Chapter 5

Manufacturing process with disruption under quadratic demand for deteriorating inventory
5.0 Introduction

Each and every supply chain or logistics structure is subject to disruption. This disruption may be due to uncertainty or unintended events like instrument breakdown, unavailability of raw materials, some crisis, natural calamities or strikes. In this chapter, we first establish the production time when there is no disruption and afterwards when system gets disrupted. After disruption in the production, we investigate whether it has been resulted in any amount of shortages or not. To maintain the goodwill of the company, an additional replenishment is suggested if there are any shortages. For disrupted production system, disruption time is calculated. Also in the case of shortages, when to replenish and how much to replenish that is also shown in the article. Moreover, quadratic demand for products is analyzed in this chapter. This type of demand initially increases with time up to some extent and then it starts to decrease. In addition, the units in the inventory system are subject to deterioration at a constant rate. Numerical example and related graphical studies are given to validate the results. Effects of variations in inventory parameters on production time are shown for manufacturers to take advantageous decisions.

One model is formulated under the following section:

Model 5.1 Manufacturing process with disruption under quadratic demand for deteriorating inventory.
5.1 Manufacturing Process with Disruption under Quadratic Demand for Deteriorating Inventory

5.1.1 Assumptions and Notations

The following additional assumptions and notations other than those given in A.1 and N.1 are used to formulate the proposed model:

5.1.1.1 Assumptions

1. Production rate is constant.
2. The items in inventory deteriorate at a constant rate.
3. There is no repair or replacement of deteriorated items during the cycle time.

5.1.1.2 Notations

\[ R(t) : a(1 + bt - ct^2) \]  where \( a > 0 \) is scale demand, \( 0 \leq b < 1 \) denotes linear rate of change of demand with respect to time, \( 0 \leq c < 1 \) denotes quadratic rate of change of demand.  \( (5.1.1) \)

\[ P \] : Production rate

\[ H \] : Time horizon

\[ Q_r \] : Ordering quantity when shortages occur

\[ T_p \] : Production stops at this time when there is no disruption

\[ T_d \] : Production System disrupts at this time

\[ T_p^d \] : Production stops at this time when there is disruption in the system
**5.1.2 Mathematical Model**

**5.1.2.1 Model: Production without disruption**

In this model, we optimize production system run, when production is smooth i.e. no disruption. Here, production continues till time $T_p$. Then production stops and inventory level reaches to zero at cycle time $H$ due to demand and deterioration as shown in figure 5.1.1.

\[
\begin{align*}
    T_r & : \text{Time of placing order to overcome shortages} \\
    \frac{dI_1(t)}{dt} + \theta I_1(t) & = P - a \left(1 + bt - ct^2\right); \quad 0 \leq t \leq T_p; \quad I_1(0) = 0 \quad (5.1.2) \\
    \frac{dI_2(t)}{dt} + \theta I_2 T(t) & = -a \left(1 + bt - ct^2\right); \quad T_p \leq t \leq H, \quad I_2(H) = 0 \quad (5.1.3)
\end{align*}
\]

Figure 5.1.1  Production system without disruption
Then solution of differential equation (5.1.2) and (5.1.3) is given by

\[ I_1(t) = \frac{1}{\theta^3} \left( (a - P) \theta^2 - \theta ab - 2ac \right) e^{-\theta t} + \left( P - a \left( 1 + bt - ct^2 \right) \right) \theta^2 \]

\[ I_2(t) = \frac{1}{\theta^3} \left( a \left( \left( 1 + bH + cH^2 \right) \theta^2 - 2c - \theta \left( b - 2cH \right) \right) e^{\theta(H-t)} \right) \]

According to figure 5.1.1, \( I_1(T_p) = I_2(T_p) \), which gives

\[ T_p = \frac{1}{ac\theta H^2 - aH (b\theta + 2c) - \theta P} \left[ \left( a^2 cH^4 (b\theta + c) \right) \right. \]

\[ \left. -2 \left( a \left( b - \theta \right) c + \frac{1}{2} b^2 \theta + c\theta P \right) aH^3 + \right. \]

\[ \left. aH^2 (b\theta + 2c) (P - 2a) + P^2 \right. \]

\[ \left. + ac\theta H^3 - aH^2 (b\theta + c) - a\theta H - p \right) ^{1/2} \]

