Inventory models for deteriorating items for fixed lifetime under two-level trade credits
4.0 Introduction

In this Chapter, the retailer's optimal policies are developed when product has fixed lifetime and also the units in inventory are subject to deterioration at a constant rate. Each and every inventory product gets deteriorated over the time as per its nature and such deteriorating products have its maximum lifetime as well. This study will be mainly applicable to pharmaceuticals, drugs, beverages and dairy products etc. To boost the demand, offering a credit period is considered as the promotional tool. The retailer passes credit period to the buyers which is received from the supplier. The objective is to maximize the total profit per unit time of the retailer with respect to optimal retail price of an item and purchase quantity during the optimal cycle time.

Three models are formulated under the following sections (viz.):

- **Model 4.1** Optimal policies for deteriorating items with maximum lifetime and two-level trade credits.
- **Model 4.2** Optimal policies for trended demand and deteriorating items with maximum lifetime under two-level trade credits.
- **Model 4.3** Optimal credit period and lot-size for deteriorating items with fixed lifetime and quadratic demand under two-level trade credits.

4.1 Optimal Policies for Deteriorating Items with Maximum Lifetime and Two-Level Trade Credits

4.1.1 Assumptions and Notations

The following additional assumptions and notations other than those given in A.1 and N.1 are used to formulate the proposed model:
4.1.1.1 Assumptions

1. The inventory system under study deals with deteriorating items having expiry rate. The deterioration rate tends to 1 when time tends to maximum lifetime $m$. Following Sarkar (2012b), Chen and Teng (2012) as well as Wang et al. (2014), the functional form for deterioration rate is

$$\theta(t) = \frac{1}{1 + m - t}; \quad 0 \leq t \leq T \leq m$$  \hspace{1cm} (4.1.1)

2. There is no repair or replacement of deteriorated items during the cycle time.

3. The credit period $N$ offered to the customers by the retailer results revenue inflow in $[N, T + N]$ (Teng and Goyal (2007))

4. When $M \leq T + N$, the retailer would pay interest during $[M, T + N]$ at the rate $I_c$ for unsold stock. When $M > T + N$, the retailer will settle the account at $M$ and does not incur any interest charges during the cycle.

5. The retailer generates revenue by selling items and earns interest during $[0, M]$. Thus, for $N \leq M$, the retailer accrues revenue and earns interest during $[N, M]$ with rate $I_e$.

4.1.1.2 Notations

- $a$: Constant demand rate
- $I_e$: Interest earned per $ per year
- $I_c$: Interest charged per $ for unsold stock per annum by the supplier
\( M \): Credit period offered by the supplier to the retailer (in days or years)

\( N \): Credit period offered by the retailer to the customer (in days or years)

\( \theta(t) \): Time varying deterioration rate at time \( t \), where \( 0 \leq \theta(t) \leq 1 \)

\( m \): Maximum lifetime (in years) of the deteriorating item

\( I(t) \): Inventory level at any instant of time \( t \), \( 0 \leq t \leq T \)

\( Q \): Procurement quantity per cycle (in units) (a decision variable)

\( \pi(T) \): Retailer’s total profit per unit time (in $)

### 4.1.2 Mathematical Model

The retailer’s initial inventory of \( Q \) units depletes to zero at \( t = T \) due to combined effect of demand and time-dependent deterioration. Hence, the rate of change of inventory level at any instant of time \( t \) is governed by the differential equation

\[
\frac{dI(t)}{dt} = -a - \theta(t)I(t), \quad 0 \leq t \leq T.
\]  

(4.1.2)

with \( I(T) = 0 \). The solution of differential equation (4.1.2) is

\[
I(t) = a(1+m-t)\ln\left(\frac{1+m-t}{1+m-T}\right).
\]  

(4.1.3)

Consequently, the retailer’s order quantity is

\[
Q = I(0) = a(1+m)\ln\left(\frac{1+m}{1+m-T}\right).
\]  

(4.1.4)
The sales revenue is

\[ SR = s \int_0^T a \, dt = s \, a \, T. \]

The ordering cost is \( OC = A. \)

The purchase cost of \( Q \)-unit is \( PC = CQ. \)

The holding cost is

\[ HC = h \int_0^T I(t) \, dt \]

\[ = ah\left[ \frac{1}{2} \ln\left( \frac{1 + m}{1 + m - T} \right)(m^2 + 1) + \ln\left( \frac{1 + m}{1 + m - T} \right) m - \frac{1}{2} T - \frac{1}{2} mT + \frac{1}{4} T^2 \right]. \]

Next, we compute interest earned and interest charges for the retailer in following two cases.

