 \mu \sigma C_\rho (\bar{T}_1 - \bar{T}_0)} \quad \alpha = \frac{QL^2}{(\bar{T}_1 - \bar{T}_0) k}
\] (6.2.20)
The non-dimensional forms of the equations (6.2.13), (6.2.14), (6.2.15), (6.2.16),
(6.2.17), (6.2.18) and (6.2.19) are as follows:

\[
\frac{d^2u}{dy^2} + M \frac{dh}{dy} = -P \tag{6.2.21}
\]

\[
\frac{dp}{dy} + g + h \frac{dh}{dy} = 0 \tag{6.2.22}
\]

\[
\frac{d^2h}{dy^2} + M \frac{du}{dy} = 0 \tag{6.2.23}
\]

\[
\frac{d^2\theta}{dy^2} + Pr Ed \left( \frac{du}{dy} \right)^2 + Pr Ec (E + Mu)^2 + \alpha = 0 \tag{6.2.24}
\]

\[
J = E + Mu = -\frac{dh}{dy} \tag{6.2.25}
\]

\[
\int_0^1 J dy = 0 \tag{6.2.26}
\]

The relevant boundary conditions are

\[
y = 0: u = 0, \theta = 0, h = 0 \tag{6.2.27}
\]

\[
y = 1: u = U, \theta = 1, h = 1 \tag{6.2.28}
\]

\[
y = 0: \quad p_1 = 0 \tag{6.2.29}
\]

The solution of the equation (6.2.22) subject to the boundary condition (6.2.29) is

\[
p_1 + g y + \frac{h^2}{7} = 0 \tag{6.2.30}
\]

The equation (6.2.23) yields,

\[
\frac{dh}{dy} + Mu = C_2, C_2 \text{ being a constant} \tag{6.2.31}
\]

The equation (6.2.26) gives,

\[
E = -\left( \frac{dh}{dy} + Mu \right) = -C_2 \tag{6.2.32}
\]
The equation (6.2.31) and (6.2.32) yield,
\[
\frac{dh}{dy} + Mu = -E \quad (6.2.33)
\]
The equation (6.2.21) and (6.2.33) together give
\[
\frac{d^2u}{dy^2} - M^2u = ME - P \quad (6.2.34)
\]
The solution of the equation (6.2.34) subject to the boundary conditions (6.2.27) and (6.2.28) is
\[
u = A_1 \sinh(My) + A_5 e^{My} - A_6 e^{-My} - A_2 \quad (6.2.35)
\]
The equation (6.2.25) and (6.2.26) give
\[
E = A_1(1 - \cosh M) + A_5(1 - e^M) + A_6(1 - e^{-M}) + MA_2 \quad (6.2.36)
\]
Using the equation (6.2.35) and subject to the boundary conditions, on solving the equation (6.2.23) we have,
\[
h = A_1(l - y + ye^M - e^{My}) + A_5(l - y + ye^M - e^{My}) + A_6(l - y + ye^M - e^{My}) + ly \quad (6.2.37)
\]
The equation (6.2.25) gives
\[
J = A_1(l - \cosh M + M \sinh My) + A_5(1 - e^M + Me^My) + A_6(1 - e^{-M} - Me^{-My}) + I \quad (6.2.38)
\]
The solutions of the equation (6.2.24) subject to the boundary conditions (6.2.27) and (6.2.28) are
\[ \theta = A_{29} \{1 - \cosh 2My - (1 - \cos 2M)y\} + \frac{A_{31}}{2} (y - y^2) + A_{30} \{1 - e^{-2My} - (1 - e^{-2M})y\} + \]
\[ A_{31} \{1 - e^{-2My} - (1 - e^{-2M})y\} + A_{32} \{1 - e^{-2My} - (1 - e^{-2M})y\} + A_{33} \{1 - e^{My} - (1 - e^{M})y\} + \]
\[ A_{34} \{1 - e^{2My} - (1 - e^{2M})y\} + A_{35} \{1 - e^{-2My} - (1 - e^{-2M})y\} + A_{36} \{1 - \cosh 2My - (1 - \cosh 2M)y\} - \]
\[ A_{37} (y \sinh M - \sinh My) + y \]

