Knowledge is that, which liberates the individual soul
In fact, the knowledge is not the correct word for “विद्या“ but it is
used in common usage.
Appendix A

Unified physical model for SDLFI

A.1 Physical model for SDLFI

As discussed in our previous publication (Ram Prakash et al., 2002) it is well understood that the transit time effects at and near the two plasma oscillation frequencies dominate the dynamic sheath properties. In order to study the ion transit time effect near the ion plasma frequency in a positive ion-rich sheath, let us reproduce some mathematical steps of Rosa model (1971) of sheath in response to an applied rf voltage. The model treatment is based on plane sheath approximations and that the electron density is negligible in side the sheath and that the electric field is also negligible at the plasma sheath edge. Under these approximations, the governing equations are reduced to Poisson's equation;

$$\frac{\partial E}{\partial x} = \frac{en_i}{\varepsilon_0}$$  \hspace{1cm} (A.1)

and the total current density equation;

$$J = j_i - j_e + \varepsilon_0 \frac{\partial E}{\partial t}$$ \hspace{1cm} (A.2)

Here $E$ is the sheath associated electric field, $e$ is the electronic charge on the ions, $n_i$ is the ion density, $\varepsilon_0$ is the permittivity of the vacuum, $J$ is the total current density, $j_i$ is the ion current density and $j_e$ is the electron current density.

Eq. A.1 describes the ion space charge distribution under negligible effect of the electron density. Continuity equation of the total current density is invoked as,
A.1 Physical model for SDLFI

\[ \frac{\partial J}{\partial x} = 0 \]  

\((A.3)\)

For frequency well below the electron plasma frequency it is argued that the electron current through the sheath is always in equilibrium with instantaneous field to obey the continuity equation of the form,

\[ \frac{\partial j_e}{\partial x} = 0 \]  

\((A.4)\)

Using Eqs. A.1 - A.4 under specified approximations and after some mathematical manipulation, Llewellyn type equation is derived in Lagrangian variables to describe the kinetic of ion sheath diode,

\[ \frac{\partial^3 x(t, t_0)}{\partial t^3} = \frac{e}{e_0 m_i} (J + j_e) \]  

\((A.5)\)

Here \( t_0 \) is the initial time of the ions at the sheath edge, \( t \) is the time taken by the ions to reach the position \( x \) and that is measured with respect to initial time, \( m_i \) is the ionic mass.

Eq. A.5 corresponds to a dynamic situation where the temporal variation of the particles' acceleration becomes important. Hence the exact solution demands the knowledge of three initial values for physical variables like position, velocity (current) and acceleration. The equilibrium initial values for position, velocities and accelerations at \( t = t_0 \) are \( x = 0, v_i0 = c_a, a_i0 = 0 \) respectively.

From Eq. A.5, the equilibrium solution gives the sheath width \( (X_0) \) and normalized amplitude of the potential drop inside the sheath \( (\eta_0 = eV_G/kT_e) \) of the dc sheath at the wall and these are given below,

\[ X_0 = \left( 1 + \frac{\lambda^2}{6} \right) c_s T_0 \omega_{pi} \]

and \( \eta_0 = \left( 1 + \frac{\lambda^2}{4} \right) \frac{\lambda^2}{2} \), \( c_s \) is the acoustic speed in bulk plasma, \( \lambda = \omega_{pi} T_0 \), \( \omega_{pi} \) is the ion plasma oscillation frequency, \( T_0 \) is defined as the ion transit time and \( V_G \) is the applied voltage at the grid wall.

Now, assuming the fluctuations of external (applied) or internal (generated by SDLFI) origin of rf voltage to ion sheath to vary as \( \propto \exp(i\omega t) \), the perturbed solution of position
A.1 Physical model for SDLFI

can be derived from Eq. A.5. Initial perturbed values of position, velocity and acceleration
of the ions are determined as $x_1 = 0, v_{i1} = c_s$ and $a_{i1} = -\frac{i\epsilon}{\epsilon_0 m_i \omega} (J_1 + J_{e1})$ respectively.

