Integrated inventory policies for stock-dependent demand under trade credit
2.0 Introduction

The classical EOQ model is developed under the assumption that buyer must settle the payment due immediately when units are received in the inventory system. To attract the buyers, the vendor uses promotional tool viz permissible delay period when the buyer’s order quantity is more than pre-specified quantity. In this chapter, we discuss four models in which we analyze integrated inventory policy for vendor-buyer when demand is stock-dependent and trade credit is linked to order quantity. The joint total profit is maximized to determine buyer’s order quantity and the number of shipments from the vendor to the buyer during one cycle.

Four models are formulated under the following sections (viz.):

Model 2.1 Optimal integrated inventory policy for stock-dependent demand when trade credit is linked to order quantity.

Model 2.2 Optimal integrated inventory policy for deteriorating items with stock-dependent demand and trade credit linked to order quantity.

Model 2.3 Optimal pricing, shipments and ordering policies for single supplier single-buyer inventory system with price sensitive stock-dependent demand and order-linked trade credit.

Model 2.4 Optimal inventory policies for single-supplier single-buyer deteriorating items with price-sensitive stock-dependent demand and order linked trade credit.
2.1 Optimal Integrated Inventory Policy for Stock-Dependent Demand When Trade Credit is Linked to Order Quantity

In this section, we analyze an integrated single-vendor single-buyer inventory model when the demand rate is stock-dependent, the production rate is finite and proportional to the demand rate and trade credit is permitted only if buyer orders more units than the pre-specified order quantity by the vendor. The joint total profit per unit time is maximized with respect to order quantity and number of shipments from vendor to the buyer. A computational procedure is outlined to find the best optimal solution. The numerical examples and sensitivity analysis are given to validate the developed model.

2.1.1 Assumptions and Notations

The following additional assumptions and notations other than those given in A.1 and N.1 are used to formulate the proposed model:

2.1.1.1 Assumptions

1. The buyer qualifies for trade credit offer if order is equal or larger than the pre-specified quantity $Q_d$ by the vendor. Otherwise, the buyer must use cash-on-delivery option.

2. During the credit period, the buyer earns interest at the rate $I_{be}$ per unit on the generated revenue. At the end of the credit period, the buyer settles the payments due against the purchases made and incurs an opportunity cost at a rate of $I_{bp}$ for unsold items in stock.
2.1.1.2 Notations

\[ R(I(t)) : \text{Stock-dependent demand rate; } \alpha + \beta I(t) \text{ where } \alpha > 0 \text{ is constant scale demand and } 0 < \beta < 1 \text{ is stock-dependent parameter} \]

\[ A_v : \text{Vendor's setup cost per set-up (in $)} \]
\[ A_b : \text{Buyer's ordering cost per order (in $)} \]
\[ C_p : \text{Production cost per unit (in $)} \]
\[ C_b : \text{Buyer's purchase cost per unit (in $)} \]
\[ s : \text{The unit retail price to customers (in $), where } s > C_b > C_p \]
\[ I_v : \text{Vendor's inventory holding cost rate per unit per annum (in $), excluding interest charges} \]
\[ I_b : \text{Buyer's inventory holding cost rate per unit per annum (in $), excluding interest charges} \]
\[ I_{vp} : \text{Vendor's opportunity cost / $ / unit time} \]
\[ I_{bp} : \text{Buyer's opportunity cost / $ / unit time} \]
\[ \rho : \text{Capacity utilization which is ratio of demand to the production rate; } \rho < 1 \text{ and known constant} \]
\[ M : \text{Allowable credit period for the buyer offered by the vendor (in days or years)} \]
\[ Q_d : \text{Pre-specified order quantity to qualify for offer of trade credit (in units)} \]
\( T_d \): The time length when \( Q_d \) units are depleted to zero (in days or years)

\( TVP \): Vendor’s total profit per unit time (in $)

\( TBP \): Buyer’s total profit per unit time (in $)

\( \pi \): (Sum of \( TVP \) and \( TBP \)) joint total profit per unit time (in $)

\( Q \): Buyer’s order quantity per order (in units) (a decision variable)

\( n \): Number of shipments from vendor to the buyer (a decision variable)

### 2.1.2 Mathematical Model

The rate of change of inventory at any instant of time can be described by the differential equation

\[
\frac{dI(t)}{dt} = -(\alpha + \beta I(t)), \quad 0 \leq t \leq T; \quad I(0) = Q \quad \text{and} \quad I(T) = 0
\]

Using \( I(T) = 0 \), the solution of the differential equation (Shah, 2011) is

\[
I(t) = \frac{\alpha}{\beta} \left( e^{\beta(T-t)} - 1 \right), \quad 0 \leq t \leq T.
\]

The units to be purchased is \( Q = I(0) = \frac{\alpha}{\beta} \left( e^{\beta T} - 1 \right) \).

#### 2.1.2.1 Vendor’s Total Profit per Unit Time

The total profit per unit time for the vendor comprises of sales revenue, set-up cost, holding cost and opportunity cost as follows:

1. Sales revenue : The total sales revenue per unit time is \( (C_b - C_p) \frac{Q}{T} \)
2. Set-up cost : \( nQ \) units are manufactured in one production run by the vendor. Therefore, the set-up cost per unit time is \( \frac{A_v}{nT} \)
(3) Holding cost: Using Joglekar (1988), the vendor’s average inventory per unit time is

\[ C_p(I_v + I_{vp}) \left[ (n-1)(1-\rho) + \rho \right] \frac{\alpha(e^{\beta T} - \beta T - 1)}{\beta^2 T} \]

(4) Opportunity cost: If pre-specified quantity \( Q_d \) or more units are ordered by the buyer, the credit period of \( M \)-units is permissible to settle the account. In this scenario, vendor endures a capital and payment received. Equivalently, when \( T \geq T_d \), the delay in payment is permissible and corresponding opportunity cost per unit time is \( \frac{C_b I_{vp} Q M}{T} \). On the other hand, when \( T < T_d \) the vendor receives payments on delivery and so no opportunity cost will occur.

Hence, the total profit per unit time for the vendor is

\[ TVP(n) = \begin{cases} 
TVP_1(n), & T < T_d \\
TVP_2(n), & T \geq T_d 
\end{cases} \tag{2.1.1} \]

where

\[ TVP_1(n) = \left( \frac{C_b - C_p}{T} \right) Q - \frac{A_v}{nT} - \frac{C_p(I_v + I_{vp}) \left[ (n-1)(1-\rho) + \rho \right] \alpha(e^{\beta T} - \beta T - 1)}{\beta^2 T} \tag{2.1.2} \]

\[ TVP_2(n) = \left( \frac{C_b - C_p}{T} \right) Q - \frac{A_v}{nT} - \frac{C_p(I_v + I_{vp}) \left[ (n-1)(1-\rho) + \rho \right] \alpha(e^{\beta T} - \beta T - 1)}{\beta^2 T} - \frac{C_b I_{vp} Q M}{T} \tag{2.1.3} \]
2.1.2.2 Buyer's Total Profit per Unit Time

The total profit per unit time for the buyer comprises of sales revenue, ordering cost, holding cost, opportunity cost and interest earned. These costs are computed as follows:

(1) Sales revenue: The total sales revenue per unit time is \( (s - C_b) \frac{Q}{T} \)

(2) Ordering cost: The ordering cost per unit time is \( \frac{A_b}{T} \)

(3) Holding cost: The buyer's holding cost (excluding interest charges) per unit time is

\[
\frac{C_b I_b \alpha (e^{\beta T} - \beta T - 1)}{\beta^2 T}
\]

(4) Opportunity cost: Based on the lengths of \( T \) and \( T_d \), the following four cases arises (i) \( 0 < T < T_d \) (ii) \( T_d \leq T \leq M \) (iii) \( T_d \leq M \leq T \) (Figure 2.1.1) and (iv) \( M \leq T_d \leq T \). The cases (iii) and (iv) are similar.

Opportunity cost per unit time:

\[
= \begin{cases} 
\frac{C_b I_{bp} Q}{T}, & 0 < T < T_d \\
0, & T_d \leq T \leq M \\
\frac{C_b I_{bp}}{\beta^2 T} - \alpha \left( e^{\beta (T - M)} - \beta (T - M) - 1 \right), & T_d \leq M \leq T \text{ or } M \leq T_d \leq T
\end{cases}
\]
Interest earned: As discussed in opportunity cost, interest earned per unit time in all the four cases is as follows.