### 6.3 CO-EFFICIENT OF SKIN FRICTION

The shear stress at any point in the fluid is given by

\[ \bar{\tau} = \mu \frac{du}{dy} = \frac{H_0}{4\pi L} \sqrt{\frac{\mu}{\sigma}} \frac{du}{dy} = \tau_0 \frac{du}{dy}, \quad \tau_0 = \frac{H_0}{4\pi L} \sqrt{\frac{\mu}{\sigma}} \quad (6.3.1) \]

The co-efficient of skin friction at any point is given by

\[ \tau = \frac{\bar{\tau}}{\tau_0} = \frac{du}{dy} \quad (6.3.2) \]

The skin-friction in the non-dimensional form at the plates \( y = 0 \) and \( y = 1 \) are:

\[ \tau_1 = \frac{du}{dy}_{y=0} = A_1 M + A_3 M + A_4 M \quad (6.3.3) \]

\[ \tau_2 = \frac{du}{dy}_{y=1} = A_1 M \cosh M + A_3 M e^M + A_4 M e^{-M} \quad (6.3.4) \]

### 6.4 RATE OF HEAT TRANSFER

The co-efficient of the rate of heat transfer at any point in fluid in terms of Nusselt number \( Nu \) is given by

\[ Nu = \frac{L}{T_w} \frac{d\bar{T}}{dy} = - \frac{d\theta}{dy} \quad (6.4.1) \]

The Nusselt number in the non-dimensional form at the plates \( y = 0 \) and \( y = 1 \) are:
6.5 RESULTS AND DISCUSSION

In order to get physical insight into the problem, we have carried out numerical calculations for non dimensional velocity field, temperature field, induced magnetic field, current density, skin friction and the rate of heat transfer in terms of Nusselt number by assigning specific values to the different parameters involved in the problem. The effects of these values are demonstrated through different graphs and the results are physically interpreted.

Figures 6.2 and 6.3 exhibit the variation of the velocity field against y under the influence of Hartmann number and pressure gradient. These two figures indicate that the fluid velocity increases in the negative direction of x-axis in a thin layer adjacent to the stationary plate and remains almost stationary before it sharply increases up to its maximum value \( U=1 \) at the moving plate. Moreover it is observed that the applied magnetic field as well as the pressure gradient leads to a fall in the magnitude of the velocity of the fluid indicating the fact that the flow field is retarded due to the imposition of the transverse magnetic field and pressure gradient.
It is observed from figure 6.4 that due to increase in the strength of the applied magnetic field, the induced magnetic field \( h \) increases steady and slowly and this phenomenon is in good agreement with the fact that the induced magnetic field appears due to application of the magnetic field.

Figure 6.5 depict the change of behavior of the current density \( J \) against \( y \) under Hartmann number \( M \). As expected it is seen from this figure that the current density rises due to increase in the strength of the applied magnetic field. Further the effect of applied magnetic field on density of the current is more in the fluid adjacent to the moving plate.

Figures 6.6-6.9 respectively exhibit the variation of the temperature field \( \theta \) versus \( y \) under the effect of the Hartmann number \( M \), the Prandtl number \( Pr \), Eckert number \( Ec \) and the pressure gradient \( P \). These figures show that the fluid temperature falls down due to imposition of the transverse magnetic field, but it rises up due to increase in the values of Prandtl number, dissipative heat (Eckert number) and the pressure gradient. Further it is noticed that the effect of the pressure gradient on the temperature field is not so pronounced.

The skin friction at either plate under the influence of transverse magnetic field and the pressure gradient is demonstrated in figures 6.10 and 6.11. It is clear from these figures that the magnitude of the drag per unit area at both the plates rises due to the imposition of the transverse magnetic field. As the effect of pressure gradient on the skin friction is concerned, it is seen that the viscous drag at the stationary plate is reduced due to it, but there is no significant effect of pressure gradient on this magnitude on the moving plate.

Figures 6.12-6.15 indicate that the application of the transverse magnetic field results in a steady decrease in the magnitude of the Nusselt number at the stationary
plate whereas reverse behavior of the Nusselt number at the other plate under the magnetic field is noticed. The figures 6.13 and 6.15 reveal that an increase in the Prandtl number causes the coefficient of heat transfer from either plate to the fluid to increase.