Subsequently, the perturbed potential drop across the sheath can be derived to express the
correlation between the potential and current perturbations. This is given below,

$$V_1 = \frac{C_s}{\epsilon_0 \omega^2} \Psi (\lambda, \theta) (J_1 + J_{e1})$$

Here, $\omega$ is the rf frequency, $\theta = \omega \tau_w$ and subscript "1" denotes the perturbed variables and

$$\Psi (\lambda, \theta) = i \theta + \frac{\lambda^2}{\omega^2} \left( i \theta^3 - \frac{1}{6} + i \theta \{ \exp (-i \theta) + 1 \} + 2 \{ \exp (-i \theta) - 1 \} \right)$$

Finally, the sheath admittance $(Y = J_1/V_1)$ can be evaluated in the following form,

$$\frac{Y}{\epsilon_0 \omega^2 m_i / c_s} = \exp (\eta^*_0 - \eta_0) - \frac{\theta^2}{\lambda^2} \Psi^{-1} (\lambda, \theta) \quad (A.6)$$

The floating potential $(\eta^*_0)$ on the grid electrode is defined as follows;

$$\eta^*_0 = \frac{1}{2} \ln \left( \frac{m_i}{2 \pi m_e} \right)$$

It is now clear (from A.6) that the response dynamics of a rf sheath can be discussed in
terms of an equivalent circuit whose active elements can be determined from this equation
by considering sheath admittance $Y = Ge + Gi + iBi$.

Here $Ge = \left( \frac{\epsilon_0 \omega_{pi}^2}{c_s} \right) \exp (\eta^*_0 - \eta_0)$, independent of rf frequency, corresponds to sheath conductance by electrons. The value of $Ge$ depends upon $\eta_0$ or the applied dc biasing voltage to the grid electrode. This is to note that the value of $Ge$ can be made increasingly small for large negative biasing to the grid electrode. Here, $Gi + iBi = \left( \frac{\epsilon_0 \omega_{pi}^2}{c_s} \right) \psi^{-1} (\lambda, \theta)$

which is the combination of $Gi$, the ion contribution of sheath conductance and $Bi$, the
sheath susceptance originating from the ion dynamics.

Now, for large negative voltage to the grid electrode, the first term on RHS of Eq. A.6 is
approximated to zero. Then for low frequency rf source (\(\omega < \tau^{-1}_w\)) condition, the remaining
second term of A.6 is binomially expanded and simplified to find out an explicit expression
for the general impedance $(Z(\omega) = 1/Y)$ of ion-rich sheath as given below,
Figure A.1: Schematic diagram of an equivalent series resonant LCR circuit associated with ion-rich sheath, showing an ion-rich sheath on a wall with applied rf source of signal $\omega$ (a), and the associated circuit elements and an internal rf source of frequency $\omega$ (b).

$$Z(\omega) = \frac{c_s^2 \lambda^2 	au_{fe}^2}{12\epsilon_0} + i\omega \left(-\frac{c_s^2 \tau_{fe}^2 \lambda^2}{40\epsilon_0}ight) - i\frac{X_0}{\omega \left[\epsilon_0 \left(1 + \frac{\lambda^2}{6}\right)\right]}.$$  

Now, if we compare this expression with the known form of the impedance of the conventional series resonant LCR electronic circuit, the equivalent sheath associated capacitance ($C_{ulf}$), inductance ($L_{ulf}$), and resistance ($R_{ulf}$) for low frequency rf source of internal origin or of external origin can be directly written as,

$$C_{ulf} = \frac{\epsilon_0 \left(1 + \frac{\lambda^2}{6}\right)}{X_0}, \quad L_{ulf} = -\frac{c_s^2 \tau_{fe}^2 \lambda^2}{40\epsilon_0}, \quad \text{and} \quad R_{ulf} = \frac{c_s^2 \lambda^2 \tau_{fe}^2}{12\epsilon_0}.$$  

Here, the subscript "ulf" refers to the internal low frequency rf source.