Interest earned per unit time:

\[
\begin{align*}
0, & \quad 0 < T < T_d \quad \text{(because payment is to be made on delivery)} \\
= \frac{s I_{be}}{T} \left( \int_0^T R(I(t)) t \, dt + Q(M - T) \right), & \quad T_d \leq T \leq M \quad \text{(figure 2.1.2)} \\
= \frac{s I_{be}}{T} \left( \int_0^M R(I(t)) t \, dt \right), & \quad T_d \leq M \leq T \quad \text{or} \quad M \leq T_d \leq T \quad \text{(figure 2.1.3)}
\end{align*}
\]
Figure 2.1.2 Interest earned by buyer when $T_d \leq T \leq M$

Figure 2.1.3 Interest earned by buyer when $T_d \leq M \leq T$
Hence, the buyer’s total profit per unit time is

\[
TBP(T) = \begin{cases} 
TBP_1(T), & 0 < T < T_d \\
TBP_2(T), & T_d \leq T \leq M \\
TBP_3(T), & T_d \leq M \leq T \\
TBP_4(T), & M \leq T_d \leq T 
\end{cases}
\] (2.1.4)

where

\[
TBP_1(T) = \left(\frac{s - C_b}{T}\right) - \frac{A_b}{T} - \frac{C_b I_b \alpha}{\beta^2 T} \left(e^{\beta T} - \beta T - 1\right) - \frac{C_b I_{bp} Q}{T} 
\] (2.1.5)

\[
TBP_2(T) = \left(\frac{s - C_b}{T}\right) - \frac{A_b}{T} - \frac{C_b I_b \alpha}{\beta^2 T} \left(e^{\beta T} - \beta T - 1\right) + \frac{S_{be}^T}{T} \left(\int_0^T R(I(t))dt + Q(M - T)\right) 
\] (2.1.6)

\[
TBP_3(T) = TBP_4(T) = \left(\frac{s - C_b}{T}\right) - \frac{A_b}{T} - \frac{C_b I_b \alpha}{\beta^2 T} \left(e^{\beta T} - \beta T - 1\right) \\
- \frac{C_b I_{bp} \alpha}{\beta^2 T} \left(e^{\beta(T-M)} - \beta(T - M) - 1\right) \\
+ \frac{S_{be}^\alpha}{\beta^2 T} \left(e^{\beta T} - (1 + \beta M)e^{\beta(T-M)}\right) 
\] (2.1.7)

### 2.1.2.3 Joint Total Profit per Unit Time

In integrated system, the vendor and buyer decide to take joint decision which maximizes the profit of the supply chain. The joint total profit per unit time for the integrated system is

\[
\pi(n,T) = \begin{cases} 
\pi_1(n,T) = TVR_1(n) + TBP_1(T), & 0 < T < T_d \\
\pi_2(n,T) = TVP_2(n) + TBP_2(T), & T_d \leq T \leq M \\
\pi_3(n,T) = TVP_2(n) + TBP_3(T), & T_d \leq M \leq T \\
\pi_4(n,T) = TVP_2(n) + TBP_3(T), & M \leq T_d \leq T. 
\end{cases}
\] (2.1.8)
where

\[
\pi_1(n,T) = (s - C_p - C_b I_{bp}) \frac{Q}{T} - \frac{A}{T} - \frac{1}{T}(\phi + \psi) \alpha \left( e^{\beta T} - \beta T - 1 \right)
\]

(2.1.9)

\[
\pi_2(n,T) = \left( s - C_p - (C_b I_{vp} - sI_{be})M \right) \frac{Q}{T} - sI_{be}Q - \frac{A}{T} - \frac{1}{T}(\phi + \psi) \alpha \left( e^{\beta T} - \beta T - 1 \right) + \frac{sI_{be}}{T} \int_0^T R(I(t)) dt
\]

(2.1.10)

\[
\pi_3(n,T) = \left( s - C_p - C_b I_{vp}M \right) \frac{Q}{T} - \frac{A}{T} - \frac{1}{T}(\phi + \psi) \alpha \left( e^{\beta T} - \beta T - 1 \right) - \frac{C_b I_{bp}}{\beta^2 T} \left( e^{\beta(T-M)} - \beta(T-M) - 1 \right) + \frac{sI_{be}}{\beta^2 T} \alpha \left( e^{\beta T} - (1 + \beta M) e^{\beta(T-M)} \right)
\]

(2.1.11)

where

\[
\bar{A} = A_b + \frac{A_v}{n}
\]

\[
\phi = \frac{C_p \left( I_v + I_{vp} \right) \left[ (n-1)(1-\rho) + \rho \right]}{\beta^2}
\]

\[
\psi = \frac{C_b I_{bp}}{\beta^2}
\]

### 2.1.2.4 Computational Procedure

For fixed \( T \), we note that \( \pi(n,T) \) is a concave function of \( n \) because

\[
\frac{\partial^2 \pi(n,T)}{\partial n^2} = -\frac{2A_v}{n^3 T} < 0.
\]

Therefore, to find optimum number of shipments \( n^* \), we will have a local optimal solution. The optimum value of cycle time can be obtained by setting \( \frac{\partial \pi}{\partial T} = 0 \) for fixed \( n \).
Algorithm:

Step 1: Set parametric values.

Step 2: Compute $T_d$ using $\frac{1}{\beta} \ln \left(1 + \frac{\beta Q_d}{\alpha}\right)$ for given value of $Q_d$.

Step 3: Set $n = 1$.

Step 4: Knowing $T_d$ and $M$, compute $T$ by solving $\frac{\partial \pi_j}{\partial T} = 0$ for $j = 1, 2, 3$.

Step 5: Find corresponding profit $\pi_j$ for $j = 1, 2, 3$.

Step 6: Increment $n$ by 1.

Step 7: Repeat steps 4 - 6 until $\pi(n-1, T(n-1)) \leq \pi(n, T(n)) \geq \pi(n+1, T(n+1))$.

Once the optimal solution $\left(n^*, T^*\right)$ is obtained, the optimal order quantity can be computed.

### 2.1.2.5 Numerical Examples and Interpretations

**Example 2.1.1:** Consider, $\alpha = 10,000$ units, $\beta = 10\%$, $\rho = 0.7$, $C_b = $10/unit, $C_p = $5/unit, $A_v = $400/setup, $A_b = $50/order, $I_v = 10\%$/unit/annum, $I_b = 10\%$/unit/annum, $I_{bp} = 8\%$/unit/annum, $I_{be} = 5\%/$$/annum, $I_{vp} = 2\%$/unit/annum, $s = $25/unit and $M = 30$ days.

The optimal shipments and ordering units with buyer, vendor and joint profit for different values of $Q_d$ are exhibited in Table 2.1.1.
Table 2.1.1: Optimal solutions for different $Q_d$

<table>
<thead>
<tr>
<th>$Q_d$</th>
<th>$Q^*$</th>
<th>$n^*$</th>
<th>$T^*$ (days)</th>
<th>Profit($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Buyer</td>
</tr>
<tr>
<td>1,000</td>
<td>1,369</td>
<td>5</td>
<td>49.65</td>
<td>1,50,150</td>
</tr>
<tr>
<td>2,000</td>
<td>1,369</td>
<td>5</td>
<td>49.65</td>
<td>1,50,150</td>
</tr>
<tr>
<td>3,000</td>
<td>3,000</td>
<td>2</td>
<td>124.46</td>
<td>1,50,051</td>
</tr>
<tr>
<td>4,000</td>
<td>4,000</td>
<td>2</td>
<td>124.46</td>
<td>1,50,051</td>
</tr>
<tr>
<td>5,000</td>
<td>1,369</td>
<td>5</td>
<td>49.65</td>
<td>1,50,150</td>
</tr>
<tr>
<td>6,000</td>
<td>1,369</td>
<td>5</td>
<td>49.65</td>
<td>1,50,150</td>
</tr>
</tbody>
</table>

From Table 2.1.1, it is seen that the vendor's total profit and joint total profit of the system increases with increase in $Q_d$ and then further increase in pre-specified units lower their profits whereas for the buyer, it is opposite trend. It is seen that the buyer's optimal order quantity $Q^*$ is equal to $Q_d$ and less than $Q_d$ when $Q_d \geq 5,000$. Thus, vendor is advised to set effective threshold. If the threshold set by the vendor is too high, the buyer will be reluctant to order a quantity greater than the threshold to take advantage of delayed payments.
The concavity of joint total profit is shown in figure 2.1.4.

![Figure 2.1.4 Concavity of joint profit w. r. t. \(n\) and \(T\)](image)

**Example 2.1.2:** Consider the data given in Example 2.1.1. We study the effect of delayed payments for \(Q_d = 3,000\) units.

Table 2.1.2: Optimal solutions for different \(M\) \((Q_d = 3,000)\)

<table>
<thead>
<tr>
<th>(M) (days)</th>
<th>(Q^*)</th>
<th>(n^*)</th>
<th>(T^*) (days)</th>
<th>Case</th>
<th>Buyer</th>
<th>Vendor</th>
<th>Joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3,000</td>
<td>2</td>
<td>126.36</td>
<td>(M \leq T_d \leq T)</td>
<td>1,49,796</td>
<td>49,136</td>
<td>1,98,932</td>
</tr>
<tr>
<td>30</td>
<td>3,000</td>
<td>2</td>
<td>124.46</td>
<td>(M \leq T_d \leq T)</td>
<td>1,50,051</td>
<td>49,074</td>
<td>1,99,125</td>
</tr>
<tr>
<td>40</td>
<td>3,000</td>
<td>2</td>
<td>121.63</td>
<td>(M \leq T_d \leq T)</td>
<td>1,50,319</td>
<td>49,008</td>
<td>1,99,327</td>
</tr>
<tr>
<td>50</td>
<td>3,000</td>
<td>2</td>
<td>117.81</td>
<td>(M \leq T_d \leq T)</td>
<td>1,50,603</td>
<td>48,938</td>
<td>1,99,541</td>
</tr>
<tr>
<td>60</td>
<td>3,000</td>
<td>2</td>
<td>112.89</td>
<td>(M \leq T_d \leq T)</td>
<td>1,50,904</td>
<td>48,863</td>
<td>1,99,767</td>
</tr>
</tbody>
</table>
From Table 2.1.2, it is observed that longer credit period increases buyer’s total profit and joint profit of the supply chain. The longer credit period reduces vendor’s total profit because payment will be received late for the purchases made.

