"Those, who have achieved the knowledge of self (Vijnan) through the path of Vedant, have grasped the exact meaning of life. Then these individual beings have achieved wisdom and have transcended to the state of pure knowledge by associating the individual being (Atman) to the cosmic existence (Supreme Atman). Such enlightened individual beings then reside in the egoless state of supreme being (infinite Brahman)"
Chapter 3

Theoretical model of transient sheath dynamics

Abstract This chapter includes new understanding and interpretation of the observed ion implantation current profile in transient sheath experiments. Two time-scale structure of the profile is clearly noted in constant phase of applied pulse voltage at the electrode. But no complete theoretical explanation is available as yet. We treat the positive (ion-rich) sheath as an equivalent series LCR electrical circuit and use Kirchoff’s law to arrive at a second order differential equation for theoretical description of the defined circuit. The Child law describes the motion of transient sheath and monitors current in the circuit. Numerical solution of the defined driven circuit equation yields almost the same current profile as observed in the transient sheath experiments.

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3.1 Motivation

As discussed in chapter 2, the sheath equivalent electrical circuit behavior offers a complementary but physically viable idea for equilibrium Child sheath description. In fact, the self-consistent analysis of the Child sheath and the derivation of conventional scaling of the sheath width motivates to believe that the sheath is associated with dual behavior. The duality of the sheath behavior could be described as its (sheath) dual role to act as a source as well as the sink to self-consistently decide the equilibrium current and width associated
with the sheath. Thus the hydrodynamic view of the sheath and its equivalent electrical circuit analog are supposed to be equivalent to each other. The present chapter considers this view of the Child sheath to develop theoretical model for transient sheath description having relevance to plasma based technology of material processing.

3.2 Introduction

This is now widely recognized that, the basics of boundary-plasma interaction physics has a major role to play in the development of plasma-based technology of industrial applications. Specifically, the basic understanding of the transient sheath dynamics offers a valuable source of knowledge for practical realization of the surface engineering of materials. The importance of the plasma sheath physics, in general, is reported to have wide range applications in many other branches of physics and engineering [1]. Considerable amount of research work has been carried out and reported to describe the static [2 - 5] and collective dynamic [6 - 8] properties of the localized space charge sheath layers. Nevertheless, not much attention has been paid to develop basic theoretical [9, 10] and experimental [11, 12] understanding of the transient sheath dynamics.

Variable capacitor model [9, 12] of the transient sheath dynamics has, indeed, physical relevance to the transient sheath physics. However, this model is physically and mathematically incomplete and inconsistent.

In principle, the transient sheath capacitance remains no more constant in time. However, as discussed in [13], the evaluation of the displacement current related to the dynamic view of the Child sheath law is physically inconsistent [12]. Following the Maxwell's arguments for Ampere's law correction [13], the displacement current should be included in the basic definition and specification of the Child current itself. To our best of knowledge, none of the earlier model calculations [12] is found suitable to produce clear understanding of the high initial transient in the ion implantation current profile.

Our main motivation behind this contribution is to put forward a complete and consistent physical model to describe and understand the basic scientific principles involved in governing the temporal transition behavior of the ion implantation current associated with an expanding matrix sheath. Many critiques may argue that the used basic theoretical laws are well-known. Nevertheless, we argue that the dual behavior of the Child sheath description, as a source and sink both has not been reported earlier. The Child sheath potential
acts as a source emf to drive the current in its own equivalent series LCR electrical circuit that acts as a load (sink) to decide the sheath associated temporal current distribution. This is to further state that although the assumed continuum sheath model may not be rigorous [1, 14], the main emphasis is laid on the physical importance of the dual behavior of the expanding Child sheath in constant phase of the applied pulse voltage. This is where the originality of our proposed theoretical model lies.

3.3 Physical model and mathematical formulations

The proposed physical model treats the localized positive space charge sheath region near a boundary wall to behave as an equivalent series LCR electrical circuit [6, 15, 16] with negative inductance [6, 15, 17]. This may be considered as a general feature of any non-neutral space charge cloud of finite size with excess positive charge. Such clouds like solitons, double layers and vortices etc., are produced by nonlinear excitation and saturation mechanisms of the plasma instabilities [2]. Similar clouds are formed in vicinity of the boundary wall as a natural course of boundary - plasma interaction processes.

