Chapter 4

Longitudinal Development of EAS

The depth of shower maximum $X_{\text{max}}$ is related to the hadronic interaction characteristics and primary mass composition. This chapter deals with the examination of model dependence regarding the shower parameters $X_{\text{max}}$ and Elongation rate.

Simulations of 1000 vertical showers each for proton and iron primaries with energies $10^{14}\text{eV}$, $10^{15}\text{eV}$, $10^{16}\text{eV}$, $10^{17}\text{eV}$, $10^{18}\text{eV}$, $10^{19}\text{eV}$, and $10^{20}\text{eV}$ are done for available H.E. models namely QGSJET-01, QGSJET-II, DPMJET-2.55, EPOS-1.66, SIBYLL-2.1, VENUS-4.12, and NEXUS-3 using CORSIKA-6735. GHEISHA-2002d low energy model is taken. With VENUS the upper limit of primary energy is $2 \times 10^{16}\text{eV}$ [1], and it is not desirable to over-stretch the H.E. model used [2]. Hence, with VENUS, showers are simulated up-to primary energy $10^{16}\text{eV}$. To study the dependence of H.E. interaction models on the longitudinal development of extensive air showers, the depth of shower maximum $X_{\text{max}}$ distribution, mean $<X_{\text{max}}>$, standard deviation(RMS) $\sigma_{X_{\text{max}}}$, the ratio RMS of $X_{\text{max}}$ to $<X_{\text{max}}>$, Skewness and Kurtosis of the $X_{\text{max}}$ distribution are considered. Elongation rate ($ER_{10}$) and longitudinal deposition of energy are also studied.

4.1 The shower maximum and $X_{\text{max}}$

The longitudinal shower size profile and the lateral particle distribution of electro-magnetic shower can be obtained from cascade theory [3, 4, 5]. On average, the longitudinal profile of a shower induced by a given primary cosmic ray is a universal function of the depth of atmosphere traversed. The depth of shower maximum for shower generated by a particle
with primary energy $E$ can be given by

$$X_{\text{max}} \approx X_R \ln \left( \frac{E_0}{E_c} \right).$$  \hspace{1cm} (4.1)

Here, $X_R$ is the radiation length of the matter, and $E_c$ is the critical energy. This is the characteristic amount of matter traversed over which a high-energy electron loses all but $1/e$ of its energy by bremsstrahlung. The radiation length $X_R$ is also $7/9$th of the mean free path for pair production by a high-energy photon. Accounting for the energy distribution of electrons in a photon-induced shower this expression becomes

$$X_{\text{max}} \approx X_R \left[ \ln \left( \frac{E_0}{E_c} \right) + \frac{1}{2} \right].$$  \hspace{1cm} (4.2)

Again the energy spectrum of secondary particles in a cascade can be approximately given by the power law $dN/dE \sim E^{(1+s)}$, where $s$ denotes the shower age parameter. The shower age is defined as

$$s \approx 3X/(X + 2X_{\text{max}}).$$  \hspace{1cm} (4.3)

Greisen [6] developed a parametrization of the mean longitudinal shower size profile of charged particles

$$N_c(X) = \frac{0.31}{\sqrt{\ln(E_0/E_c)}} \exp \left[ (1 - \frac{3}{2} ln s) \frac{X}{X_R} \right].$$  \hspace{1cm} (4.4)

On average, the longitudinal profile of a shower induced by a given primary cosmic ray is a universal function of the depth of atmosphere traversed. The function proposed by Gaisser and Hillas [7] is widely used to fit measured shower profiles

$$N(X) = N_{\text{max}} \left( \frac{X - X_0}{X_{\text{max}} - X_0} \right)^{\frac{X_{\text{max}} - X}{X_{\text{max}} - X_0}} \exp \left( \frac{X_{\text{max}} - X}{\lambda} \right).$$  \hspace{1cm} (4.5)

where $X_0$ is the depth of first interaction, the main source of shower-to-shower fluctuations. $\lambda$ an energy dependent floating parameter in the fit, generally fixed to 70 g/cm$^2$ [8].