**Example 2.1.3:** In this example, we carry out sensitivity analysis to find the critical inventory parameters. The changes in the optimum cycle time and joint profit are studied by varying inventory parameters as $-20\%, -10\%, 10\%$ and $20\%$, one at a time. The variations in cycle time are exhibited in figure 2.1.5 and objective function is displayed in figure 2.1.6.

![Variations in cycle time](image)

Figure 2.1.5 Variations in cycle time

From the figure 2.1.5, it is observed that increase in set-up cost, ordering cost increases cycle time moderately and increase in retail price and display parameter increases cycle time drastically. Cycle time have negative impact of scale demand, production cost, buyer's purchase cost, buyer's inventory holding cost, buyer's opportunity cost.
It is observed from figure 2.1.6 that joint profit has significant positive impact of scale demand and retail price set by the buyer. It is evident that both the player should take advantage of demand increase and setting agreeable selling price. Production cost of supplier reduced joint total profit. It is advised to the supplier to use advanced technology which reduces this production cost. Other inventory parameters have very small contribution in increasing profit of the supply chain.

**Example 2.1.4:** In Table 2.1.3, we compare independent vs. joint decision, for pre-specified quantity $Q_d = 3,000$ units at which buyer qualities for getting delay period facility.
Table 2.1.3: Optimal solution of independent and scenario

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Total Shipments</th>
<th>Buyer</th>
<th>Vendor</th>
<th>Joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Shipments</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ordering Quantity</td>
<td>1,548</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cycle Time (days)</td>
<td>56</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Annual Profit ($)</td>
<td>1,50,205</td>
<td>48,828</td>
<td>1,99,033</td>
</tr>
<tr>
<td>Integrated</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Shipments</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ordering Quantity</td>
<td>3,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cycle Time (days)</td>
<td>124</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Annual Profit ($)</td>
<td>1,50,050</td>
<td>49,074</td>
<td>1,99,124</td>
</tr>
<tr>
<td></td>
<td>Readjusted Total Annual Profit ($)</td>
<td>1,50,274</td>
<td>48,850</td>
<td>1,99,124</td>
</tr>
</tbody>
</table>

where

Buyer's profit:

\[ \pi(n,T) \times \frac{TBP(P,T)}{TBP(P,T) + TVP(n)} = 1,99,124 \times \frac{1,50,205}{(1,50,205 + 48,828)} = 1,50,274 \]

Supplier's profit:

\[ \pi(n,T) \times \frac{TVP(n)}{TBP(P,T) + TVP(n)} = 1,99,124 \times \frac{48,828}{(1,50,205 + 48,828)} = 48,850 \]

Table 2.1.3 shows that the total annual profit under joint decision $1,99,125 ($=1,50,051+$49,074) which is greater than the total profit under independent decision $1,99,033 ($=1,50,205+$48,828). It establishes that joint decision is advantageous to both the players. The last row of Table 2.1.3 is about readjustment of the profits (Goyal (1976)) to encourage players for joint decision.
2.2 Optimal Integrated Inventory Policy for Deteriorating Items with Stock-Dependent Demand and Trade Credit Linked to Order Quantity

In this section, we study effect of constant rate of deterioration in model 2.1.

2.2.1 Assumptions and Notations

The following additional assumptions and notations other than those given in A.1, N.1 and section 2.1.1 are used to formulate the proposed model.

2.2.1.1 Assumptions

1. The units in inventory are subject to deterioration at a constant rate, \( \theta, 0 < \theta < 1 \). The deteriorated units can neither be repaired nor replaced during the cycle time.

2.2.2 Mathematical Model

The rate of change of inventory at any instant of time can be discussed by differential equation

\[
\frac{dI(t)}{dt} = -\left(\alpha + \beta I(t) + \theta I(t)\right), 0 \leq t \leq T; \ I(0) = Q \text{ and } I(T) = 0.
\]

Using \( I(T) = 0 \), the solution of the differential equation is

\[
I(t) = \frac{\alpha}{\theta + \beta} \left(e^{(\theta + \beta)T} - 1\right), \ 0 \leq t \leq T.
\]

The units to be purchased \( Q = I(0) = \frac{\alpha}{\theta + \beta} \left(e^{(\theta + \beta)T} - 1\right). \)

2.2.2.1 Vendor’s Total Profit per Unit Time

The total profit per unit time for the vendor comprises of sales revenue, set-up cost, holding cost and opportunity cost as follows:
(1) Sales revenue: The total sales revenue per unit time is \( (C_b - C_p) \frac{Q}{T} \)

(2) Set-up cost: \( nQ \) units are manufactured in one production run by the vendor. Therefore, the set-up cost per unit time is \( \frac{A_v}{nT} \).

(3) Holding cost: Using Joglekar (1988), the vendor’s average inventory per unit time is

\[
C_p (I_v + I_{vp}) \left[ \frac{(n-1)(1-\rho) + \rho}{(\theta + \beta)^2 T} \alpha (e^{(\theta + \beta)T} - (\theta + \beta)T - 1) \right].
\]

(4) Opportunity cost: If \( Q_d \) or more units are ordered by the buyer, the credit period of \( M \) units is permissible to settle the account. In this scenario, vendor endures a capital and payment received. Equivalently, when \( T \geq T_d \), the delay in payment is permissible and corresponding opportunity cost per unit time is \( \frac{C_b I_{vp} Q M}{T} \). On the other hand, when \( T < T_d \) the vendor receives payments on deliver and so no opportunity cost will occur.

Hence, the total profit per unit time for the vendor is

\[
TVP(n) = \begin{cases} 
TVP_1(n), & T < T_d \\
TVP_2(n), & T \geq T_d 
\end{cases}
\]
where

$$
TVP_1(n) = \left( C_b - C_p \right) \frac{Q}{T} - \frac{A_v}{nT} \left[ C_p (I_v + I_{vp}) [(n-1)(1-\rho) + \rho \alpha (e^{(\theta+\beta)T} - (\theta + \beta)T - 1) \right]
$$

$$
TVP_2(n) = \left( C_b - C_p \right) \frac{Q}{T} - \frac{A_v}{nT} \left[ C_p (I_v + I_{vp}) [(n-1)(1-\rho) + \rho \alpha (e^{(\theta+\beta)T} - (\theta + \beta)T - 1) \right]
$$

$$
\quad - \frac{C_b I_{vp} QM}{T}
$$

\hspace{1cm} (2.2.2)

\hspace{1cm} (2.2.3)

### 2.2.2.2 Buyer’s Total Profit per Unit Time

The total profit per unit time for the buyer comprises of sales revenue, ordering cost, holding cost, opportunity cost and interest earned. These costs are computed as follows:

1. **Sales revenue**: The total sales revenue per unit time is $(s - C_b) \frac{Q}{T}$

2. **Ordering cost**: The ordering cost per unit time is $\frac{A_b}{T}$

3. **Holding cost**: The buyer’s holding cost (excluding interest charges) per unit time is $\frac{C_b I_b \alpha (e^{(\theta+\beta)T} - (\theta + \beta)T - 1)}{(\theta + \beta)^2 T}$

4. **Opportunity cost**: Based on the lengths of $T, M$ and $T_d$, the following four cases arises (i) $0 < T < T_d$ (ii) $T_d \leq T \leq M$ (iii) $T_d \leq M \leq T$ (Figure 2.1.1) and (iv) $M \leq T_d \leq T$. The cases (iii) and (iv) are similar.
Opportunity cost per unit time:

\[
\begin{cases}
    \frac{C_b I_{bp} Q}{T}, & 0 < T < T_d \\
    0, & T_d \leq T \leq M \\
    \frac{C_b I_{bp}}{(\theta + \beta)^2 T} \alpha \left( e^{(\theta + \beta)(T - M)} - (\theta + \beta)(T - M) - 1 \right), & T_d \leq M \leq T \text{ or } M \leq T_d \leq T
\end{cases}
\]

Interest earned: As discussed in opportunity cost interest earned per unit time in all the four cases is as follows.

Interest earned per unit time

\[
= \begin{cases}
    0, & 0 < T < T_d \text{ (because payment is to be made on delivery)} \\
    \frac{sI_{be}}{T} \left( R \left( I(t) \right) t_{dt} + Q(M - T) \right), & T_d \leq T \leq M \text{ (figure 2.1.2)} \\
    \frac{sI_{be}}{T} \left( M \right) \int_{0}^{T} R \left( I(t) \right) t_{dt}, & T_d \leq M \leq T \text{ or } M \leq T_d \leq T \text{ (figure 2.1.3)}
\end{cases}
\]

Hence, the buyer's total profit per unit time is

\[
TBP(T) = \begin{cases}
    TBP_1(T), & 0 < T < T_d \\
    TBP_2(T), & T_d \leq T \leq M \\
    TBP_3(T), & T_d \leq M \leq T \\
    TBP_4(T), & M \leq T_d \leq T
\end{cases}
\]

where

\[
TBP_1(T) = \frac{(s - C_b)Q}{T} - \frac{A_b}{T} - \frac{C_b I_{bp} \alpha}{(\theta + \beta)^2 T} \left( e^{(\theta + \beta)T} - (\theta + \beta)T - 1 \right) - \frac{C_b I_{bp} Q}{T}
\]
Joint Total Profit per Unit Time

