CHAPTER III

PRELIMINARIES

This brief chapter introduces definitions and notations that are used across Chapters IV, V and VI. Some well-known basic results on intervals that are relevant to the present work are also stated.

3.1 DEFINITIONS

DEFINITION 1: *Domain*

A set D with a total order is called a domain. For any $x, y \in D, x \leq y$ if x appears before y in the ordering.

DEFINITION 2: *Interval*

Given a domain D and $x, y \in D$, a subset [x, y] of D defined by $\{z \mid x \leq z \leq y\}$ is called an interval.

DEFINITION 3: *Endpoint*

Given a domain D and a set B of brackets {'[', ']'}, an element (x, b) of D x B is called an endpoint. An endpoint (x, '[') is called a left endpoint and an endpoint (x, ']') is called a right endpoint.

The set of left and right endpoints over a domain D will be denoted by LE(D) and RE(D) respectively. An element (L, R) of $LE(D) \times RE(D)$ can be associated with an interval [x, y], where L = (x, '[') and R = (y, ']').

In the set B, a total order $\leq$ is defined as '[' $\leq$ ']'. Accordingly, a total order $\leq$ is obtained in the set of endpoints as follows:

$$(x, b_1) \leq (y, b_2) \text{ if } x < y$$

or

$$\text{if } x = y \text{ and } b_1 \leq b_2$$

e.g. $(3,]) \leq (4, [)$

$(2, [) \leq (2,])$

DEFINITION 4: *Interval Transaction*

An ordered pair (tid, I) is called an interval transaction; tid is a transaction identifier and I is an interval. For a given interval transaction T = (tid, I), intv(T) denotes the interval I.

DEFINTION 5: *Endpoint of an interval transaction*

If T is an interval transaction, then the endpoints of intv(T) are said to be the endpoints of T.

DEFINITION 6: *Interval Transaction Database*

An interval transaction database TDB is a set of interval transactions.

DEFINITION 7: *Endpoint in an interval transaction database*

An endpoint is said to be in an interval transaction database TDB if it is an endpoint of some interval transaction in TDB.

DEFINITION 8: *Interval in an interval transaction database*

An interval I is said to be in an interval transaction database TDB if $I = intv(T)$ for some interval transaction T in TDB.

3.2 SOME USEFUL RESULTS

1. [a, b] is non-empty iff $a \leq b$

2. Suppose [a, b] is non-empty, then

(a) $[a, b] \subseteq [c, d]$ iff $c \leq a \leq b \leq d$

(b) $[a, b] \subset [c, d]$ iff either $c < a$ and $b \leq d$ or $c \leq a$ and $b < d$

3. If the intersection of k intervals is non-empty, then it is an interval with endpoints from the k intervals.

The proofs of these well-known results are provided in Appendix I