It seems to be reasonable, to define these clouds of the non-neutral space charge distributions, in general, as the natural non-neutral plasmas (NNP) with positive (negative) polarities depending on the richness of the positive (negative) space charges [18]. Physical shapes and sizes of these clouds are governed by the boundary conditions and the level of nonlinearity considered in theoretical descriptions. A bi-potential structure of positive NNP around a negatively biased grid in double plasma device experiments is destabilized by the asymmetry in the bi-potential structure [19]. Theoretically, the terminal point of the presheath scale defines the sheath edge as a singular point.

Indeed a few efforts have been made to find out the conditions for obtaining smooth solutions of sheath/presheath potential distributions [3, 4, 20, 21]. But the questions pertaining to the problem of sheath edge singularity have become a subject of continuous debates and discussions. However, only a few specific cases of chosen plasma systems and force field configurations are considered [3, 4, 20 and references therein]. In reality the problem of sheath edge singularity still remains to be resolved to full satisfaction.

Now, using the Kirchoff's law, the electrical circuit equation for a dynamic sheath
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An equivalent series LCR circuit with negative inductance can be written as follows,

\[ |L_{sh}| \frac{d^2 I}{dt^2} - R_{sh} \frac{dI}{dt} - \frac{I}{C_{sh}} = -\frac{d\varepsilon_{sh}}{dt}. \]  

(3.1)

Here, \( L_{sh}, R_{sh} \) and \( C_{sh} \) represent for the inductance, resistance and capacitance of the ion-rich sheath respectively. The notation \( \varepsilon_{sh} \) denotes for the source emf and corresponds to the applied negative potential at the electrode \((V_W)\). In constant phase of the applied voltage, the source emf is supposed to be constant in time but the sheath potential gradient that motivates the ions to move, changes in time due to sheath edge motion. The negativity of the inductance implies that the induced emf due to negative sheath inductance supports the primary source of current variation. For example, if the current is decreasing in time (negative rate of current change) it will further decrease in time till the occurrence of current reversal. On the other hand, if the current is increasing in time (positive rate of current change), it will further increase till the saturation point is reached.

Let us express the total time derivative of the source emf as

\[ \frac{d\varepsilon_{sh}}{dt} = (\frac{d\varepsilon_{sh}}{dx}) (\frac{dx}{dt}) \approx (\varepsilon_{sh}/x) (\frac{dx}{dt}). \]

This kind of dimensional representation can be assumed for a transient sheath, so as to allow the inclusion of the effect of sheath edge motion on the ion implantation current. In fact, this assumption is based on the idea of an equivalent linear analog of the Child sheath associated potential drop over its own global width having time variation due to sheath edge motion. This is to note that the value of initial linear potential gradient is considered to be unity for the sake of simplicity. Subsequently, the sheath width and the associated electric field variations in time are decided by sheath edge motion. Using the aforesaid simplicity and the negative sign of the rate of current change term in Eq. 3.1, following form of a dissipative series LCR electrical circuit equation is obtained,

\[ |L_{sh}| \frac{d^2 I}{dt^2} + R_{sh} \frac{dI}{dt} - \frac{I}{C_{sh}} = -\frac{\varepsilon_{sh}}{x} \frac{dx}{dt}. \]  

(3.2)

In fact, the time varying sheath electric field arising due to sheath edge motion acts as a source of its own equivalent electrical circuit as discussed above. It is assumed that the transient sheath dynamics follows the Child-Langmuir law [9] during whole time period of the ion implantation processes. It is given below as,

\[ \frac{4}{3} \varepsilon_0 \left( \frac{2e}{m_e} \right)^{1/2} V_w^{3/2} \frac{dx}{x^2} = e_n (\frac{dx}{dt} + u_0) + \varepsilon_0 \frac{dE_{sh}}{dt}, \]  

(3.3)
where $\varepsilon_0$ is the permittivity of the free space, $e$ is electronic charge, $n_0$ is the ion matrix sheath density. Notation $m_i$ represents the ionic mass of the plasma under consideration. $u_0$ denotes for the minimum speed with which the ions enter the transient sheath and $E_{sh}$ is the sheath associated electric field. In the Child limit of applied wall potential the space charge sheath induced velocity of the ions exceeds the value of $u_0$, which equals the sonic speed of bulk plasma medium. Hence, it will be neglected in subsequent mathematical calculations. This is justified by numerical depiction (Fig. 3.2) of the Mach value associated with sheath edge motion. Here, the space coordinate $x$ represents the position of moving sheath edge with zero reference value at the wall of immersed electrode in plasma.