(A) Suppose \( M \geq N \)

Case 1 \( M \geq T + N \)

Here, the retailer has sold all the items before the permissible time \( M \), so the interest charge is zero i.e. \( IC_1 = 0. \) The retailer spawns revenue from the beginning of the cycle and settles the account at time \( N \). So the retailer’s interest earned per cycle is

\[ IE_1 = s I_e \left[ \int_N^{T+N} \int_N^T a \, dv \, dt + \left( M - T - N \right) \int_0^T a \, dt \right]. \]
Case 2 $M \leq T + N$

Here, the retailer lacks the fund to settle the account at $M$ because customer will settle account at time $T + N$. So, the retailer will pay interest charges as

$$IC_2 = CI_c \int_{M}^{T+N} I(t-N) dt.$$ 

and interest earned on the generated revenue at the rate $I_e$ during $[N, M]$, which is given by

$$IE_2 = s I_e a \left[ \frac{1}{2} T^2 + M - N \right].$$

Hence, the total profit per unit time for retailer is

$$\pi(T) = \begin{cases} 
\pi_1(T), & 0 \leq T \leq M - N \\
\pi_2(T), & T \geq M - N 
\end{cases} \quad (4.1.5)$$

where $\pi_1(T) = \frac{1}{T} \{ \text{SR} - \text{PC} - \text{OC} - \text{HC} - IC_1 + IE_1 \} \quad (4.1.6)$

and

$$\pi_2(T) = \frac{1}{T} \{ \text{SR} - \text{PC} - \text{OC} - \text{HC} - IC_2 + IE_2 \} \quad (4.1.7)$$
(B) Suppose \( M \leq N \)

Here, the retailer does not generate any revenue from the customer. So the interest earned by the retailer is \( IE_3 = 0 \). The interest is charged for all the items and is given by

\[
IC_3 = CI_c \left[ (N - M)Q + \int_0^T \int_0^{T+N} I(t-N) dt \right].
\]

The total profit per unit time for retailer is

\[
\pi_3(T) = \frac{1}{T} \{ SR - PC - OC - HC - IC_3 + IE_3 \}.
\]

The goal is to maximize the total profit per unit time with respect to cycle time when items in inventory are deteriorating and having maximum lifetime. The non-linearity of the objective functions in the equations (4.1.6 - 4.1.8) does not allow us to obtain the closed form solution. We analyze the model with numerical values for the inventory parameters in the next section.

4.1.2.1 Numerical Examples

The necessary condition to optimize profit function is to set \( \frac{\partial \pi_1(T)}{\partial T} = 0 \) and follow the steps given below to select the best solution for the retailer.

Step 1: Assign values to all inventory parameters.

Step 2: For \( M \geq N \), solve \( \frac{\partial \pi_1}{\partial T} = 0 \) and \( \frac{\partial \pi_2}{\partial T} = 0 \).
If $M \geq T + N$, then compute total profit per unit time from equation (4.1.6) otherwise compute $\pi_2$ from equation (4.1.7). Knowing optimum cycle time $T$, retailer can determine order quantity using equation (4.1.4).

Step 3: For $M < N$, the retailer’s replenishment time can be calculated by setting $\frac{\partial \pi_3}{\partial T} = 0$. Obtain the total profit per unit time $\pi_3$ from equation (4.1.8) and order quantity from equation (4.1.4).

We consider following examples to validate the mathematical formulation.

**Examples 4.1.1**: Consider $A = $150 per order, $C = $6 per unit, $a = 1,000$ units, $s = $15 per unit, $I_e = 10\% / \$ / \text{year}$, $I_c = 15\% / \$ / \text{year}$, $M = 0.8$ year and $N = 0.4$ year, $m = 2$ years. Then optimum cycle time is 0.2175 years and corresponding profit is $8,239.78$. Clearly, here $M \geq T + N$ is seen. The retailer’s purchase is 226 units. The concavity of the profit function is exhibited in figure 4.1.1.