The depth of shower maximum contains the information about the mass of the primary CR initiating the shower as well as about the properties of hadronic interactions involved in the process of cascade evolution. The average value $X_{\text{max}}$ depends on the primary energy $E$ and on the number of nucleons $A$ of the primary as given in the Eq.4.6,

$$<X_{\text{max}}>= \alpha (lnE - lnA) + \beta,$$  \hspace{1cm} (4.6)
where $\alpha$ and $\beta$ depend on the details of hadronic interactions so far as a fixed primary is considered. Their values are very sensitive to changes in cross-section, multiplicity and elasticity [9]. Eq.4.6 can be derived from the simple generalized Heitler model of cascade formation due to hadronic primaries, but it is in good agreement with the description of the $X_{\text{max}}$ evolution predicted by hadronic models currently in use.

### 4.2 Elongation Rate

The shift of the location of the depth of maximum development $X_{\text{max}}$ with the energy of shower generating primary is called the elongation and its rate of change per decade of primary energy is termed as the elongation rate [10]. Thus elongation rate $ER_{10} = \frac{dX_{\text{max}}}{d\log E}$. The elongation rate of em showers is energy-independent,

$$ER_{\text{em}} = \log X_R \approx 85 g/cm^2,$$  \hspace{1cm} (4.7)

where $X_R$ is the radiation length. The elongation rate, the absolute position of the mean $X_{\text{max}}$ and fluctuations in $X_{\text{max}}$ about the mean carry information about the primary composition. As the interpretation is hadronic model dependent, model systematic uncertainties can be assessed by comparing the predictions of a variety of hadronic models.

Again, Eq.4.6 can be expressed as,

$$<X_{\text{max}}> = ER_{10}(\log E - \log A) + X_{\text{init}},$$  \hspace{1cm} (4.8)

where $X_{\text{init}}$ is the depth of first interaction [7].

There is no strong model dependence on elongation rate in the energy range $10^{14} eV - 10^{18} eV$ [11].

### 4.3 Model Dependence of $X_{\text{max}}$

The longitudinal development of EAS depends on the inelastic cross-sections $\sigma_{\text{inel}}$ of the primary particles with air nuclei, the multiplicity $\mu$ giving the average number of particles production in an interaction, and the inelasticity $K$ — the fraction of energy transferred to the secondary produced in an interaction. The development of the cascade occurs earlier and $X_{\text{max}}$ occurs in smaller slant depth with an increase in the inelastic cross-sections. Also an increased inelasticity results the particles to lose more energy, so that the shower
4.3. MODEL DEPENDENCE OF $X_{\text{MAX}}$

reaches its maximum earlier in the atmosphere. Similarly, an increase in multiplicity results in more particles in the first interactions, so that the produced secondaries are less energetic. Hence, the shower maximum occurs early in the atmosphere. Rate of change of $X_{\text{MAX}}$ as function of the changes of $\mu$ and $K$ can be given as [12]

$$\frac{\Delta X_{\text{MAX}}}{X_{\text{MAX}}} \approx -\frac{1}{2} \frac{\Delta \mu}{\mu} - \frac{1}{10} \frac{\Delta K}{K}. \tag{4.9}$$

A similar relation for the dependence of $X_{\text{MAX}}$ on inelastic proton-air cross-sections can be given as

$$\frac{\Delta X_{\text{MAX}}}{X_{\text{MAX}}} \approx -\frac{5}{7} \frac{\Delta \sigma_{p-\text{air}}^{\text{inel}}}{\sigma_{p-\text{air}}^{\text{inel}}}. \tag{4.10}$$

Another sensitive parameter is $\sigma_{X_{\text{MAX}}}$, expressing quantitatively the shower to shower fluctuations of $X_{\text{MAX}}$. It depends mainly on the cross section and less strongly on the elasticity. This makes fluctuations in $X_{\text{MAX}}$, a good parameter to study hadronic cross sections at ultra-high energies [9].