In integrated system, the vendor and buyer decide to take joint decision which maximizes the profit of the supply chain. The joint total profit per unit time for the integrated system is

$$\pi(n,T) = \begin{cases} 
\pi_1(n,T) = TVP_1(n) + TBP_1(T), & 0 < T < T_d \\
\pi_2(n,T) = TVP_2(n) + TBP_2(T), & T_d \leq T \leq M \\
\pi_3(n,T) = TVP_3(n) + TBP_3(T), & M \leq T \leq T_d \\
\pi_4(n,T) = TVP_4(n) + TBP_4(T), & M \leq T \leq T_d.
\end{cases}$$

where

$$\pi_1(n,T) = \left(s - C_p - C_b I_{bp}\right) \frac{Q}{T} - \frac{A}{T} - \frac{1}{T}(\phi + \psi)\alpha \left(\frac{e^{(\theta + \beta)T}}{T} - (\theta + \beta)T - 1\right)$$

$$\pi_2(n,T) = \left(s - C_p - \left(C_b I_{vp} + sI_{be}\right)M\right) \frac{Q}{T} - sI_{be}Q - \frac{A}{T} - \frac{1}{T}(\phi + \psi)\alpha \left(\frac{e^{(\theta + \beta)T}}{T} - (\theta + \beta)T - 1\right) + \frac{sI_{be}}{T} \int_0^T R(I(t))dt.$$
\[ \pi_3(n,T) = (s - C_p - C_b I_{v \mu} M) \frac{Q}{T} - \frac{\bar{A}}{T} \frac{1}{T}(\phi + \psi) \alpha \left( e^{(\theta + \beta)T} - (\theta + \beta)T - 1 \right) \]
\[ - \frac{C_b I_{bp} \alpha}{(\theta + \beta)^2 T} \left( e^{(\theta + \beta)(T-M)} - (\theta + \beta)(T-M) - 1 \right) \]
\[ + \frac{s I_{be} \alpha}{(\theta + \beta)^2 T} \left( e^{(\theta + \beta)T} - (1 + (\theta + \beta)M) e^{(\theta + \beta)(T-M)} \right) \]  

(2.2.11)

where

\[ \bar{A} = A_b + \frac{A_v}{n} \]
\[ \phi = \frac{C_p (I_v + I_{vp}) [(n-1)(1-\rho) + \rho]}{(\theta + \beta)^2} \]
\[ \psi = \frac{C_b I_b}{(\theta + \beta)^2} \]

2.2.2.4 Computational Procedure

For fixed \( T \), we note that \( \pi(n,T) \) is a concave function of \( n \) because

\[ \frac{\partial^2 \pi(n,T)}{\partial n^2} = -\frac{2A_v}{n^3 T} < 0. \]

Therefore to find optimum number of shipments \( n^* \), we will have a local optimal solution. The optimum value of cycle time can be obtained by setting \( \frac{\partial \pi}{\partial T} = 0 \) for fixed \( n \).

Algorithm:

Using algorithm given in section 2.1.2.4 with \( T_d = \frac{1}{\theta + \beta} \ln \left( 1 + \frac{(\theta + \beta)Q_d}{\alpha} \right) \), optimal solution \( (n^*, T^*) \) and hence optimal order quantity is obtained.
2.2.2.5 Numerical Examples and Interpretations

Example 2.2.1: Consider, \( \alpha = 10,000 \text{ units} \), \( \beta = 10\% \), \( \theta = 10\% \), \( \rho = 0.7 \),
\( C_b = \$10 / \text{unit} \), \( C_p = \$5 / \text{unit} \), \( A_v = \$400 / \text{setup} \), \( A_b = \$50 / \text{order} \),
\( I_v = 10\% / \text{unit/annum} \), \( I_b = 10\% / \text{unit/annum} \), \( I_{bp} = 8\% / \text{unit/annum} \),
\( I_{be} = 5\% / \$/\text{annum} \), \( I_{vp} = 2\% / \text{unit/annum} \), \( s = \$25 / \text{unit} \) and \( M = 30 \text{ days} \).

The optimal shipments and ordering units with buyer, vendor and joint profit for different values of \( Q_d \) are exhibited in Table 2.2.1.

<table>
<thead>
<tr>
<th>( Q_d ) (units)</th>
<th>( Q^* ) (units)</th>
<th>( n^* )</th>
<th>( T^* ) (days)</th>
<th>( \text{Profit($)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Buyer</td>
</tr>
<tr>
<td>1,000</td>
<td>1,367</td>
<td>5</td>
<td>49</td>
<td>1,50,149</td>
</tr>
<tr>
<td>2,000</td>
<td>1,367</td>
<td>5</td>
<td>49</td>
<td>1,50,149</td>
</tr>
<tr>
<td>3,000</td>
<td>3,000</td>
<td>2</td>
<td>124</td>
<td>1,50,054</td>
</tr>
<tr>
<td>4,000</td>
<td>4,000</td>
<td>2</td>
<td>124</td>
<td>1,50,054</td>
</tr>
<tr>
<td>5,000</td>
<td>1,367</td>
<td>5</td>
<td>49</td>
<td>1,50,149</td>
</tr>
<tr>
<td>6,000</td>
<td>1,367</td>
<td>5</td>
<td>49</td>
<td>1,50,149</td>
</tr>
</tbody>
</table>

From Table 2.2.1, it is seen that vendor has to set appropriate threshold to take advantage of delayed payment as discussed in section 2.1. Along with that here effect of deterioration can be seen in joint total profit. Deterioration decreases the joint total profit mostly in all cases.
The concavity of joint total profit is shown in figure 2.2.1.

**Example 2.2.2:** Consider the data given in Example 2.2.1. We study the effect of delayed payments for \( Q_d = 3,000 \) units.

**Table 2.2.2: Optimal solutions for different \( M (Q_d = 3,000) \)**

<table>
<thead>
<tr>
<th>( M ) (days)</th>
<th>( Q^* ) (units)</th>
<th>( n^* ) (days)</th>
<th>( T^* ) (days)</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Buyer</td>
</tr>
<tr>
<td>20</td>
<td>3,000</td>
<td>2</td>
<td>126</td>
<td>1,49,799</td>
</tr>
<tr>
<td>30</td>
<td>3,000</td>
<td>2</td>
<td>124</td>
<td>1,50,055</td>
</tr>
<tr>
<td>40</td>
<td>3,000</td>
<td>2</td>
<td>122</td>
<td>1,50,324</td>
</tr>
<tr>
<td>50</td>
<td>3,000</td>
<td>3</td>
<td>81</td>
<td>1,50,743</td>
</tr>
</tbody>
</table>
From Table 2.2.2, it is observed that longer credit period increases buyer's total profit and joint profit of the supply chain. The longer credit period reduces vendor’s total profit because payment will be received late for the purchases made.

**Example 2.2.3:** In this example, we carry out sensitivity analysis to find the critical inventory parameters. The changes in the purchase quantity and joint profit are studied by varying inventory parameters as $-20\%, -10\%, 10\%$ and $20\%$, one at a time. The results are exhibited in figure 2.2.2 and figure 2.2.3.

![Figure 2.2.2 Variations in purchase quantity](image)

From figure 2.2.2, the purchase quantity is very sensitive to the retail price set by the buyer and scale demand. Increase in buyer's purchase price forces him to put smaller order. The production cost also decreases buyer's order quantity. Similar effect is observed for buyer's holding charge fraction, interest charged by the supplier for the unsold stock and holding charge fraction of the vendor. This suggests that buyer should make an attempt to sell the items procured well in
time i.e. before the permissible delay period. This will help buyer to save in terms of charges to be paid on un-sold items and interest on storing system.

![Figure 2.2.3 Variations in joint profit](image)

It is observed from figure 2.2.3 that joint profit has significant positive impact of scale demand and retail price set by the buyer. It is evident that both the player should take advantage of demand increase and setting agreeable selling price. Production cost of supplier reduced joint total profit. It is advised to the supplier to use advanced technology which reduces this production cost. Other inventory parameters have very small contribution in increasing profit of the supply chain.

**Example 2.2.4:** In Table 2.2.3, we compare independent vs. joint decision, for pre-specified quantity $Q_d = 3,000$ units at which buyer qualifies for getting delay period facility.
Table 2.2.3: Optimal solution of independent and integrated scenario

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Buyer</th>
<th>Vendor</th>
<th>Joint</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Independent</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Shipments</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ordering Quantity (units)</td>
<td>1,568</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cycle Time (days)</td>
<td>56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Annual Profit ($)</td>
<td>1,50,206</td>
<td>48,828</td>
<td>1,99,034</td>
</tr>
<tr>
<td><strong>Integrated</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Shipments</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ordering Quantity (units)</td>
<td>3,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cycle Time (days)</td>
<td>124</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Annual Profit ($)</td>
<td>1,50,055</td>
<td>49,070</td>
<td>1,99,125</td>
</tr>
<tr>
<td>Readjusted Total Annual Profit ($)</td>
<td>1,50,275</td>
<td>48,850</td>
<td>1,99,125</td>
</tr>
</tbody>
</table>

where

**Buyer’s profit:**

\[
\pi(n, T)\times \frac{TBP(P,T)}{\left[TBP(P,T) + TVP(n)\right]} = 1.99,125 \times \frac{1,50,206}{(1,50,206 + 48,828)} = 1,50,275
\]

**Supplier’s profit:**

\[
\pi(n, T)\times \frac{TVP(n)}{\left[TBP(P,T) + TVP(n)\right]} = 1.99,125 \times \frac{48,828}{(1,50,206 + 48,828)} = 48,850
\]

Table 2.2.3 shows that the total annual profit under joint decision $1,99,125 (=1,50,055+49,070) which is greater than the total profit under independent decision $1,99,034 (=1,50,206+48,828). It establishes that joint decision is advantageous to both the players. The last row of Table 2.2.3 is about readjustment of the profits (Goyal (1976)) to encourage players for joint decision.
2.3 Optimal Pricing, Shipments and Ordering Policies for Single Supplier Single-Buyer Inventory System with Price Sensitive Stock-Dependent Demand and Order-Linked Trade Credit

In this section, we make selling price to be decision variable in contrast to fixed selling price in model 2.1.