![Figure 4.1.1 Concavity of total profit w. r. t. cycle time $T$ for $M \geq T + N$]
Examples 4.1.2: Take \( N = 0.72 \) year and all other inventory parameters are as given in Example 4.1.2. The cycle time obtained is 0.1267 year. We have \( M < T + N \) so corresponding profit per unit time for the retailer is $8,559.39 by purchasing 129.45 units. The concavity of the profit per unit time is exhibited in figure 4.1.2.

![Figure 4.1.2 Concavity of total profit w. r. t. cycle time \( T \) for \( M < T + N \)](image)

Examples 4.1.3: To demonstrate the scenario \( M < N \), consider \( M = 0.6 \) year and \( N = 0.8 \) year. Then \( \frac{d\pi_3(t)}{dt} = 0 \), gives cycle time \( T = 0.2260 \) year. The profit is $7,516.68 and purchase quantity is 234.99 units. Figure 4.1.3 shows that profit obtained is concave.

![Figure 4.1.3 Concavity of total profit w. r. t. cycle time \( T \) for \( M < N \)](image)
Next, we study the variations in cycle time (figure 4.1.4) and profit realization (figure 4.1.5) by changing inventory parameters as -40%, -20%, 20% and 40%.

The observations are as follows:

![Figure 4.1.4 Sensitivity analysis for cycle time ($T$)](image)

1. (Figure 4.1.4) The retailer’s cycle time is very sensitive to the credit period offered to the customer. Increase in $N$ increases cycle time. The increase in ordering cost increases cycle time. Increase in demand and delay period $M$ forces retailer to order frequently. Increase in maximum life of deteriorating items increases cycle time. Cycle time is negatively sensitive to cycle time. Other critical inventory parameters are interest charged and interest earned. Both are decreasing cycle time significantly.
Figure 4.1.5 Sensitivity analysis for total Profit

(2) (Figure 4.1.5) The retailer’s total profit per unit time increases sharply when demand and selling price of an item increases. Settling the account at a later date $M$ is also beneficial to the retailer. Though maximum lifetime of product is uncontrollable it can be controlled to increase the profit. The retailer can adopt advance facility to extend life of the product. Increase in purchase cost decreases profit drastically.

The retailer must maintain the balance between the offer of delayed payment, he receives from the supplier and that is passed to the customers. By placing orders frequently the retailer will increase the ordering cost. So the trade-off is also required to combat between ordering cost and credit period $M$.

Figure 4.1.6 shows how smaller delay period encourages decision maker to buy more quantity.
4.2 Optimal Policies for Trended Demand and Deteriorating Items with Maximum Lifetime under Two-Level Trade Credits

Here, we analyze optimal policies for trended demand

4.2.1 Assumptions and Notations

Optimal policies for trended demand and deteriorating items with maximum lifetime under two-level trade credits is developed under the following additional assumptions and notations other than those given in A.1, N.1 and section 4.1.1.

4.2.1.1 Notations

\[ R(t) = a(1+bt) \] where \( a > 0 \) is scale demand and \( 0 < b < 1 \) denotes the rate of change of demand.
4.2.2 Mathematical Model

The retailer’s opening inventory of $Q$ units reduces to zero at $t = T$ due to joint effect of demand and time-dependent deterioration. Hence, the rate of change of inventory level at any instant of time $t$ is governed by the differential equation

$$\frac{dI(t)}{dt} = -a(1+bt) - \theta(t)I(t), \quad 0 \leq t \leq T.\quad (4.2.1)$$

With $I(T) = 0$, the solution of differential equation (4.2.1) is

$$I(t) = a(1+m-t)\left[(1+b(1+m))\ln\left(\frac{1+m-t}{1+m-T}\right) + b(t-T)\right].\quad (4.2.2)$$

Accordingly, the retailer’s order size is

$$Q = I(0) = a(1+m)\left[(1+b(1+m))\ln\left(\frac{1+m}{1+m-T}\right) - bT\right].\quad (4.2.3)$$

The sales revenue is

$$SR = s \int_0^T a(1+bt)dt = saT\left(1 + \frac{bT}{2}\right).$$

The ordering cost is $OC = A$.