In fact, the Child sheath equation was initially derived for conventional diode under the assumptions of zero electric fields on both sides of the diode sheath. There is no doubt in it to admit that the contradiction lies in this assumption where the finite directed ion velocity at the sheath edge is demanded to fulfill the Bohm criterion of sheath formation. However, as discussed in [3, 20] this contradiction is qualitatively resolved by an ad-hoc assumption that some residual electric field of Debye sheath scale leaks through the sheath edge and extends to a large scale length termed as the presheath scale. But quantitative description of the presheath scale potential distribution [21] is still not completely investigated. Without going into critical debates on the plasma-sheath transition layer problem [22] and rigorousness of the assumed continuum model [1, 14] we stress to elucidate the dual character of the expanding Child sheath to govern the temporal relaxation behavior of the ion implantation current profile in constant phase of the applied pulse voltage.

Following the idea of Chester [23] the dynamical aspects of the Child sheath are described by Eq. 3.3, which is reduced to the following form by replacing $E_{sh} = -V_W/x$, in Eq. 3.3

$$\frac{\delta}{\delta x} \left( \frac{2\varepsilon_0}{m_i} \right)^{1/2} \frac{V_W^{3/2}}{x^2} = e\eta_0 \left( \frac{dx}{dt} + u_0 \right) + \epsilon_0 \frac{V_W}{x^2} \frac{dx}{dt}. \tag{3.4}$$

Following the assumptions of Wood [9] and Liberman [24], the above equation is solved to yield the value of $x$ as a function of time is given as

$$x = \sqrt{\epsilon_0 V_W / (\epsilon n_0)} \left\{ \left[ F(t) + G(t) \right]^{\frac{1}{3}} + \left[ F(t) - G(t) \right]^{\frac{1}{3}} \right\}, \tag{3.5}$$

where

$$F(t) = (5\sqrt{2})/2 \left[ 1 + (4/15)\omega_{pi} t \right], \quad G(t) = \sqrt{[F(t)]^2 + 1}, \quad \omega_{pi} = \sqrt{(n_0 e^2)/(\epsilon_0 m_i)}$$

is the ion oscillation frequency of the matrix sheath. The normalized form of $x$ can be derived by
3.3 Physical model and mathematical formulations

dividing it by the Debye length to yield

\[ \xi = x / \lambda_d = \sqrt{eV_w} / (T_e) \left\{ \left[ F(t) + G(t) \right] + \left[ F(t) - G(t) \right] \right\} \]

Here, at \( t = 0 \) the value of \( x \) is denoted by \( x(0) \) and its normalized value by \( \xi(0) \), which is defined as the initial ion matrix sheath thickness or simply the initial sheath edge position. This is found out as a real root of \( x \) by putting \( t = 0 \) in (3.5) and hence it automatically comes out to be

\[ x(0) = \left( \sqrt{2eV_w / e \rho_0} \right), \xi(0) = \left( \sqrt{2eV_w / T_e} \right) \]  

(3.6)

The same expression can be derived from the Poisson equation as discussed in [25] under constant ion matrix sheath approximation. Now, the displacement rate of the moving sheath edge can be evaluated and given as,

\[ \frac{dx}{dt} = \frac{K}{3} \left\{ (F + G)^{-3/2} + (F - G)^{-3/2} \right\} \left( \frac{dF}{dt} + \frac{dG}{dt} \right) \]

where \( F \equiv F(t), G \equiv G(t), K = \sqrt{eV_w / e \rho_0} \). Using Eqs. 3.5 - 3.7, the driven series LCR electrical circuit can be described by the following form of (3.2),

\[ |L_{sh}| \frac{d^2I}{dt^2} + R_{sh} \frac{dI}{dt} - \frac{I}{C_{sh}} = -\frac{2\sqrt{2} \rho_{pl} AV_w}{9G} \left[ \frac{(F - G)^{-3/2} + 1}{(F - G)^{-3/2} - 1} \right] \]

(3.8)

This is basically the required equation for theoretical description of the dynamic sheath in terms of electrical circuit network of series LCR circuit.

