Preface

Numerical Analysis is an emerging area of research nowadays. Basically, industrial problems are modelled into mathematical problems, which are often partial differential equations (PDE) with certain initial, boundary or initial-boundary conditions. Finding exact solutions of these models is quite formidable and sometimes impossible. To overcome this difficulty, we proceed to find approximate solutions of these models with the help of numerical methods. During a long period of time, attempts have been taken to develop techniques and methods to find approximate solutions of these models and to examine whether the approximate solution converges to the exact solution or not. The error estimates between the exact and the approximate solutions have been studied extensively in the recent past. New endeavours have been made to find methods and techniques which require minimum time in evaluating the models with maximum accuracy. These methods, techniques and tools constitute the subject of "Scientific Computation". In late eighties, a robust method, namely, wavelet method came into existence. The study of this field requires deep knowledge of Functional Analysis and Linear Algebra besides Numerical Analysis.
Numerical solution of a PDE problem requires a discretization method that reduces the problem to a problem of finding the solution of an equivalent system of algebraic equations (SAE) in a finite number of unknowns. PDE's involve functions that model spatially distributed, and possibly time varying, physical quantities such as temperature, velocity, or displacement, and differential operators that model the physical processes which determine the static or dynamic behaviour of these quantities. Discretization methods represent the solution function $u$ by an approximation $v$ defined by a finite number of parameters. Then the differential operator and the constraints such as initial and boundary conditions are approximated by algebraic operations involving these parameters. This results in a SAE whose exact solution determines $v$. A discretization method is effective, if the truncation error $u - v$ tends to zero 'rapidly' as the number of discretization parameters increases. It is then only required to obtain an approximate solution $w$ of the SAE such that the algebraic error $v - w$ is comparable to the truncation error $u - v$. Since the total error $e = u - w$ satisfies $e = \text{truncation error} + \text{algebraic error}$, therefore, $v - w \approx u - v$, and then additional computation is more effectively utilized by increasing the number of discretization parameters to decrease the truncation error.

There are two types of wavelet methods in practice, namely, Wavelet-Galerkin Method and Wavelet-Collocation Method. In this thesis, we attempt numerical solutions of PDE problems by using wavelet-Galerkin method. This thesis contains six chapters.
Chapter 1 is introductory. All the preliminary results and concepts related to this thesis are discussed here. Section 1.2 recalls the distribution theory and Sobolev spaces which is an integral and important part in the wavelet solutions of PDE problems. Section 1.3 describes wavelets and multiresolution analysis briefly. In Section 1.4, we give a detailed account of the variational formulations of PDE problems including their approximations in finite dimension. Section 1.5 is devoted to a short introduction to the classical variational methods and the finite difference method for the solutions of PDE problems. In Chapter 2, all the theoretical results are summarized. In section 2.2, a brief introduction to Daubechies wavelets is presented. Section 2.3 discusses the problem of approximation of various function (Sobolev) spaces, useful for Galerkin solution of PDE problems, in Daubechies wavelet bases, using two approaches. In Chapter 3, we solve second order elliptic problems using the wavelet-Galerkin method investigated here. Section 3.2 discusses the variational formulations of the problems to be solved. In sections 3.3 and 3.4, we solve the problems using two different techniques. In Chapter 4, we solve fourth order elliptic problems using the wavelet-Galerkin method similar to second order problems in Chapter 3. In Chapter 5, we investigate a method for the solution of parabolic problems in one space dimension by using semidiscrete approximations. The space direction is discretized by wavelet-Galerkin method and the time direction is discretized by classical finite difference schemes. In Section 5.2, we formulate the problems to be solved.
Section 5.3 discusses the spatial approximation of the problems using wavelet-Galerkin method. In Section 5.4, we give an outline of the time discretization of the problems using classical finite difference schemes. Section 5.5 consists of numerical experiments of some test examples which contains problems of diffusion, diffusion-reaction, convection-diffusion and convection-diffusion-reaction with various types of boundary conditions. In Chapter 6, we solve hyperbolic problems in one space dimension by using semidiscrete approximations similar to parabolic problems in Chapter 5. Here also, the space direction is discretized by wavelet-Galerkin method and the time direction is discretized by some classical finite difference schemes, other than those used in Chapter 5. In this chapter, we perform numerical tests for some wave problems.