2.3.1 Assumptions and Notations

The following additional assumptions and notations other than those given in A.1, N.1 and section 2.1.1 are used to formulate the proposed model:

2.3.1.1 Notations

\[ R(I(t), s) : \text{Price-sensitive Stock-dependent demand rate; } (\alpha + \beta I(t))s^{-\eta} \]

where \( \alpha > 0 \) denotes constant scale demand, \( 0 < \beta < 1 \) denotes stock-dependent parameter, \( \eta > 1 \) denotes price-elasticity and \( s \) denotes retail price of the product per unit by the buyer. (\( s \) is a decision variable)

2.3.2 Mathematical Model

The rate of change of inventory at any instant of time can be discussed by differential equation

\[
\frac{dI(t)}{dt} = -(\alpha + \beta I(t))s^{-\eta}, 0 \leq t \leq T; \quad I(0) = Q \quad \text{and} \quad I(T) = 0.
\]

Using \( I(T) = 0 \), the solution of the differential equation is

\[
I(t) = \frac{\alpha}{\beta} \left( e^{\beta s^{-\eta} (T-t)} - 1 \right), \quad 0 \leq t \leq T.
\]
The units to be purchased is \( Q = I(0) = \frac{\alpha}{\beta} \left( e^{\beta s^{-\eta} T} - 1 \right) \).

### 2.3.2.1 Vendor’s Total Profit per Unit Time

The total profit per unit time for the vendor comprises of sales revenue, set-up cost, holding cost and opportunity cost as follows:

1. **Sales revenue**: The total sales revenue per unit time is \( (C_b - C_p) \frac{Q}{T} \)
2. **Set-up cost**: \( nQ \) units are manufactured in one production run by the vendor. Therefore, the set-up cost per unit time is \( \frac{A_v}{nT} \)
3. **Holding cost**: Using Joglekar (1988), the vendor’s average inventory per unit time is
   \[
   \frac{C_p (I_v + I_{vp}) \left[ (n-1)(1-\rho) + \rho \right]}{T \beta^2} \left( e^{\beta s^{-\eta} T} s^{-\eta} - \sigma T \right)
   \]
4. **Opportunity cost**: If \( Q_d \) or more units are ordered by the buyer, the credit period of \( M \) units is permissible to settle the account. In this scenario, vendor endures a capital and payment received. Equivalently, when \( T \geq T_d \), the delay in payment is permissible and corresponding opportunity cost per unit time is \( \frac{C_b I_{vp} Q M}{T} \). On the other hand, when \( T < T_d \) the vendor receives payments on deliver and so no opportunity cost will occur.
Hence, the total profit per unit time for the vendor is

\[
TVP(n,s) = \begin{cases} 
  TVP_1(n,s), & T < T_d \\
  TVP_2(n,s), & T \geq T_d 
\end{cases}
\tag{2.3.1}
\]

where

\[
TVP_1(n,s) = \frac{(C_b - C_p)Q}{T} - \frac{A_v}{nT} - \frac{C_p(I_v + I_{vp})[(n-1)(1-\rho) + \rho]}{T \beta^2} 
\left( e^{\beta s^{-\eta}T} \eta - s^\eta - \beta T \right)
\]

\[
TVP_2(n,s) = \frac{(C_b - C_p)Q}{T} - \frac{A_v}{nT} - \frac{C_p(I_v + I_{vp})[(n-1)(1-\rho) + \rho]}{T \beta^2} 
\left( e^{\beta s^{-\eta}T} \eta - s^\eta - \beta T \right) 
- \frac{C_b I_{vp}QM}{T}
\]

\[
\tag{2.3.2}
\tag{2.3.3}
\]

### 2.3.2.2 Buyer’s Total Profit per Unit Time

The total profit per unit time for the buyer comprises of sales revenue, ordering cost, holding cost, opportunity cost and interest earned. These costs are computed as follows:

1. **Sales revenue**: The total sales revenue per unit time is \( (s - C_b) \frac{Q}{T} \)

2. **Ordering cost**: The ordering cost per unit time is \( \frac{A_b}{T} \)

3. **Holding cost**: The buyer’s holding cost (excluding interest charges) per unit time is

\[
\frac{C_b I_b \alpha \left( e^{\beta s^{-\eta}T} \eta - s^\eta - \beta T \right)}{T \beta^2}
\]
(4) Opportunity cost : Based on the lengths of $T, M$ and $T_d$, the following four cases arises (i) $0 < T < T_d$ (ii) $T_d \leq T \leq M$ (iii) $T_d \leq M \leq T$ (figure 2.1.1) and (iv) $M \leq T_d \leq T$. The cases (iii) and (iv) are similar.

Opportunity cost per unit time:

$$\frac{C_b I_{bp} Q}{T}, \quad 0 < T < T_d$$
$$0, \quad T_d \leq T \leq M$$
$$\frac{C_b I_{bp}}{T \beta^2} \left( s^n e^{-M \beta s^{-n}} + M M^{-s^n} + \beta M - s^n - \beta T \right), \quad T_d \leq M \leq T \text{ or } M \leq T_d \leq T$$

Interest earned : As discussed in opportunity cost interest earned per unit time in all the four cases is as follows.

Interest earned per unit time:

$$\frac{s I_{be}}{T} \left( \int_0^T R(I(t), s) t \ dt + Q(M - T) \right), \quad T_d \leq T \leq M \text{ (figure 2.1.2)}$$
$$\frac{s I_{be}}{T} \left( \int_0^M R(I(t), s) t \ dt \right), \quad T_d \leq M \leq T \text{ or } M \leq T_d \leq T \text{ (figure 2.1.3)}$$

Hence, the buyer’s total profit per unit time is

$$TBP(s, T) = \begin{cases} 
TBP_1(s, T), & 0 < T < T_d \\
TBP_2(s, T), & T_d \leq T \leq M \\
TBP_3(s, T), & T_d \leq M \leq T \\
TBP_4(s, T), & M \leq T_d \leq T 
\end{cases}$$

(2.3.4)
where

$$TBP_1(s,T) = \frac{(s-C_b)Q}{T} - \frac{A_b}{T} - \frac{C_b I_b}{T \beta^2} \alpha \left( e^{b s^{-\eta} T} s^{-\eta} - s^{-\eta} - \beta T \right) - \frac{C_b I_b p Q}{T}$$ \hfill (2.3.5)

$$TBP_2(s,T) = \frac{(s-C_b)Q}{T} - \frac{A_b}{T} - \frac{C_p I_b}{T \beta^2} \alpha \left( e^{b s^{-\eta} T} s^{-\eta} - s^{-\eta} - \beta T \right)$$

$$+ \frac{s I_{be}}{T} \left[ R(I(t),s) dt + Q(M-T) \right]$$ \hfill (2.3.6)

$$TBP_3(s,T) = TBP_4(s,T) = \frac{(s-C_b)Q}{T} - \frac{A_b}{T} - \frac{C_b I_b}{T \beta^2} \alpha \left( e^{b s^{-\eta} T} s^{-\eta} - s^{-\eta} - \beta T \right)$$

$$- \frac{C_b I_{bp}}{T \beta^2} \alpha \left( s^{-\eta} e^{-M \beta s^{-\eta}} + \beta s^{-\eta} T + \beta M - s^{-\eta} - \beta T \right)$$

$$- \frac{s I_{be} \alpha}{T \beta^2} \left( e^{b s^{-\eta} T} s^{-\eta} + s^{-\eta} e^{-M \beta s^{-\eta}} + \beta s^{-\eta} T + e^{-M \beta s^{-\eta}} + \beta s^{-\eta} M \beta \right)$$ \hfill (2.3.7)

### 2.3.2.3 Joint Total Profit per Unit Time

In integrated system, the vendor and buyer decide to take joint decision which maximizes the profit of the supply chain. The joint total profit per unit time for the integrated system is

$$\pi(n,s,T) = \begin{cases} 
\pi_1(n,s,T) = TVP_1(n,s) + TBP_1(s,T), & 0 < T < T_d \\
\pi_2(n,s,T) = TVP_2(n,s) + TBP_2(s,T), & T_d \leq T \leq M \\
\pi_3(n,s,T) = TVP_2(n,s) + TBP_3(s,T), & T_d \leq M \leq T \\
\pi_4(n,s,T) = TVP_2(n,s) + TBP_3(s,T), & M \leq T_d \leq T.
\end{cases}$$ \hfill (2.3.8)
where

$$\pi_1(n,s,T) = (s - C_p - C_b I_{bp}) \frac{Q}{T} - \frac{\bar{A}}{T} - \frac{1}{T} (\phi + \psi) \alpha \left( e^{\beta s^{-\eta} T} s^\eta - s^\eta - \beta T \right)$$

(2.3.9)

$$\pi_2(n,s,T) = (s - C_p - (C_b I_{vp} - sI_{be}) M) \frac{Q}{T} - sI_{be} Q - \frac{\bar{A}}{T}$$

$$- \frac{1}{T} (\phi + \psi) \alpha \left( e^{\beta s^{-\eta} T} s^\eta - s^\eta - \beta T \right) + \frac{sI_{be} T}{T} \int_0^T R(I(t),s) t dt$$