3.3.1 Analytical and numerical calculations

Let us first identify the nature of governing differential equations of two different time scales. For this purpose, we rewrite (3.1, 3.2) in the following normalized form with total time derivative of the source emf:

\[ \frac{d^2I}{d\chi^2} + \left( \frac{T}{\tau_C} \right) \frac{dI}{d\chi} - \left( \frac{T}{\tau_R} \right)^2 I = - \left( \frac{T}{\tau_S} \right) \frac{d\xi_{sh}}{d\chi}. \]

(3.9)

Here, \( I = I/I_B, I_B \) defines the Bohm current, \( \chi = \tau / T \) where \( T \) denotes for general time scale of arbitrary choice, \( \tau_C \sim |L_{sh}| / R_{sh} \) denotes for the time constant and \( \tau_R \sim \sqrt{|L_{sh}| C_{sh}} \) denotes for the resonance time scale of the defined series LCR electrical circuit. The normalized quantity \( \xi_{sh} = (\varepsilon_{sh} / V_W) \) represents the dimensionless source emf. The notation \( \tau_S = I_B |L_{sh}| / V_W \sim \tau_C (I_B R_{sh} / V_W) \) has time dimension. Now, for experimental parameters as given in [12], these characteristic times obey the following scaling, \( \tau_R \approx \tau_S \gg \tau_C \) having validity (as shown later) within order of magnitude.
3.3 Physical model and mathematical formulations

Now, the ion current on fast time scale of \( T = \tau_C \ll \tau_R \approx \tau_S \) can be described by the following equation,

\[
\frac{d^2 I}{dx^2} + \frac{dI}{dx} = 0.
\]  

(3.10)

This is well known that this equation yields an exponentially damping solution and determines the rate of initial transient current change in the observed ion implantation current profile.

Similarly, the ion current on the slow time scale of \( T = \tau_R \approx \tau_S \gg \tau_C \) is described by the following equation,

\[
\frac{d^2 I}{dx^2} + \left( \frac{\tau_R}{\tau_C} \right) \frac{dI}{dx} - I = -\frac{d\Phi}{dx}.
\]  

(3.11)

In limiting case of \( dI/dx \to 0 \), the similar ion current is obtained as that derived in [12]. This is obvious to note that, the defined slow time scale is sensitive even to the weak but finite rate of current change in determining the shape of the ion current profile due to presence of a multiplying factor of large value than unity. This is to further note that, the first and second order terms of the rate of current change are common in Eqs. 3.10 and 3.11 and govern smooth transition of the ion current profile without any singularity.

From basic conventional circuit theory of a series LCR electrical circuit with time varying source term [16], the analytical solution of (3.8) can be derived and expressed in the following form

\[
I = A_1e^{s_1t} + A_2e^{s_2t} + P(t),
\]  

(3.12)

where \( s_1 = \left\{ -R_{sh}/(2|L_{sh}|) + \sqrt{[R_{sh}/(2|L_{sh}|)]^2 + 1/(|L_{sh}|C_{sh})} \right\} \) and

\[
s_2 = \left\{ -R_{sh}/(2|L_{sh}|) - \sqrt{[R_{sh}/(2|L_{sh}|)]^2 + 1/(|L_{sh}|C_{sh})} \right\}.
\]

This is to note that the derivation of the analytical form of particular integral \( P(t) \) is complicated due to time dependence of the source term and hence, (3.8) has been numerically solved. Following form of (3.8) has been used for numerical calculation:

\[
\frac{d^2 j}{d\tau^2} + \frac{10 dj}{3\lambda d\tau} - \frac{40}{\lambda^2} J = -\frac{40}{8\lambda^8} H(\tau) \Phi.
\]  

(3.13)