This is to comment that the sheath equivalent electrical circuit behavior of the diode-like positive ion-rich sheath should be observable only when ac source modulation of either external origin or of internal origin is introduced to the sheath. This seems to occur at the plasma-sheath boundary in the form of rf probe sheath resonance behavior at or near the ion plasma oscillation frequency. The schematic diagram of equivalent series resonant LCR circuit behavior of ion-rich sheath is shown in Fig. A.1.
A.2 Unified empirical model for SDLFI

As discussed by us (Ram Prakash et al., 2002), it is clear that the positive ion rich sheath reveals its diode-like equivalent electrical circuit behavior when it is perturbed by some rf source of either external or of internal origin. In low frequency limit of the rf voltage, $L, C, R$ elements characterize the equivalent circuit, which render the positive ion-rich sheath to behave as a resonant circuit.

Figs. A.2 - A.4 provide quantitative values of the associated $L, C$ and $R$ elements respectively for different values of the plasma parameters. Fig. A.5 as shown in next page depicts the variation of sheath admittance versus the normalized rf frequency. From this, one can determine the resonance frequency of the specified circuit. Variation of resonant frequency with applied voltage is shown in Fig. A.6. Using the known values of $L_{uf}, C_{uf}, R_{uf}$ for realistic plasma parameters of DP device produces these plots. It is found that the circuit resonant frequencies are in reasonably good agreement (within order of magnitude) with ion-beam plasma model prediction by Dwivedi and co-authors (1999). Thus the matching of the resonant eigen mode frequencies predicted by circuit model as well as by ion-beam plasma model makes a case for forwarding the following hypothesis to unify both the models to discuss the self-consistent underlying physics of SDLFI.
According to our hypothesis, we argue that the ion-beam plasma model provides a physical basis of source mechanism to generate an internal low frequency rf source. Subsequently, the internal rf source couples with ion-rich sheath diode to transform the system into an active "monotron" oscillator. This is what, we believe, is the phenomenon occurring in the experimental situations of negatively biased grid in DP with external or internal source of the sheath potential asymmetry. Nevertheless, more experimental and theoretical investigations will be needed to arrive at some final conclusions about the physical reality of our hypothesis.

The consideration of kinetic treatment for bounce oscillations of the trapped ions in negative potential well around the biased grid may indeed provide a more suitable basis to
A.3 The proposed hypothesis

Figure A.6: Variation of the normalized resonant frequency ($\omega$) and $\lambda$ with grid-biasing voltage ($V_g$); * corresponds to empirical ion transit time model as suggested by Barrett and Greaves [1989]. In their model, a discrepancy was arising due to improper mathematical treatment, which was overcome by introducing a fitting parameter $\beta$ in the concerned equation (in our case, we have chosen $\beta = 0.67$). ▲ corresponds to circuit model calculations. × corresponds to real experimental values for double plasma device with $V_s$ (source biasing voltage) = 10 V. The vertical bars represent the error of 10%. ■ corresponds to $\lambda$ values.

Comment on the grid biasing threshold value for occurrence of SDLFI. This is not included in the model theoretical treatment of the ion-beam plasma model of Dwivedi and co-authors (1999). Since the grid biasing potential appears in the equivalent circuit formulation of the diode-like ion-rich sheath and calculated resonant frequency agrees with the experimental findings, it is intuitively felt that our hypothesis lies on sound physical footing to offer a valuable scope for comprehensible understanding of the basic physics of SDLFI in terms of circuit resonance too. This part of work is included in chapter 3 of Ph.D thesis titled "Study of the sheath behavior and spectroscopic measurements in plasmas" of Ram Prakash submitted to Dr. B. R. Ambedkar University, Agra in February, 2002.

References

Prakash, R., Sarma, A., Dwivedi, C. B., Deka, U., Singha, B., Bujarbarua, S. and Upad-
A.3 The proposed hypothesis