5.1.2.2 Model: II Production with disruption

In this case, we optimize production system run when production is disrupted due to unplanned event. Due to this, production rate is changed by \( \Delta P \). Production begins disrupted at the time \( T_d \). Production will be increased or decreased as per positive or negative \( \Delta P \) respectively.
Then the governing differential equations are

\[
\frac{dI_1(t)}{dt} + \theta I_1(t) = P - a \left(1 + bt - ct^2\right); \quad 0 \leq t \leq T_d, \quad I_1(0) = 0
\]  
(5.1.6)

\[
\frac{dI_2(t)}{dt} + \theta I_2(t) = P + \Delta P - a \left(1 + bt - ct^2\right); \quad T_d \leq t \leq T_{pd}
\]  
(5.1.7)

\[
\frac{dI_3(t)}{dt} + \theta I_3(t) = -a \left(1 + bt - ct^2\right); \quad T_{pd} \leq t \leq H; \quad I_3(H) = 0
\]  
(5.1.8)

\(I_1(t)\) remains same as per equation (5.1.4).
Solving Equation (5.1.7) using the boundary condition \( I_1(T_d) = I_2(T_d) \), we get

\[
I_2(t) = \frac{e^{-\theta(t-T_d)}}{\theta^3} \left[ \left( P + \Delta P - a \left(1 + bt - ct^2\right)\right) \theta^2 + a \theta(b - 2ct) + 2a c \right] e^{\theta(t-T_d)} - \theta^2 \Delta P 
\] (5.1.9)

Solving equation (5.1.8) using boundary condition \( I_3(H) = 0 \), we get

\[
I_3(t) = \frac{1}{\theta^3} \left[ \left( -1 - Hb + cH^2 \right) \theta^2 + (b - 2cH) \theta + 2c \right] e^{\theta(H-T_p^d)} + \left( -1 - bt + ct^2 \right) \theta^2 + (b - 2ct) \theta + 2c \right] e^{\theta(t-T_p^d)} + 2 \theta^2 \left( 1 + bT_p^d - c(T_p^d)^2 \right) + \left( 4cT_p^d - 2b \right) \theta - 4c 
\] (5.1.10)

If production system satisfies the demand of items after disruption then we find optimal production time \( T_p^d \) such that at time \( H \), entire stock will be sold-out and inventory level will be zero.

Using Condition \( I_2(T_p^d) = I_3(T_p^d) \), we can obtain production time \( T_p^d \).
And if production system after disruption does not satisfy the demand of items then there will be shortages in the system. In this situation, we will find the replenishment time $T_r$ of placing the order and respective order quantity $Q_r$.

We can obtain replenishment time $T_r$ using the condition $I_2(T_r) = 0$.

At obtained $T_r$, we can find order quantity $Q_r$ using $Q_r = I_3(T_r)$.

### 5.1.1.3 Numerical Examples and Sensitivity Analysis

**Example: 1** Taking values $H = 1$ year, $P = 350$ units, $\Delta P = -200$ units, $a = 200$, $b = 0.5$, $c = 0.15$, $\theta = 15\%$ and $T_d = 0.25$ year.

We have $T_p = 0.7069788488$, $T_p^d = 0.8940309846$, $Q_r = 16.52950962$ and $T_r = 0.7205812350$. 

![Figure 5.1.3 Production system after disruption](image)
Following figures show sensitivity of inventory parameters.

**Figure 5.1.4** $T_d$ Vs. $T_p^d$

**Figure 5.1.5** $a$ Vs. $T_p^d$

**Figure 5.1.6** $b$ Vs. $T_p^d$

**Figure 5.1.7** $c$ Vs. $T_p^d$

**Figure 5.1.8** $\theta$ Vs. $T_p$

**Figure 5.1.9** $T_d$ Vs. $T_r$
From figure 5.1.4 we can observe that $T_p^d$ is directly proportional to $T_d$. It is evident from figure 5.1.5, figure 5.1.6 and figure 5.1.7 that increase in scale demand $a$ and linear rate of change of demand $b$ decrease $T_p^d$ while increase in quadratic rate of change of demand $c$ increases $T_p^d$. Figure 5.1.8 shows the obvious fact that larger deterioration rate increase production time when there is no disruption. From figure 5.1.9, we can say that in case of shortages, instantaneous replenishment time gets extended according to increase in disruption time.

**Conclusions**

The production system experiences different types of interruptions. This study determines optimal production time initially when system runs smoothly without disruption and afterwards when system has some sort of disruption due to any reason. It is also taken care that if shortages occur due to disruption then how much new instantaneous replenishment should be made. Quadratic demand for deteriorating inventory units is studied. Graphical analysis is provided for a numerical example. Managers can take decisions from the results e.g. what should be the optimal production time if deterioration rate changes or disruption starts or demand changes. Future research can be done by incorporating various demand structures like stock dependent demand, price sensitive demand or fuzzy demand. Different preservation technologies can also be implemented to reduce deterioration.