**Skewness and Kurtosis**

A distribution is said to be skewed if it is not symmetric about the mean. The distribution is said to skewed right or positively skewed if the right tail is longer than the left, i.e., rise is steeper and is followed by a flatter fall. While, the opposite where the left tail longer is called negatively skewed or left skewed distribution. The skewness of a data set containing $n$ data points with values $x_i$ is denoted by $g_1$, where

$$g_1 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^3 \left(\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2\right)^{3/2} = \frac{m_3}{(m_2)^{3/2}} = \frac{m_3}{\sigma^3}. \tag{4.11}$$

Here, $\bar{x}$ is the sample mean, $m_2$ is the second moment about the sample mean $\bar{x}$ and is called sample variance ($\sigma^2$). Where $\sigma$ is the standard deviation and indicates the spread of the distribution about the mean. The term $m_3$ is the sample third moment.
Again measure of the ‘peakedness’ of the distribution is known as Kurtosis. To provide a comparison of the shape of a given distribution to that of the normal distribution, kurtosis is defined as,

\[ g_2 = \frac{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^4}{\left( \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^2} - 3 = \frac{m_4}{(m_2)^2} - 3 = \frac{m_4}{\sigma^4} - 3. \quad (4.12) \]

Here, \( m_4 \) is the fourth moment about the sample mean. The term 3 corresponds the kurtosis of the standard normal distribution.

### 4.4 Results

Longitudinal shower profiles of 1000 showers are plotted in Figure 4.1. In Fig. 4.2, the distribution of depth of shower maximum is shown.

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Figure 4.1: Total charged particles Vs. Atmospheric depth.

Figure 4.2: Distribution of \( X_{\text{max}} \) for 1000 showers.
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Figure 4.3: Longitudinal Shower Development for iron(left)/proton(right) primary for different primary energies

Figure 4.4: Longitudinal Shower Development for proton(left)/iron(right) primary at $10^{17}$eV for different HE models
Figure 4.5: Shower to Shower Fluctuation of $X_{\text{max}}$ for proton and iron primaries at $10^{17}\text{eV}$ with different HE models

Figure 4.6: RMS of $X_{\text{max}}$ to $<X_{\text{max}}>$ ratio

In figure 4.3, longitudinal shower development are shown from simulated showers using CORSIKA-6735, for iron(left)/proton(right) primary with energies $10^{14}\text{eV}$, $10^{15}\text{eV}$, $10^{16}\text{eV}$, $10^{17}\text{eV}$, $10^{18}\text{eV}$, $10^{19}\text{eV}$, and $10^{20}\text{eV}$ for QGSJET model. It is observed that shower size increases with increase in the primary energy irrespective of the primary. For iron primaries faster shower development is noticed with respect to the proton primaries. As protons have larger interaction mean free path than iron-nuclei, higher depth is required to reach shower maximum. A nucleus can be considered as collection of independent nucleons. Thus interaction probabilities of nucleons add up to give the interaction probability of the nucleus, leading to, on average a faster development of the shower.

In the figure 4.4, longitudinal shower profile of 1000 showers each for iron and proton primaries at fixed primary energy of $10^{17}\text{eV}$ are plotted for different High Energy models. It is observed that irrespective of the H.E. model considered, shower size at shower maximum for proton primary is higher than that for iron primary. Position of shower maximum for proton is at a greater depth compared to the same for iron shower.

Shower to Shower Fluctuation of depth of shower maximum for proton and iron primaries at $10^{17}\text{eV}$ are shown in the figure 4.5. It is seen that for proton shower fluctuation is more than for iron shower.
Figure 4.7: Energy Deposition for iron(left)/proton(right) primary at $10^{17}$eV

Figure 4.8: Prediction of $<X_{\text{max}}>$ with different HE models: $10^{14}$-$10^{17}$eV
In Figure 4.6, the ratio of RMS of depth of shower maximum, $\sigma_{X_{\text{max}}}$ to mean depth of shower maximum, $<X_{\text{max}}>$ are plotted against the primary energy. This ratio is higher for proton and lower for iron. This ratio decreases as primary energy increases for both the primaries. The difference of this ratio between proton and iron showers also decreases with the increase in the primary energy.