The purchase cost of $Q$-unit is $PC = CQ$.

The holding cost is; $HC = h \int_0^T I(t)dt$. 
Subsequently, we need to work out interest earned and interest charges for the retailer in following two cases.

(A) Suppose \( M \geq N \)

Case 1 \( M \geq T + N \)

Here, the retailer has sold all the stuff earlier than the permissible time \( M \), so the interest charge is zero i.e. \( IC_1 = 0 \). The retailer spawns revenue from the commencement of the cycle and patch up the account at time \( N \). So the retailer's interest earned per cycle is

\[
IE_1 = \int_{T}^{T} \int_{0}^{0} R(u) du \, dt + (M - T - N) \int_{0}^{T} R(t) dt.
\]

Case 2 \( M \leq T + N \)

Here, the retailer lacks the fund to clear up the account at \( M \) as customer will settle account at time \( T + N \). So, the retailer will pay interest charges as

\[
IC_2 = C \int_{M}^{T+N} I(t-N) dt.
\]

and interest earned on the generated revenue at the rate \( I_e \) in \([N, M]\) is given by

\[
IE_2 = \int_{\frac{1}{2}T^2 + \frac{1}{6}bT^3 + \frac{1}{2}b(M-N)^2 + M-N}^{\frac{1}{2}T^2 + \frac{1}{6}bT^3 + \frac{1}{2}b(M-N)^2 + M-N}.
\]
Hence, the total profit per unit time for retailer is

\[
\pi(T) = \begin{cases} 
\pi_1(T), & 0 \leq T \leq M - N \\
\pi_2(T), & T \geq M - N
\end{cases}
\]  

(4.2.4)

where \( \pi_1(T) = \frac{1}{T} \{SR - PC - OC - HC - IC_1 + IE_1\} \)  

(4.2.5)

and \( \pi_2(T) = \frac{1}{T} \{SR - PC - OC - HC - IC_2 + IE_2\} \)  

(4.2.6)

(B) Suppose \( M \leq N \)

Here, the retailer does not produce any revenue from the customer. So the interest earned by the retailer is \( IE_3 = 0 \). The interest is charged for all the items and is given by

\[
IC_3 = C I_c \left[ (N - M)Q + \int_{N}^{T+N} I(t - N) dt \right].
\]

The total profit per unit time for retailer is

\[
\pi_3(T) = \frac{1}{T} \{SR - PC - OC - HC - IC_3 + IE_3\}
\]  

(4.2.7)

The goal is to make total profit per unit time maximum with respect to cycle time when deteriorating items have maximum lifetime. The non-linearity of the objective functions in the equations (4.2.5) - (4.2.7) does not permit us to get the closed form solution. We examine the model with numerical values for the inventory parameters in the next part.
4.2.2.1 Numerical Examples

The necessary condition to optimize profit function is to set \[ \frac{\partial \pi_1(T)}{\partial T} = 0 \] and go along the steps given below to select the best solution for the retailer.

Step 1: Allocate values to all inventory parameters.

Step 2: For \( M \geq N \), work out \( \frac{\partial \pi_1}{\partial T} = 0 \) and \( \frac{\partial \pi_2}{\partial T} = 0 \).

If \( M \geq T + N \), then calculate total profit per unit time from equation (4.2.5) otherwise calculate \( \pi_2 \) from equation (4.2.6). Knowing most advantageous cycle time \( T \), retailer can determine order quantity using equation (4.2.3).

Step 3: For \( M < N \), the retailer’s replenishment time can be calculated by setting \( \frac{\partial \pi_3}{\partial T} = 0 \). Obtain the total profit per unit time \( \pi_3 \) from equation (4.2.7) and order quantity from equation (4.2.3).

We consider examples to validate the mathematical formulation.