6.6 CONCLUSIONS

Our investigation leads to the following conclusions:

i) The fluid motion is retarded due to imposition of the transverse magnetic field and is accelerated under the pressure gradient.

ii) The fluid temperature falls due to imposition of the transverse magnetic field, but it rises due to dissipative heat and pressure gradient.

iii) The magnitude of viscous drag at either plate rises when the strength of the magnetic field is increased.

iv) The application of the transverse magnetic field results in a fall in the magnitude of the rate of heat transfer from the stationary plate to the fluid, whereas there is a rise in the rate of heat transfer from the moving plate to the fluid due to increase in the strength of the applied magnetic field.
Figure 6.2: Velocity field $u$ versus $y$ for $P=1$, $Ec=0.4$, $Pr=7$ and $\alpha=0.8$

Figure 6.3: Velocity field $u$ versus $y$ for $M=10$, $Ec=0.4$, $Pr=7$ and $\alpha=0.8$
Figure 6.4: Induced Magnetic field $h$ versus $y$ for $P = 1$, $Ec = 0.4$, $Pr = 7$ and $\alpha = 0.8$

Figure 6.5: Current Density $J$, versus $y$ for $P = 1$, $Ec = 0.4$, $Pr = 7$ and $\alpha = 0.8$
Figure 6.6: Temperature $\theta$ versus $y$ for $P = 1$, $Pr= 7$, $Ec= 0.4$ and $\alpha = 0.8$

Figure 6.7: Temperature $\theta$ versus $y$ for $P = 1$, $M = 10$, $Ec= 0.4$ and $\alpha = 0.8$
Figure 6.8: Temperature $\theta$ versus $y$ for $P = 1, M = 10, Pr = 7$ and $\alpha = 0.8$

Figure 6.9: Temperature $\theta$ versus $y$ for $M = 10, Pr = 7, Ec = 0.4$ and $\alpha = 0.8$
Figure 6.10: Skin friction $\tau_1$ versus $P$ for $Ec = 0.4$, $Pr = 7$ and $\alpha = 0.8$

Figure 6.11: Skin friction $\tau_2$ versus $P$ for $Ec = 0.4$, $Pr = 7$ and $\alpha = 0.8$
Figure 6.12: Nusselt number $\text{Nu}_1$ versus $P$ for $Ec = 0.4$, $Pr = 7$ and $\alpha = 0.8$

Figure 6.13: Nusselt number $\text{Nu}_1$ versus $P$ for $M = 10$, $Ec = 0.4$ and $\alpha = 0.8$
Figure 6.14: Nusselt number $\text{Nu}_2$ versus $P$ for $Pr = 7$, $Ec = 0.4$ and $\alpha = 0.8$

Figure 6.15: Nusselt number $\text{Nu}_2$ versus $P$ for $M=10$, $Ec = 0.4$ and $\alpha = 0.8$
6.7 NOMENCLATURE

B is the magnetic induction vector

Cp is the specific heat at constant pressure

\( \vec{E} \) is the electric field vector

E is the dimensionless electric field

\( \overline{E} \) is the dimensional electric field

Ec is the Eckert number

\( \vec{g} \) is the acceleration due to gravity

g is the dimensionless acceleration due to gravity

\( H_0 \) is the intensity of the applied magnetic field

\( \vec{H} \) is the magnetic field intensity vector

\( \vec{H}_i \) is the induced magnetic field vector

\( \overline{H}_i \) is the dimensional induced magnetic field

h is the dimensionless induced magnetic field

\( \hat{i}, \hat{j}, \hat{k} \) are unit vectors along the co-ordinate axes

\( \vec{J} \) is the current density vector

J is the dimensionless current density

\( \overline{J} \) is the dimensional current density

L is the distance between the parallel plates.