Energy deposition for iron and proton initiated showers with primary $10^{17}$eV are plotted in the figure 4.7. It is noticed that proton induced showers encounter higher level of fluctuations during the early development of shower cascade while entering the atmosphere. It is seen that some of the protons enters the upper atmosphere up to $\approx 200\text{g/cm}^2$ without any interaction. These fluctuations lead to the fluctuations in the occurrence of the shower maximum for proton initiated showers. On the other-hand iron-nuclei interact with the air molecules almost at the same depth while entering the atmosphere, resulting lesser fluctuations in the depth of shower maximum.

Prediction of mean depth of shower maximum of proton and iron induced showers with primary energies $10^{14}$–$10^{17}$eV for different H.E. models available in CORSIKA are plotted against the primary energy in the figure 4.8. Data from various experiments namely Yakutsk [13], Casa-Blanka, Tunka, and Ling Wang are also plotted. It is observed
that irrespective of High Energy Hadronic Interaction Models, experimental data fall in between the simulated data of proton and iron, upper set of curves referring to proton and the lower set to iron primaries. All the models are compatible with the experimental data which appear to show that primary mass composition is becoming heavier beyond the knee and near $10^{17}$eV. Similar trend is also observed in figure 4.9, where model predictions of mean depth of shower maximum with QGSJET and DPMJET for primary energies $10^{17}$–$10^{20}$eV along with HIRES [14], Fly’s Eye [15] and Pierre Auger Observatory [16] data are plotted. The systematic uncertainties in depth of shower maximum are 12g/cm$^2$ for Auger, 20g/cm$^2$ for Yakutsk, and 6g/cm$^2$ for HiRes [17].

Table 4.1: Average depth of the shower maximum for primary protons and iron nuclei calculated with CORSIKA, using different interaction models

| Energy $E_0$[eV] | Average depth of the shower maximum $X_{max}$ [g/cm$^2$] |
|------------------|------------------|------------------|------------------|
|                  | DPMJET p  Fe     | EPOS p  Fe       | NEXUS p  Fe      |
| $10^{14}$        | 495.7±3.2 370.5±1.7 | 490.5±3.2 357.5±1.4 | 483.6±2.9 362.1±1.6 |
| $10^{15}$        | 564.8±2.9 425.0±1.5  | 557.9±2.8 437.4±1.4  | 553.0±2.7 438.9±1.6  |
| $10^{16}$        | 628.0±2.7 521.5±1.5  | 607.9±2.5 509.1±1.5  | 615.0±2.4 514.4±1.6  |
| $10^{17}$        | 687.4±2.4 581.5±1.5  | 670.0±2.5 566.7±1.5  | 677.0±2.3 582.0±1.5  |
| $10^{18}$        | 762.3±2.3 651.1±1.4  | 745.2±2.5 641.7±1.6  | 752.4±2.7 640.4±1.6  |
| $10^{19}$        | 812.8±2.8 683.7±1.5  | 806.5±2.7 703.7±1.6  | 807.1±3.0 708.3±1.7  |

Average depth of shower maximum of 1000 simulated showers each in the energy range $10^{14}$–$10^{17}$eV with primary protons and iron nuclei using different H.E. models are tabulated in the Table 4.1. It is seen that the maximum difference in $< X_{max} >$ is 39.7g/cm$^2$ (DPMJET & QGSJET, $10^{19}$eV) for proton and 27.8g/cm$^2$ (DPMJET & NEXUS, $10^{19}$eV) for iron.
The RMS ($\sigma_{X_{\text{max}}}$) of the distribution of the depth of the shower maximum for protons and iron nuclei within the energy range $10^{14}$–$10^{17}$eV are plotted in the Fig. 4.10. It is observed that, RMS values are much higher for proton induced showers than the respective iron induced showers. The differences in the RMS among the models are also higher for proton primary than that for corresponding iron primary.