Examples 4.2.1: Consider \( A = $150 \) per order, \( C = $6 \) per unit, \( h = $2.5/\text{unit/year}, a = 1,000 \) units, \( b = 10\% \), \( s = $15 \) per unit, \( I_e = 10\% / $ / \text{year}, I_c = 15\% / $ / \text{year}, M = 0.8 \text{ year} \) and \( N = 0.4 \text{ year}, m = 2 \text{ years} \). Then optimum cycle time is 0.2314 years and corresponding profit is $8,337.05. Clearly, here \( M \geq T + N \) is seen. The retailer’s purchases 244 units. The concavity of the profit function is demonstrated in figure 4.2.1.
Examples 4.2.2: Take $N = 0.72$ year and all other inventory parameters are as given in Example 4.2.1. The cycle time obtained is 0.1401 year. We have $M < T + N$ so consequent profit per unit time for the retailer is $8,620.49$ by purchasing 144.46 units. The concavity of the profit per unit time is demonstrated in figure 4.2.2.
Examples 4.2.3: To exhibit the scenario $M < N$, consider $M = 0.6$ year & $N = 0.8$ year. Then $\frac{d\pi_3(t)}{dt} = 0$, gives cycle time $T = 0.2401$ year. The profit is $7,609.09$ and purchase quantity is 253.27 units. Figure 4.2.3 confirms that profit obtained is concave.

Figure 4.2.3 Concavity of total profit w. r. t. cycle time $T$ for $M < N$

Next, we examine the variations in cycle time (figure 4.2.4) and profit realization (figure 4.2.5) by altering inventory parameters as -40%, -20%, 20% and 40%.

The interpretations are as follows:

Figure 4.2.4 Sensitivity analysis for cycle time ($T$)
(3) (Figure 4.2.4) The retailer’s cycle time is very responsive to the credit time he offers to his buyer. By increasing $N$, we observe significant effect on cycle time. Increase in ordering cost increases cycle time. Increase in scale demand and delay period $M$ drives retailer to order repeatedly. Increase in maximum life of deteriorating items increases cycle time. Cycle time is negatively sensitive to purchase cost. Other critical inventory parameters are interest earned and interest charged. Interest earned decreases cycle time drastically and interest charged marginally. When cycle time decreases, retailer has to order frequently and hence ordering cost increases in such cases.

![Figure 4.2.5 Sensitivity analysis for total Profit](image)

(4) (Fig. 4.2.5) If there is an increase in demand and selling price, retailer's total profit per unit time increases sharply. Settling the account at a later date $M$ is also advantageous to the retailer. Though maximum lifetime of product is uncontrollable, it can be controlled to improve the profit. The
retailer can implement advance facility to lengthen life of the product. We observed a remarkable decrease in profit if purchase cost is increased. The retailer must care for the balance between the offers of delayed payment, he obtains from the supplier and that is passed on to the customers. If retailer places order often, then ordering cost gets increased. So the trade-off is also necessary to combat among ordering cost and credit period $M$.

4.3 Optimal Credit Period and Lot-Size for Deteriorating Items with Fixed Lifetime and Quadratic Demand under Two-Level Trade Credits

Here, time-credit dependent demand is considered.

4.3.1 Assumptions and Notations

Optimal credit period and lot-size for deteriorating items with fixed lifetime and quadratic demand under two-level trade credits is developed under the following additional assumptions and notations other than those given in A.1, N.1, section 4.1.1 and 4.2.1

4.3.1.1 Assumptions

1. For retailer, default risk increases if longer credit period is presented to the customers. Here, the rate of default risk giving the credit period $N$ is taken to be $F(N) = 1 - e^{-\beta N}$ where $\beta > 0$ is constant.
4.3.1.2 Notations

\[ R(t, N) := -a \left(1 + bt - ct^2\right) e^{\alpha N} \]

where \( a > 0 \) is scale demand, \( 0 \leq b < 1 \) denotes the linear rate of change of demand with respect to time, \( 0 \leq c < 1 \) denotes the quadratic rate of change of demand and \( \alpha \) is mark up for trade credit.