M is the Hartmann number

\( \overline{p} \) is the fluid pressure

\( \overline{p}_0 (x) \) is the mean pressure

\( \overline{p}_1 (y) \) is the perturbed pressure
Pr is the Prandtl number

\( \overline{P} \) is the pressure gradient

\( P \) is the pressure gradient

\( \overline{P} \) is the dimensionless pressure gradient

\( p_0 \) is the dimensionless mean pressure

\( p_i \) is the dimensionless perturbed pressure

\( Q \) is the constant heat source

\( \mathbf{q} \) is the velocity vector

\( \overline{T} \) is the temperature

\( \overline{T}_w \) is the temperature at the either plate

\( \overline{u} \) is the \( x \)-component of the fluid velocity

\( U \) is the Plate velocity

\( (\overline{x},\overline{y},\overline{z}) \) are cartesian coordinates

\( (x,y,z) \) are dimensionless coordinates

\( \eta \) is the magnetic diffusivity

\( \mu_0 \) is the magnetic permeability

\( \sigma \) is the electrical conductivity

\( \rho \) is the fluid density

\( \mu \) is the co-efficient of viscosity

\( \kappa \) is the thermal conductivity of the fluid

\( \varphi \) is the dissipation of energy due to viscosity

\( \theta \) is the dimensionless temperature

\( \alpha \) is the dimensionless heat source
APPENDIX

\[ A_1 = \frac{U}{\sinh M}, \quad A_2 = \frac{MB - P}{M^2}, \quad A_3 = \frac{1 - e^{-M}}{2 \sinh M}, \quad A_4 = \frac{1 - e^M}{2 \sinh M}, \quad A_5 = A_2 A_3, \]

\[ A_6 = A_2 A_4, \quad A_7 = \frac{\Pr \cdot EcA_2^2 M^2}{2}, \quad A_8 = \Pr \cdot EcA_4^2 M^2, \quad A_9 = \Pr \cdot EcA_6^2 M^2, \]

\[ A_{10} = 2 \Pr \cdot EcA_4 A_5 M^2, \quad A_{11} = 2 \Pr \cdot EcA_5 A_6 M^2, \quad A_{12} = 2 \Pr \cdot EcA_1 A_6 M^2, \quad A_{13} = \Pr \cdot EcE^2, \]

\[ A_{14} = \frac{\Pr \cdot EcA_2^2 M^2}{2}, \quad A_{15} = \Pr \cdot EcA_4^2 M^2, \quad A_{16} = \Pr \cdot EcA_6^2 M^2, \quad A_{17} = \Pr \cdot EcA_2^2 M^2, \]

\[ A_{18} = 2 \Pr \cdot EcA_4 A_5 M^2, \quad A_{19} = 2 \Pr \cdot EcA_5 A_6 M^2, \quad A_{20} = 2 \Pr \cdot EcA_1 A_6 M^2, \]

\[ A_{21} = 2 \Pr \cdot EcA_5 A_6 M^2, \quad A_{22} = 2 \Pr \cdot EcA_2 A_5 M^2, \quad A_{23} = 2 \Pr \cdot EcA_2 A_6 M^2, \]

\[ A_{24} = 2 \Pr \cdot EcE A_1 M, \quad A_{25} = 2 \Pr \cdot EcE A_4 M, \quad A_{26} = 2 \Pr \cdot EcE A_6 M, \quad A_{27} = 2 \Pr \cdot EcE A_2 M, \]

\[ A_{28} = A_{13} - A_{14} + A_{21} - A_{27}, \quad A_{29} = \frac{A_7}{4 M^2}, \quad A_{30} = \frac{A_8}{4 M^2} + \frac{A_{10}}{8 M^2} + \frac{A_{15}}{4 M^2} + \frac{A_{18}}{8 M^2}, \]

\[ A_{31} = \frac{A_9}{4 M^2} + \frac{A_{16}}{4 M^2} + \frac{A_{19}}{8 M^2}, \quad A_{32} = \frac{A_{12}}{8 M^2}, \quad A_{33} = \frac{A_{22}}{M^2} + \frac{A_{25}}{M^2}, \quad A_{34} = \frac{A_{23}}{M^2} - \frac{A_{26}}{M^2}, \]

\[ A_{35} = \frac{A_{10}}{4} + \frac{A_{11}}{4} + \frac{A_{12}}{4} - \frac{A_{18}}{4} - \frac{A_{28}}{2} + \alpha, \quad A_{36} = \frac{A_{14}}{4 M^2}, \quad A_{37} = \frac{A_{20}}{M^2} - \frac{A_{24}}{M^2}, \]

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