To have a better view and to study the difference in RMS of the depth of shower maximum distributions due to different models, figure 4.10 is split into Fig. 4.11 for proton primary and Fig. 4.12 for iron primary.
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Figure 4.11: RMS of depth of shower maximum for proton primary at $10^{14} - 10^{17}$eV with different models

Figure 4.12: RMS of depth of shower maximum for iron primary at $10^{14} - 10^{17}$eV with different models
It is observed in Fig. 4.11 that for proton primary RMS for VENUS is the highest, while that for NEXUS is the lowest, where QGSJET-II & DPMJET are intermediate. RMS value decreases with increase in primary energy. Again in Fig. 4.12, it is observed that for iron primary RMS value for EPOS is the lowest and is increasing slightly with primary energy. For all the models RMS value is almost constant with primary energy, except DPMJET, showing erratic result.

The skewness of the distribution of the depth of the shower maximum for protons and iron nuclei within the energy range $10^{14} - 10^{17}$eV are plotted in the figure 4.13 and figure 4.14 respectively. It is observed that skewness shows higher values (right skewed) for proton induced showers than for iron induced showers. This is due to the fact that proton has more probability than iron to enter deep into the atmosphere without undergoing any interaction.

It is observed in Fig. 4.13 that NEXUS shows more skewness than the rest. All the distributions are right skewed and the skewness decrease with increase in primary energy.
Again for iron primary, Fig. 4.14 shows that skewness for DPMJET and NEXUS are more than the rest. For all the models skewness decreases slightly with increasing primary energy.

In figure 4.15 & 4.16, kurtosis of the distribution of the depth of the shower maximum for protons and iron nuclei respectively within the energy range $10^{14}$–$10^{17}$eV are plotted. Kurtosis for proton induced showers with a given primary energy is higher than that for corresponding iron primary induced showers. Thus the distribution of depth of shower maximum for proton induced showers are more peaked than that of the iron induced showers. The distribution of depth of shower maximum for iron induced showers are closer to that of a standard normal distribution. Also relative differences in the kurtosis due to H.E. models are lesser for iron primary than that for proton.

It is observed in Fig. 4.15 that NEXUS shows more kurtosis than the rest except at $10^{15}$eV, where SIBYLL shows a higher value. Peakedness of the distribution of depth of shower maximum decreases with increase in primary energy. Again in Fig. 4.16, it is observed that for iron primaries that skewness values for DPMJET and NEXUS are above the the rest. DPMJET prediction is the highest except at $10^{14}$eV. The depth of shower maximum distribution due to NEXUS at $10^{15}$eV and DMPJET at $10^{14}$ & $10^{15}$eV are more peaked than the Gaussian. Slopes of the curves depict poor energy dependence.
Figure 4.15: Kurtosis of depth of shower maximum for iron/proton primary at $10^{14} - 10^{17}$eV with different models

Figure 4.16: Kurtosis of depth of shower maximum for iron/proton primary at $10^{14} - 10^{17}$eV with different models
4.5 Conclusion and Remarks

In this chapter we have studied the longitudinal development of EAS simulated with different primary energy, mass composition & interaction models. The longitudinal profile for the different models follow the same pattern and there is no significant difference amongst the predictions of various models. So far depth of shower maximum distribution of EAS is concerned, there is no significant difference amongst the predictions made by the different hadronic interaction models. Fluctuations in $X_{\text{max}}$ distribution is measured by RMS value, $\sigma_{X_{\text{max}}}$. It is found that, this value is much higher for proton induced showers compared with iron induced showers, and the value decreases with increasing primary energy. So far as the model dependence is concerned, for proton showers, VENUS shows the highest $\sigma_{X_{\text{max}}}$, and NEXUS, the lowest. For iron induced showers $\sigma_{X_{\text{max}}}$ is almost independent of both primary energy and model. Further the difference in RMS values between proton & iron induced showers decreases with increasing primary energy. Asymmetry of the $X_{\text{max}}$ distribution as measured by the skewness parameter shows a right handed skewness for all the models and for both the primaries. The parameter kurtosis measures the peakedness of $X_{\text{max}}$ distribution as compared to Gaussian distribution. The skewness and kurtosis of the $X_{\text{max}}$ distribution for proton induced showers are higher than the iron induced showers. For proton showers, these parameters decrease with primary energy, but for iron showers, they are almost constant. No systematic model difference is observed in the simulated results of skewness and kurtosis. The model NEXUS is found to show relatively higher values of these parameters.
References


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