The values of inductance, capacitance and resistance are expressed in terms of plasma parameters and source parameters [6, 15]. Here the time is normalized by ion oscillation time scale as \( \tau = tw_{ps}, \ j = I/(A_0ec_s) \), where \( A \) is the electrode surface area, \( c_s \) is the
plasma acoustic speed, $\lambda = t_i \omega_{pi}$, $t_i$ is the ion transit time. The applied potential is normalized as $\Phi = eV_W/T_e$ and
\[
H(\tau) = \frac{2\sqrt{2}}{[9G(\tau)]} \left( \frac{[F(\tau) - G(\tau)]^{-\frac{3}{2}} + 1}{[F(\tau) - G(\tau)]^{-\frac{3}{2}} - 1} \right).
\]

Now, Eq. 3.13 has been solved numerically as an initial value problem using Runge Kutta-4 [26] method for the plasma parameters and electrode voltage as given in [12]. The choice of the initial current has been made in accordance to the maximum value of the observed current profile. The initial value of the rate of current change ($-10.7$) has been considered as a fitting parameter and is chosen arbitrarily so as to reproduce a best possible fit to the observed current profile as revealed in Fig. 3.1. All calculations are performed for both the sets of experimental parameters [12]. The results are found to be in good agreement in both the cases. However, for graphical depiction of the nature of obtained results, we present the numerical results for only one set of the parameters.

3.3.2 Properties of sheath equivalent electrical circuit

The different characteristic (e-folding) time scales of the equivalent series LCR electrical circuit are estimated for the applied electrode wall voltage, $V_W = 3010$ V, Ar plasma density, $n_0 = 9.4 \times 10^{14}$ m$^{-3}$ and electron temperature, $T_e = 2$ eV [12] and are given as below,

- transient time scale (time constant of the circuit) $= 2|L_{sh}|/R_{sh} \sim 1$ $\mu$s,
- capacitor charging time scale $= R_{sh}C_{sh} \sim 14$ $\mu$s,
- resonance time scale $= 2\pi \sqrt{|L_{sh}|C_{sh}} \sim 17$ $\mu$s,
- the ion transit time scale for steady state sheath $\sim 10$ $\mu$s.

The total impedance ($Z_{sh}$) associated with sheath equivalent series LCR electrical circuit is given as $Z_{sh} = \sqrt{R_{sh}^2 + (\omega |L_{sh}| + 1/(\omega C_{sh}))^2}$. Now, one can see that the transient (fast variation) and the steady state (slow variation) time scales of the sheath equivalent series LCR electrical circuit differs by an order of magnitude. These two time scales as characterized by the circuit theory are in good agreement with the experimental current profile as shown in Fig. 3.1.

The presence of low frequency (LF) fluctuations (time scale $\sim 0.1$ $\mu$s) in the ion implantation current indicates that the circuit is active throughout the implantation process. The origin of these observed LF fluctuations could be attributed to the spontaneous excitation of the current driven acoustic instability of short scale length. If the Doppler up-shift correction is assumed to dominate over the natural acoustic frequency, the acoustic oscillation time scale $\tau_{osc} \approx \left(2\pi \omega_{pi}^{-1}\right) / (Mk\lambda_{De})$. Comparing with observed oscillation time $\approx 0.1 \mu$s,
Figure 3.1: Comparison between the theoretical model calculations and the experimental results. The solid line represents our theoretical results obtained from Eq. 3.13; the fluctuating bigger dashed lines represents the experimental results [12]; the dotted line, dashed dotted line and small dash line represent the earlier theoretical models as discussed in [12].

Figure 3.2: Variation of Mach number associated with implanted ions with respect to time is shown by the dashed curve. Variation of the Mach number associated with moving sheath edge with respect to time is shown by the solid curve. The value of $u_0$ (although neglected in the calculation) is equal to sonic speed, which for the considered plasma system comes out to be $\approx 2 \times 10^3 \text{m/s}$.

The normalized acoustic wave number can be estimated for given Mach values ($M$).