(2.3.10)

$$\pi_3(n,s,T) = (s - C_p - C_b I_{vl} M) \frac{Q}{T} - \frac{\bar{A}}{T} - \frac{1}{T} (\phi + \psi) \alpha \left( e^{\beta s^{-\eta} T} s^\eta - s^\eta - \beta T \right)$$

$$- \frac{C_b I_{bp} \alpha}{\beta^2 T} \left( e^{\beta s^{-\eta} (T-M)} s^\eta - s^\eta - \beta (T-M) \right)$$

$$+ \frac{sI_{be} \alpha}{\beta^2 T} \left( e^{\beta s^{-\eta} T} s^\eta - \left( s^\eta + \beta M \right) e^{\beta s^{-\eta} (T-M)} \right)$$

(2.3.11)

where

$$\bar{A} = A_b + A_v$$

$$\phi = \frac{C_p \left( I_v + I_{vp} \right) \left[ (n-1)(1-\rho) + \rho \right]}{\beta^2}$$

$$\psi = \frac{C_b I_{lb}}{\beta^2}$$
### 2.3.2.4 Computational Procedure

For fixed $T$, we note that $\pi(n, s, T)$ is a concave function of $n$ because

$$\frac{\partial^2 \pi(n, s, T)}{\partial n^2} = -\frac{2A_v}{n^3 T} < 0.$$  Therefore to find optimum number of shipments $n^*$, we will have a local optimal solution. The optimum value of cycle time can be obtained by setting $\frac{\partial \pi}{\partial T} = 0$ and $\frac{\partial \pi}{\partial s} = 0$ simultaneously for fixed $n$.

**Algorithm:**

Step 1: Set parametric values.

Step 2: Compute $T_d$ using

$$\frac{1}{\beta s^{-\eta}} \ln \left(1 + \frac{\beta Q_d}{\alpha} \right)$$

for given value of $Q_d$.

Step 3: Set $n = 1$.

Step 4: Knowing $T_d$ and $M$, compute $T$ and $s$ by solving $\frac{\partial \pi_j}{\partial T} = 0$ and $\frac{\partial \pi_j}{\partial s} = 0$ simultaneously for $j = 1, 2, 3$.

Step 5: Find corresponding profit $\pi_j$ for $j = 1, 2, 3$.

Step 6: Increment $n$ by 1.

Step 7: Repeat step 4 and 6 until

$$\pi(n-1, s(n-1), T(n-1)) \leq \pi(n, s(n), T(n)) \geq \pi(n+1, s(n+1), T(n+1)).$$

Once the optimal solution $\left(n^*, s^*, T^* \right)$ is obtained, the optimal order quantity can be obtained.
2.3.2.5 Numerical Examples and Interpretations

Example 2.3.1: Consider, $\alpha = 10,000$ units, $\beta = 10\%$, $\eta = 1.25$, $\rho = 0.7$, $C_b = $10/unit, $C_p = $5/unit, $A_v = $400/setup, $A_b = $50/order, $I_v = 10\%$/unit/annum, $I_b = 10\%$/unit/annum, $I_{bp} = 8\%$/unit/annum, $I_{be} = 5\%$/$/unit$/annum, $I_{vp} = 2\%$/unit/annum, $s = $25/unit and $M = 30$ days.

The optimal shipments and ordering units with buyer, vendor and joint profit for different values of $Q_d$ are exhibited in Table 2.3.1.

Table 2.3.1: Optimal solutions for different $Q_d$

<table>
<thead>
<tr>
<th>$Q_d$ (units)</th>
<th>$Q^*$ (units)</th>
<th>$n^*$</th>
<th>$s^*$ ($)</th>
<th>$T^*$ (days)</th>
<th>Profit($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Buyer</td>
</tr>
<tr>
<td>100</td>
<td>74</td>
<td>11</td>
<td>29.34</td>
<td>184</td>
<td>2,661</td>
</tr>
<tr>
<td>200</td>
<td>74</td>
<td>11</td>
<td>29.34</td>
<td>184</td>
<td>2,661</td>
</tr>
<tr>
<td><strong>300</strong></td>
<td><strong>300</strong></td>
<td><strong>9</strong></td>
<td><strong>29.63</strong></td>
<td><strong>219</strong></td>
<td><strong>2,690</strong></td>
</tr>
<tr>
<td>400</td>
<td>400</td>
<td>9</td>
<td>29.63</td>
<td>219</td>
<td>2,690</td>
</tr>
<tr>
<td>500</td>
<td>74</td>
<td>11</td>
<td>29.34</td>
<td>184</td>
<td>2,661</td>
</tr>
<tr>
<td>600</td>
<td>74</td>
<td>11</td>
<td>29.34</td>
<td>184</td>
<td>2,661</td>
</tr>
</tbody>
</table>

From Table 2.3.1, it is seen that buyer’s optimal order quantity $Q^*$ is equal to $Q_d$ and less than $Q_d$ when $Q_d \geq 500$. Also vendor is advised to set proper threshold as discussed in sections 2.1 and 2.2.

The concavity of joint total profit for $(s, T)$ for obtained 9-shipments is exhibited in figure 2.3.1, for $(n, T)$ for obtained $s^* = $29.63 in figure 2.3.2 and for $(n, s)$ for obtained $T^* = 219$ days in figure 2.3.3.
Figure 2.3.1 Concavity of joint profit for $(s, T)$ for $n^*$

Figure 2.3.2 Concavity of joint profit for $(n, T)$ for $s^*$
Example 2.3.2: Consider the data given in Example 2.3.1. We study the effect of delayed payments for $Q_d = 300$ units.

Table 2.3.2: Optimal solutions for different $M$ ($Q_d = 300$)

<table>
<thead>
<tr>
<th>$M$ (days)</th>
<th>$Q^*$ (units)</th>
<th>$n^*$</th>
<th>$s^*$ ($)</th>
<th>$T^*$ (days)</th>
<th>Profit($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>300</td>
<td>9</td>
<td>29.65</td>
<td>219</td>
<td>2,684 567 3,251</td>
</tr>
<tr>
<td>30</td>
<td>300</td>
<td>9</td>
<td>29.63</td>
<td>219</td>
<td>2,689 567 3,256</td>
</tr>
<tr>
<td>40</td>
<td>300</td>
<td>9</td>
<td>29.61</td>
<td>219</td>
<td>2,690 566 3,256</td>
</tr>
<tr>
<td>50</td>
<td>300</td>
<td>9</td>
<td>29.58</td>
<td>218</td>
<td>2,693 566 3,259</td>
</tr>
<tr>
<td>60</td>
<td>300</td>
<td>9</td>
<td>29.56</td>
<td>218</td>
<td>2,696 566 3,261</td>
</tr>
</tbody>
</table>

From Table 2.3.2, it is observed that longer credit period increases buyer’s total profit and joint profit of the supply chain. The longer credit period reduces vendor’s total profit because payment will be received late for the purchases.
made. This suggests that late payment increases risk of cash shortage for the vendor.

**Example 2.3.3:** In this example, we carry out sensitivity analysis to find the critical inventory parameters. The changes in joint profit are studied by varying inventory parameters as $-20\%, -10\%, 10\%$ and $20\%$, one at a time. The results are exhibited in Figure 2.3.4.

![Figure 2.3.4 Variation in joint total profit](image)

It is observed from figure 2.3.4 that joint profit increases positively with increase in scale demand. It is evident that both the player should take advantage of demand increase and setting agreeable selling price. Production cost of supplier reduced joint total profit. It is advised to the supplier to use advanced technology which reduces this production cost. Other inventory parameters have very small perturbations in profit of the supply chain.
Example 2.3.4: In Table 2.3.3, we compare independent vs. joint decision, for pre-specified quantity \( Q_d = 300 \) units at which buyer qualifies for getting delay period facility.

Table 2.3.3: Optimal Solution of independent and integrated scenario

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Total Shipments</th>
<th>Buyer</th>
<th>Vendor</th>
<th>Joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent</td>
<td>Total Shipments</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ordering Quantity (units)</td>
<td>62</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cycle Time (days)</td>
<td>329</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Annual Profit ($)</td>
<td>2,901</td>
<td>236</td>
<td>3,137</td>
</tr>
<tr>
<td>Integrated</td>
<td>Total Shipments</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ordering Quantity (units)</td>
<td>87</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cycle Time (days)</td>
<td>219</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Annual Profit ($)</td>
<td>2,690</td>
<td>567</td>
<td>3,256</td>
</tr>
<tr>
<td></td>
<td>Readjusted Total Annual Profit ($)</td>
<td>3,011</td>
<td>245</td>
<td>3,256</td>
</tr>
</tbody>
</table>

where

\[
\text{Buyer’s profit} = \pi(n,T) \times \frac{TBP(P,T)}{[TBP(P,T)+TVP(n)]} = 3,256 \times \frac{2,901}{(2,901+236)} = 3,011
\]

\[
\text{Supplier’s profit} = \pi(n,T) \times \frac{TVP(n)}{[TBP(P,T)+TVP(n)]} = 3,256 \times \frac{236}{(2,901+236)} = 245
\]

Table 2.3.3 shows that the total annual profit under joint decision $3,256 (=\$2,690+\$567) which is greater than the total profit under independent decision $3,137 (=\$2,901+\$236). It establishes that joint decision is advantageous to both the players. The last row of Table 2.3.3 is about readjustment of the profits (Goyal (1976)) to encourage players for joint decision.
2.4 Optimal Inventory Policies for Single-Supplier Single-Buyer Deteriorating Items with Price-Sensitive Stock-Dependent Demand and Order Linked Trade Credit.