4.3.2 Mathematical Model

The retailer's opening inventory of \( Q \) units depleted to zero at the time \( t = T \) due to demand and time dependent deterioration. Hence, the rate of change of inventory at any instantaneous time \( t \) is governed by the differential equation

\[ \frac{dI(t)}{dt} + \theta(t) I(t) = -a \left(1 + bt - ct^2\right) e^{\alpha N}. \]

Using boundary condition \( I(T) = 0 \), the solution of the differential equation is given by

\[ I(T) = a e^{\alpha N} (1 + m - t) \cdot \left[ \left(1 + b(1 + m) - c(1 + m)^2\right) \ln\left(\frac{1+m-t}{1+m-T}\right) - b(T - t) + \frac{c}{2} (T^2 - t^2) + c(T - t)(1 + m) \right] \]

Consequently, the ordering quantity for the retailer is

\[ Q = I(0) = a e^{\alpha N} (1 + m) \cdot \left(1 + b(1 + m) - c(1 + m)^2\right) \ln\left(\frac{1+m}{1+m-T}\right) - bT + \frac{cT^2}{2} + cT(1 + m). \]

Sales revenue is given by \( SR = \int_{0}^{T} R(t, N) dt (1 - FN) \).
Purchase cost is given by \( PC = CQ \).

Holding cost is given by \( HC = h \int_0^T I(t) \, dt \).

Ordering cost is given by \( OC = A \).

Now we calculate retailer’s interest earned and interest charges for the following two cases

(A) \textbf{Suppose } \( M \geq N \)

\textbf{Case 1 } \( M \leq T + N \)

In this case, retailer has to pay the supplier at \( M \), as he gets the payment at \( T + N \) from the customers, so the retailer pays the interest at the rate \( I_c \) in \( [M, T + N] \) as

\[
IC_1 = CI_c \int_M^{T+N} R(t) \, dt.
\]

and earn interest on income from costumer at the rate \( I_e \) in \( [N, M] \) is given by

\[
IE_1 = sI_e \int_0^{M-N} R(t) \, dt.
\]

\textbf{Case 2 } \( M \geq T + N \)

Here, the retailer sold all the items before the permissible delay period \( M \) given by supplier, so the interest charge is zero

i.e. \( IC_2 = 0 \)
and he earns revenue at $N$ and settles the account at $M$ with the supplier. So he earns interest in $[N, M]$ as

$$IE_2 = s I_e \left[ T \int_0^T R(u) du + (M - T - N) \int_0^T R(t) dt \right].$$

Hence, the total profit per unit time for retailer is

$$\pi(T, N) = \begin{cases} 
\pi_1(T, N), & 0 \leq T \leq M - N \\
\pi_2(T, N), & T \geq M - N 
\end{cases}$$

where $\pi_1(T, N) = \frac{1}{T} \{SR - PC - HC - OC - IC_1 + IE_1\}$

and $\pi_2(T, N) = \frac{1}{T} \{SR - PC - HC - OC - IC_2 + IE_2\}$

(B) Suppose $M \leq N$

In this case, the retailer is not generating any revenue to earn interest as his permissible delay period is more than the supplier's delay period.

i.e. $IE_3 = 0.$

and pays interest charge as

$$IC_3 = CI_e \left[ (N - M) Q + \int_N^{T+N} I(t + N) dt \right].$$
The total profit per unit time for retailer is

\[ \pi_3(T,N) = \frac{1}{T} \{ SR - PC - HC - OC - IC_3 + IE_3 \} . \]

The aim is to make total profit per unit time maximum with respect to cycle time and retailer's delay period when items are deteriorating and having maximum lifetime. We examine the model with numerical values for the inventory parameters.

**Numerical Examples**

**Example 4.3.1:** Taking \( A = $150 \) per order, \( C = $6 \) per unit, \( h = $2.5/\text{unit/year} \), \( a = 1,000 \) units, \( b = 10\% \), \( c = 15\% \), \( s = $15 \) per unit, \( I_e = 10\% / \$ / \text{year} \), \( I_e = 15\% / \$ / \text{year} \), \( \alpha = 0.5 \), \( \beta = 0.2 \), \( M = 0.25 \) year and \( m = 2 \) years. Then optimum cycle time is 0.23 year and corresponding profit is $8,082.9 and \( N = 0.036 \). Clearly, here \( M \leq T + N \) is seen. The retailer's purchases 244 units. The concavity of the profit function is demonstrated in figure 4.3.1.
Figure 4.3.1 Concavity of total profit w. r. t. cycle time $T$ and retailer’s delay period $N$ for $M \leq T + N$

