Appendix A

The average deviation of non normalized Cross-correlation

We define the non normalized Cross-correlation between two time-series as

\[ c_{XY} = \frac{1}{N^2} \sum_{k=-N/2}^{N/2-1} \hat{X}_k \hat{Y}_k^* = \frac{1}{N^2} \sum_{k=-N/2}^{N/2-1} \hat{X}_k \hat{Y}_{-k} \] (A.1)

Without loss of generality the two times-series may be split into coherent and incoherent components as

\[ \hat{X}_k = \hat{X}_k^c + \hat{X}_k^{nc} \]
\[ \hat{Y}_k = A \hat{X}_k^c + \hat{Y}_k^{nc} \] (A.2)

such that the ensemble averages \(< \hat{X}_k^c \hat{X}_{-k}^{nc} >, < \hat{X}_k^c \hat{Y}_{-k}^{nc} > \) and \(< \hat{X}_k^{nc} \hat{Y}_k^{nc} > \) are all zero. Then \(< c_{XY} > = A \sum < \hat{X}_k^c \hat{X}_{-k}^c > \) and

\[ < c_{XY} >^2 = \frac{A^2}{N^4} \sum \sum < \hat{X}_k^c \hat{X}_{-k}^c > < \hat{X}_m^c \hat{X}_{-m}^c > = \frac{A^2}{N^4} \sum \sum < (\hat{X}_k^c)^2 > < (\hat{X}_m^c)^2 > \] (A.3)
where the implicit index of summation over \( k \) and \( m \) has been omitted for clarity. The average of the square,

\[
< c_{xy}^2 > = \frac{1}{N^4} \sum_k \sum_{m \neq k} \bar{X}_k \bar{Y}_{-k} \bar{X}_m \bar{Y}_{-m} \\
= \frac{1}{N^4} \left\{ A^2 \sum_{k \neq -m} < (\bar{X}_k)^2 > < (\bar{X}_m)^2 > \\
+ A^2 \sum_k < (\bar{X}_k^c)^2 > + \sum_k < (\bar{Y}_k^c)^2 > < (\bar{Y}_k^c)^2 > \\
+ A^2 \sum_k < (\bar{X}_k^c)^2 > < (\bar{X}_k^c)^2 > \\
+ \sum_k < (\bar{X}_k^c)^2 > < (\bar{Y}_k^c)^2 > \right\} \quad (A.4)
\]

Now since \(< (\bar{X}_k^c)^4 > = 2 < (\bar{X}_k^c)^2 >^2 \), the above expression can be simplified to

\[
< c_{xy}^2 > = \frac{A^2}{N^4} \sum_k < (\bar{X}_k^c)^2 > < (\bar{X}_m^c)^2 > + \frac{1}{N^4} \sum_k < (\bar{X}_k^c)^2 > < (\bar{Y}_k^c)^2 > \quad (A.5)
\]

Thus,

\[
(\Delta c_{xy})^2 = < c_{xy}^2 > - < c_{xy} >^2 = \frac{1}{N^4} \sum_{k=-N/2}^{N/2-1} < |\bar{X}_k|^2 > < |\bar{Y}_k|^2 > \quad (A.6)
\]

If \( \bar{X}_k = \bar{Y}_k \) (i.e. the two time-series are identical) then \( c_{xy} = \sigma_X^2 \) and hence

\[
(\Delta \sigma_X^2)^2 = \frac{1}{N^4} \sum_{k=-N/2}^{N/2-1} < |\bar{X}_k|^2 >^2 \quad (A.7)
\]

In the special case that the time-series is produced by a stochastic white noise (i.e. \(|\bar{X}_k|^2 \) is a constant independent of \( k \)) then

\[
\Delta \sigma_X^2 = \frac{1}{\sqrt{N}} \sigma_X^2 \quad (A.8)
\]