In this section, we study the effect of deterioration in model 2.3.

2.4.1 Assumptions and Notations

Using assumptions and notations given in A.1, N.1, sections 2.1.1, 2.2.1, and 2.3.1 the proposed model is formulated in next section.

2.4.2 Mathematical Model

The rate of change of inventory at any instant of time can be discussed by differential equation

\[
\frac{dI(t)}{dt} = -\left( (\alpha + \beta I(t))s^{-\eta} + \theta I(t) \right), \quad 0 \leq t \leq T; \quad I(0) = Q \quad \text{and} \quad I(T) = 0.
\]

Using \( I(T) = 0 \), the solution of the differential equation is

\[
I(t) = \frac{\alpha s^{-\eta}}{\beta s^{-\eta} + \theta} \left( e^{\left( \frac{\beta s^{-\eta} + \theta}{T-t} \right)} - 1 \right), \quad 0 \leq t \leq T.
\]

The units to be purchased is given by

\[
Q = I(0) = \frac{\alpha s^{-\eta}}{\beta s^{-\eta} + \theta} \left( e^{\left( \frac{\beta s^{-\eta} + \theta}{T} \right)} - 1 \right).
\]

2.4.2.1 Vendor’s Total Profit per Unit Time

The total profit per unit time for the vendor comprises of sales revenue, set-up cost, holding cost and opportunity cost as follows:
(1) Sales revenue: The total sales revenue per unit time is \( (C_b - C_p) \frac{Q}{T} \)

(2) Set-up cost: \( nQ \) -units are manufactured in one production run by the vendor. Therefore, the set-up cost per unit time is \( \frac{A_v}{nT} \)

(3) Holding cost: Using Joglekar (1988), the vendor’s average inventory per unit time is

\[
\frac{C_p(I_v + I_{vp})}{T} \left[ (n-1)(1-\rho) + \rho \right] \alpha \left( s^\eta \left( e^{(\beta s^{-\eta} + \theta)T} - \beta T \right) \right)
\]

(4) Opportunity cost: If \( Q_d \) or more units are ordered by the buyer, the credit period of \( M \) -units is permissible to settle the account. In this scenario, vendor endures a capital and payment received. Equivalently, when \( T \geq T_d \), the delay in payment is permissible and corresponding opportunity cost per unit time is \( \frac{C_b I_{vp} QM}{T} \). On the other hand, when \( T < T_d \) the vendor receives payments on deliver and so no opportunity cost will occur.

Hence, the total profit per unit time for the vendor is

\[
TVP(n,s) = \begin{cases} 
TVP_1(n,s), & T < T_d \\
TVP_2(n,s), & T \geq T_d 
\end{cases}
\]

(2.4.1)
where

\[
TVP_1(n,s) = \left(\frac{C_b - C_p}{T}\right)Q - \frac{A_v}{nT} - \frac{C_p(l_v + l_{vp})[(n-1)(1-\rho) + \rho]}{T \left(\beta^2 + 2\beta\theta s^\eta + \theta^2 s^{2\eta}\right)} \alpha \left(s^\eta \left(e^{(\beta s^{-\eta} + \theta)} - 1 - T\theta\right) - \beta T\right)
\]

(2.4.2)

\[
TVP_2(n,s) = \left(\frac{C_b - C_p}{T}\right)Q - \frac{A_v}{nT} - \frac{C_b l_{vp} QM}{T} - \frac{C_p(l_v + l_{vp})[(n-1)(1-\rho) + \rho]}{T \left(\beta^2 + 2\beta\theta s^\eta + \theta^2 s^{2\eta}\right)} \alpha \left(s^\eta \left(e^{(\beta s^{-\eta} + \theta^2)} - 1 - T\theta\right) - \beta T\right)
\]

(2.4.3)

### 2.4.2.2 Buyer’s Total Profit per Unit Time

The total profit per unit time for the buyer comprises of sales revenue, ordering cost, holding cost, opportunity cost and interest earned. These costs are computed as follows:

1. **Sales revenue**: The total sales revenue per unit time is \((s - C_b)\frac{Q}{T}\)

2. **Ordering cost**: The ordering cost per unit time is \(\frac{A_b}{T}\)

3. **Holding cost**: The buyer’s holding cost (excluding interest charges) per unit time is

\[
\frac{C_b l_b}{T \left(\beta^2 + 2\beta\theta s^\eta + \theta^2 s^{2\eta}\right)} \alpha \left(s^\eta \left(e^{(\beta s^{-\eta} + \theta^2)} - 1 - T\theta\right) - \beta T\right)
\]
(4) Opportunity cost : Based on the lengths of $T$, $M$ and $T_d$, the following four cases arises 

(i) $0 < T < T_d$ 
(ii) $T_d \leq T \leq M$ 
(iii) $T_d \leq M \leq T$

(figure 2.1.1) and (iv) $M \leq T_d \leq T$. The cases (iii) and (iv) are similar.

Opportunity cost per unit time:

$$
\begin{align*}
C_b \frac{I_{bp} Q}{T} & \quad , 0 < T < T_d \\
0 & \quad , T_d \leq T \leq M \\
\frac{C_b I_{bp}}{T} \left[ \int_{T}^{M} I(t) dt \right] & \quad , T_d \leq M \leq T \text{ or } M \leq T_d \leq T
\end{align*}
$$

Interest earned As discussed in opportunity cost interest earned per unit time in all the four cases is as follows.

Interest earned per unit time:

$$
\begin{align*}
0 & \quad , 0 < T < T_d \quad \text{(because payment is to be made on delivery)} \\
\frac{sI_{pe}}{T} \left( \int_{0}^{T} R(I(t), s) dt + Q(M - T) \right) & \quad , T_d \leq T \leq M \quad \text{(figure 2.1.2)} \\
\frac{sI_{pe}}{T} \left( \int_{0}^{M} R(I(t), s) dt \right) & \quad , T_d \leq M \leq T \text{ or } M \leq T_d \leq T \quad \text{(figure 2.1.3)}
\end{align*}
$$

Hence, the buyer’s total profit per unit time is

$$
TBP(s, T) =
\begin{align*}
TBP_1(s, T) & \quad , 0 < T < T_d \\
TBP_2(s, T) & \quad , T_d \leq T \leq M \\
TBP_3(s, T) & \quad , T_d \leq M \leq T \\
TBP_4(s, T) & \quad , M \leq T_d \leq T
\end{align*}
$$

(2.4.4)
where

\[
TBP_1(s,T) = \frac{(s - C_b)Q}{T} - \frac{A_b}{T} - \frac{C_b I_b}{T \left( \beta^2 + 2 \beta \theta s^n + \theta^2 s^2 \right)} \alpha \left( s^n \left( e^{(\beta s^{-\eta} + \theta)T} - 1 - T \theta \right) - \beta T \right) \\
- \frac{C_b I_{bp} Q}{T}
\]

(2.4.5)

\[
TBP_2(s,T) = \frac{(s - C_b)Q}{T} - \frac{A_b}{T} - \frac{C_b I_b}{T \left( \beta^2 + 2 \beta \theta s^n + \theta^2 s^2 \right)} \alpha \left( s^n \left( e^{(\beta s^{-\eta} + \theta)T} - 1 - T \theta \right) - \beta T \right) + \frac{s I_{be}}{T} \left[ \int_0^T R(I(t),s) dt + Q(M - T) \right]
\]

(2.4.6)

\[
TBP_3(s,T) = TBP_4(s,T) = \frac{(s - C_b)Q}{T} - \frac{A_b}{T} - \frac{C_b I_b}{T \left( \beta^2 + 2 \beta \theta s^n + \theta^2 s^2 \right)} \alpha \left( s^n \left( e^{(\beta s^{-\eta} + \theta)T} - 1 - T \theta \right) - \beta T \right) \\
- \frac{C_b I_{bp}}{T} \left[ \int_M^T I(t) dt \right] - \frac{s I_{be}}{T} \left[ \int_0^M R(I(t),s) dt \right]
\]

(2.4.7)

