CHAPTER III

PRELIMINARIES

This brief chapter introduces definitions and notations that are used across Chapters IV, V and VI. Some well-known basic results on intervals that are relevant to the present work are also stated.

3.1 DEFINITIONS

DEFINITION 1: Domain
A set $D$ with a total order is called a domain. For any $x, y \in D$, $x \leq y$ if $x$ appears before $y$ in the ordering.

DEFINITION 2: Interval
Given a domain $D$ and $x, y \in D$, a subset $[x, y]$ of $D$ defined by $\{z \mid x \leq z \leq y\}$ is called an interval.

DEFINITION 3: Endpoint
Given a domain $D$ and a set $B$ of brackets $\{[', ']\}$, an element $(x, b)$ of $D \times B$ is called an endpoint. An endpoint $(x, '[')$ is called a left endpoint and an endpoint $(x, ']$) is called a right endpoint.

The set of left and right endpoints over a domain $D$ will be denoted by $LE(D)$ and $RE(D)$ respectively. An element $(L, R)$ of $LE(D) \times RE(D)$ can be associated with an interval $[x, y]$, where $L = (x, '[')$ and $R = (y, ']$).
In the set B, a total order ≤ is defined as ‘[’ ≤ ‘]’. Accordingly, a total order ≤ is obtained in the set of endpoints as follows:

\[(x, b_1) \leq (y, b_2) \text{ if } x < y \]

\[\text{ or }\]

\[\cdot \text{ if } x = y \text{ and } b_1 \leq b_2\]

e.g. \((3, ]) \leq (4, ]\)

\((2, ] \leq (2, ]\)

**DEFINITION 4: Interval Transaction**

An ordered pair \((\text{tid}, I)\) is called an interval transaction; \(\text{tid}\) is a transaction identifier and \(I\) is an interval. For a given interval transaction \(T = (\text{tid}, I)\), \(\text{intv}(T)\) denotes the interval \(I\).

**DEFINITION 5: Endpoint of an interval transaction**

If \(T\) is an interval transaction, then the endpoints of \(\text{intv}(T)\) are said to be the endpoints of \(T\).

**DEFINITION 6: Interval Transaction Database**

An interval transaction database \(\text{TDB}\) is a set of interval transactions.

**DEFINITION 7: Endpoint in an interval transaction database**

An endpoint is said to be in an interval transaction database \(\text{TDB}\) if it is an endpoint of some interval transaction in \(\text{TDB}\).

**DEFINITION 8: Interval in an interval transaction database**
An interval $I$ is said to be in an interval transaction database $TDB$ if $I = \text{intv}(T)$ for some interval transaction $T$ in $TDB$.

### 3.2 SOME USEFUL RESULTS

1. $[a, b]$ is non-empty iff $a \leq b$

2. Suppose $[a, b]$ is non-empty, then
   
   (a) $[a, b] \subseteq [c, d]$ iff $c \leq a \leq b \leq d$
   
   (b) $[a, b] \subset [c, d]$ iff either $c < a$ and $b \leq d$ or $c \leq a$ and $b < d$

3. If the intersection of $k$ intervals is non-empty, then it is an interval with endpoints from the $k$ intervals.

*The proofs of these well-known results are provided in Appendix I*