**Example 4.3.2:** Using the same values as in example 4.3.1 except $M = 0.82$ year, we get optimum cycle time is 0.22 year and corresponding profit is $8,975.8$ and $N = 0.165$. Clearly, here $M \geq T + N$ is seen. The retailer's purchases 245 units. The concavity of the profit function is demonstrated in figure 4.3.2
**Example 4.3.3:** Using the same values as in example 4.3.1 except $M = 0.25$ year we get optimum cycle time is 0.22 year and corresponding profit is $8,060.3$ and $N = 0.26$. Clearly, here $M \leq N$ is seen. The retailer’s purchases 262 units. The concavity of the profit function is demonstrated in figure 4.3.3
Comparison of total profit for three cases is exhibited in figure 4.4.4. It clearly demonstrates that retailer is more beneficial when supplier’s credit period is more than retailer’s credit period and cycle time.

Now for the case \( M \geq T + N \), we examine the effects of various inventory parameters on total profit, decision variables cycle time \((T)\) & credit period offered by retailer \((N)\) by varying them as -40%, -20%, 20% and 40%. The observations are as follows.
Figure 4.3.5 Sensitivity analysis for total profit

Figure 4.3.6 Sensitivity analysis for cycle time ($T$)
Figure 4.3.7 Sensitivity analysis for credit period offered by retailer (N)

Table 4.3.1: Summary of sensitivity analyses

<table>
<thead>
<tr>
<th>Inventory Parameters</th>
<th>Total Profit</th>
<th>Cycle Time (T)</th>
<th>Credit Period offered by retailer (N)</th>
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<td>$m$</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>$\beta$</td>
<td>↓</td>
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<td>↓</td>
</tr>
<tr>
<td>$C$</td>
<td>↓</td>
<td>↑</td>
<td>↓</td>
</tr>
</tbody>
</table>

↑ shows increasing pattern  ↓ shows decreasing pattern
From figure 4.3.5 - 4.3.7 & Table 4.3.1 it is clear that by increasing scale demand, linear rate of change of demand, mark up for trade credit, unit sale price, interest earned, credit period offered by supplier to the retailer and fixed lifetime total profit gets increased while by increasing quadratic rate of change of demand, ordering cost, holding cost, mark up for default risk and purchase cost total profit gets decreased.

These observations are very obvious. Hence to increase profit for the retailer more credit period should be asked. Lifetime of products can be increased in some cases by providing them required environment and thus profit can be increased. Also retailer should try to lower all costs. Retailer can lower default risk by giving some marketing offers.

Cycle time has positive effect of linear rate of change of demand, ordering cost, interest earned, maximum lifetime, mark up for default risk and purchase cost and cycle time has negative effect of scale demand, quadratic rate of change of demand, mark up for trade credit, holding cost, unit sale price and credit period offered by supplier to retailer.

Result shows that scale demand, quadratic rate of change of demand, mark up for trade credit, unit sale price, credit period from supplier to retailer and maximum lifetime have positive impact on credit period $N$ where as linear rate of change of demand, ordering cost, holding cost, interest earned, mark up for trade credit and purchase cost have negative impact on it.
Conclusions

In this chapter, economic ordering policies is studied for constant demand, trended demand and quadratic demand under two-level trade credit to maximize the profit for the retailer when product has fixed lifetime and is deteriorating in nature. This research is applicable to the products which are deteriorate continuously and having maximum lifetime, e.g. food, beverages, dairy products, medicines, electronics goods as new technology replaces the old technology etc. It is observed from the numerical examples that the retailer should wisely choose the payment time for the settlement of the accounts to the supplier and from the customer. For the retailer total profit increases if he gets more credit period from the supplier and gives smaller credit period to his customers. By this retailer can reduce the risk of non-payment from customers. We have also considered the default risk in the last model for the retailer and finally maximized the total profit of the retailer with respect to credit period and cycle time.