### 2.4.2.3 Joint Total Profit per Unit Time

In the integrated system, the vendor and buyer decide to take joint decision which maximizes the profit of the supply with respect to retail price, cycle time and number of shipments from the vendor to the buyer. The joint total profit per unit time for the integrated system is
\[ \pi(n,s,T) = \begin{cases} 
\pi_1(n,s,T) = TVP_1(n,s) + TBP_1(s,T), & 0 < T < T_d \\
\pi_2(n,s,T) = TVP_2(n,s) + TBP_2(s,T), & T_d \leq T \leq M \\
\pi_3(n,s,T) = TVP_2(n,s) + TBP_3(s,T), & T_d \leq M \leq T \\
\pi_4(n,s,T) = TVP_2(n,s) + TBP_3(s,T), & M \leq T_d \leq T. 
\end{cases} \tag{2.4.8} \]

where

\[ \pi_1(n,s,T) = \left( s - C_p - C_b I_{bp} \right) \frac{Q}{T} - \frac{\bar{A}}{T} - \frac{1}{T} (\phi + \psi) \alpha \left( s^\eta \left( e^{(\beta s^{-\eta} + \theta)T} \right) - \beta T \right) \] \tag{2.4.9}

\[ \pi_2(n,s,T) = \left( s - C_p - (C_b I_{vp} - s I_{be}) M \right) \frac{Q}{T} - s I_{be} Q - \frac{\bar{A}}{T} \]
\[ - \frac{1}{T} (\phi + \psi) \alpha \left( s^\eta \left( e^{(\beta s^{-\eta} + \theta)T} \right) - \beta T \right) + \frac{s I_{be}}{T} \int_0^T R(I(t),s) t \, dt \] \tag{2.4.10}

\[ \pi_3(n,s,T) = \left( s - C_p - C_b I_{vp} M \right) \frac{Q}{T} - \frac{\bar{A}}{T} - \frac{1}{T} (\phi + \psi) \alpha \left( s^\eta \left( e^{(\beta s^{-\eta} + \theta)T} \right) - \beta T \right) \]
\[ - \frac{C_b I_{bp}}{T} \left( \frac{T}{M} \int_0^M I(t) \, dt \right) - \frac{s I_{be}}{T} \left( \frac{M}{M} \int_0^M R(I(t),s) t \, dt \right) \] \tag{2.4.11}

where

\[ \bar{A} = A_b + \frac{A_v}{n} \]
\[ \phi = \frac{C_p (I_v + I_{vp}) [(n-1)(1-\rho) + \rho]}{\left( \beta^2 + 2\beta \theta s^\eta + \theta^2 s^{2\eta} \right)} \]
\[ \psi = \frac{C_b I_b}{\left( \beta^2 + 2\beta \theta s^\eta + \theta^2 s^{2\eta} \right)} \]
2.4.2.4 Computational Procedure

For fixed $T$, we note that $\pi(n, s, T)$ is a concave function of $n$ because
\[
\frac{\partial^2 \pi(n, s, T)}{\partial n^2} = -\frac{2A_v}{n^3 T} < 0.
\]
Therefore to find optimum number of shipments $n^*$, we will have a local optimal solution. The optimum value of cycle time can be obtained by setting $\frac{\partial \pi}{\partial n} = 0$ and $\frac{\partial \pi}{\partial s} = 0$ simultaneously for fixed $n$.

Using algorithm given in section 2.3.2.4 with
\[
T_d = \frac{1}{\left( \beta s^{-\eta} + \theta \right) \ln \left( 1 + \frac{\left( \beta s^{-\eta} + \theta \right) Q_d}{\alpha s^{-\eta}} \right)},
\]
onoptimal solution $(n^*, s^*, T^*)$ can be determined and hence, optimal order quantity is obtained.

2.4.2.5 Numerical Examples and Interpretations

Example 2.4.1: Consider, $\alpha = 10,000$ units, $\beta = 10\%$, $\eta = 1.25$, $\rho = 0.7$,

$C_b = \$10$ / unit, $C_p = \$5$ / unit, $A_v = \$400$ / setup, $A_b = \$50$ / order,

$I_v = 10\%$ / unit / annum, $I_b = 10\%$ / unit / annum, $I_{bp} = 8\%$ / unit / annum,

$I_{be} = 5\%$ / $/ annum, I_{vp} = 2\%$ / unit / annum, $s = \$25$ / unit and $M = 30$ days.

The optimal shipments and ordering units with buyer, vendor and joint profit for different values of $Q_d$ are exhibited in Table 2.4.1.
Table 2.4.1: Optimal solutions for different $Q_d$

<table>
<thead>
<tr>
<th>$Q_d$ (units)</th>
<th>$Q^*$ (units)</th>
<th>$n^*$</th>
<th>$s^*$ ($)</th>
<th>$T^*$ (days)</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>54.93</td>
<td>13</td>
<td>30.19</td>
<td>135</td>
<td>2,619</td>
</tr>
<tr>
<td>200</td>
<td>54.93</td>
<td>13</td>
<td>30.19</td>
<td>135</td>
<td>2,619</td>
</tr>
<tr>
<td>300</td>
<td>300</td>
<td>11</td>
<td>30.14</td>
<td>168</td>
<td>2,675</td>
</tr>
<tr>
<td>400</td>
<td>400</td>
<td>11</td>
<td>30.14</td>
<td>168</td>
<td>2,675</td>
</tr>
<tr>
<td>500</td>
<td>54.93</td>
<td>13</td>
<td>30.19</td>
<td>135</td>
<td>2,619</td>
</tr>
<tr>
<td>600</td>
<td>54.93</td>
<td>13</td>
<td>30.19</td>
<td>135</td>
<td>2,619</td>
</tr>
</tbody>
</table>

From Table 2.4.1, it is seen that vendor has to set appropriate threshold to take advantage of delayed payment as discussed in section 2.3. Along with that here effect of deterioration can be seen as decreased joint total profit.

The concavity of joint total profit for $(s, T)$ for obtained 11 - shipments is exhibited in figure 2.4.1, for $(n, T)$ for obtained $s^* = $30.14$ in figure 2.4.2 and for $(n, s)$ for obtained $T^* = 168$ days in figure 2.4.3.

![Figure 2.4.1 Concavity of joint profit for $(s, T)$ for obtained $n^*$](image-url)
Example 2.4.2: Consider the data given in Example 2.4.1. We study the effect of delayed payments for $Q_d = 300$ units.
From Table 2.4.2, it is observed that buyer’s total profit and joint profit of the supply chain increases with increase in the offered credit period. The longer credit period shrinks vendor’s total profit because payment will be received late for the purchases made. This suggests that late payment increases risk of cash shortage for the vendor.

**Example 2.4.3:** In this example, we carry out sensitivity analysis to find the critical inventory parameters. The changes in joint profit are studied by varying inventory parameters as $-20\%, -10\%, 10\%$ and $20\%$, one at a time. The results are exhibited in figure 2.4.4.

---

**Table 2.4.2: Optimal solutions for different $M$ and $Q_d = 300$**

<table>
<thead>
<tr>
<th>$M$ (days)</th>
<th>$Q^*$ (units)</th>
<th>$n^*$</th>
<th>$s^*$ ($)</th>
<th>$T^*$ (days)</th>
<th>Profit($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>300</td>
<td>11</td>
<td>30.19</td>
<td>169</td>
<td>2,668</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>532</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3,200</td>
</tr>
<tr>
<td>30</td>
<td>300</td>
<td>11</td>
<td>30.14</td>
<td>168</td>
<td>2,675</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>531</td>
</tr>
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<td>11</td>
<td>30.08</td>
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<td>2,682</td>
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<td>300</td>
<td>11</td>
<td>30.02</td>
<td>166</td>
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<td>3,227</td>
</tr>
</tbody>
</table>
It is observed from figure 2.4.4 that the scale demand has positive impact on joint profit. It reveals that both the players should take advantage of demand increase and setting agreeable selling price. Production cost of supplier decreases joint total profit. It is advised to the supplier to use advanced technology which reduces this production cost. Other inventory parameters have very small perturbations in profit of the supply chain. The deterioration of units in inventory also decreases the joint total profit of the supply chain.

**Example 2.4.4:** In Table 2.4.3, we compare independent vs. joint decision, for pre-specified quantity $Q_d = 300$ units at which buyer qualifies for delayed payment period. It is observed that the buyer’s profit decreases in integrated decision while that of vendor increases significantly. This will discourage the buyer to opt for joint decision. To entice buyer for joint decision we give readjustments of profits (Goyal (1976)) in the last row of the Table.
Table 2.4.3: Optimal solution of independent and integrated scenario

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Total Shipments</th>
<th>Buyer</th>
<th>Vendor</th>
<th>Joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent</td>
<td>10</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Order Quantity (units)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Cycle Time (days)</td>
<td>258</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Annual Profit ($)</td>
<td>2,879 218 3,097</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Integrated</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Order Quantity (units)</td>
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<td></td>
</tr>
<tr>
<td>Cycle Time (days)</td>
<td>168</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Annual Profit ($)</td>
<td>2,675 531 3,206</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Readjusted Total Annual Profit ($)</td>
<td>2,980 226 3,206</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where

\[
\text{Buyer's profit} = \pi(n,T) \times \frac{TBP(P,T)}{[TBP(P,T) + TVP(n)]} = 3,206 \times \frac{2,879}{(2,879 + 218)} = 2,980
\]

\[
\text{Supplier's profit} = \pi(n,T) \times \frac{TVP(n)}{[TBP(P,T) + TVP(n)]} = 3,206 \times \frac{218}{(2,879 + 218)} = 226
\]

Table 2.4.3 shows that the total annual profit under joint decision $3,206 (=\$2,675+\$531) which is greater than the total profit under independent decision $3,097 (=\$2,879+\$218). It establishes that joint decision is advantageous to both the players.

**Conclusions**

An integrated inventory policy comprising of single-vendor single-buyer is studied when demand is stock-dependent and credit terms are linked to order quantity. The model is then analyzed by incorporating deterioration of inventory products, price-sensitive stock-dependent demand and then both. The computational
procedure is outlined to optimize joint total profit per unit time with respect to number of shipments from the vendor to the buyer and cycle time when vendor’s stock depletes to zero.

![Comparison of joint profits](image)

**Figure 2.4.5 Comparison of joint profits**

Based on the results, it is observed that joint profit for the supply chain increases in joint decision compared to independent decision but reduces that of the buyer. To attract the buyer for the joint decision vendor should set proper threshold to offer credit period. From figure 2.4.5, the suitable selection of selling price and controlling deterioration rate of units will increase the joint profit of the system.