It is clear from Fig. 3.2 that the values of $M$ for implanted ionic speeds vary from about 45 to 13. The plot represents the Mach value with which the ions reach the wall as
the observed ion current is experimentally measured at the wall. Otherwise, in principle, it should be the same throughout the sheath region by virtue of the assumed ion flux conservation in derivation of Child sheath equation. Now, the above equality yields the normalized acoustic wave number $k \lambda_{De} \approx 0.2 - 0.6$. Buneman instability is unlikely to exist because the required threshold [27] is not satisfied. The displacement of the sheath edge position is plotted against time. The position of the sheath edge at any instant of time is normalized by the Debye length. The initial normalized position of the sheath edge i.e. $\xi(0) = 54.863$.

ion implantation current profile seem to be localized at the sheath edge of the dynamical ion matrix sheath. This is what, we believe, is occurring during the transient sheath dynamics to activate the electrical circuit under consideration and then to regulate the ion implantation current profile.

Figure 3.3: Variation of the normalized sheath edge position is plotted against time. The position of the sheath edge at any instant of time is normalized by the Debye length. The initial normalized position of the sheath edge i.e. $\xi(0) = 54.863$.

Figure 3.4: Frequency response curve for the series LCR electrical circuit of our interest. The solid vertical line represents the resonance at $3.477 \times 10^6$ cycles/second.
3.4 Results and discussions

Numerically solved ion current profile from Eq. 3.13 as shown in Fig. 3.1 is found identical to that of the experimentally observed one. One can thus argue that, the electrical circuit theory offers a more suitable and clear description of the physical phenomena associated with transient sheath dynamics. Here by suitability we mean that the proposed theoretical model approach based on the dual behavior of the Child sheath as mentioned in the introduction section gives rise to the idea about the origin of the transient time scale missing in the earlier theoretical model approaches. Similarly, by clear understanding we mean that the entire temporal relaxation process is governed by the combined views of hydrodynamical and equivalent electrical circuit descriptions of the Child sheath.

Our model consideration is based on the constant values of the electrical circuit elements for applied electrode biasing voltage. As shown in Figs. 3.4 and 3.5 the sheath equivalent series LCR electrical circuit remains in low impedance near resonance value. The two time-scale behavior of the observed current profile is comprehensible under the proposed electrical circuit theory for the positive space charge sheath of transient nature. The observed LF fluctuations are attributed to the current driven acoustic oscillations with significant role of Doppler frequency shift. Of course, thorough investigations will be required to look into the details of the excitation mechanism of the LF fluctuations and its eigen mode behavior. Furthermore, the validity of Child sheath description in an unstable condition of plasma forms an additional problem for critical investigation, which is beyond the scope of the present analysis.

In brief, one can summarize that the proposed electrical circuit model of the positive

![Figure 3.5: Frequency response curve for the series LCR electrical circuit of our interest to depict the minimum value of impedance for long time duration. The solid vertical line represents the resonance at $3.477 \times 10^5$ cycles/second.](image)
ion-rich sheath provides a more suitable and clear understanding of the physical phenomena and associated behavior of the transient sheath dynamics.

3.5 Conclusions and practical remarks

In this chapter the main conclusion, which we arrive at, is that the transient behavior of the sheath can also be explained by the basic idea of current equivalence principle of the equilibrium Child sheath description as included in the previous chapter. The theoretical and numerical analysis along with experimental observation depicts that the transient sheath dynamics is governed by two-time scale transition behavior. It is thus shown that the basic physical principle of transient relaxation of the current in plasma immersed ion implantation experiments is governed by the dual behavior of the Child sheath description. It is noted that the singular behavior in the temporal relaxation of the ion implantation current doesn’t arise. Of course, the problem of sheath edge singularity in static Child sheath approximation remains to be fully resolved.

The validity of theoretical and numerical calculations is obvious as because the observed current profile is recovered. The basic conceptual framework of the model analysis seems to have practical relevance to understand the relaxation process of current in transient sheath experiments. However, more experimental and theoretical investigations are required to understand the basic physical mechanism of the driving process of the instability reported in the experimental findings. Moreover, the role of surface emission current has to be identified, which otherwise seems to be absent in the applied voltage of few kilo volts as per our model analysis, which reproduces the observed current.

References

3.5 Conclusions and practical remarks

University, Sofia, 1999), p. 191.


