CHAPTER V

SYSTEM SIZE EFFECT
AND
CRITICAL BEHAVIOUR IN NUCLEAR MULTIFRAGMENTATION
5.1 Introduction

Studies on intermediate energy A+A collision are of special significance from the point of view that such collisions are associated with abundant multifragment production [1]. The connection between the multifragmentation (MF) as decay mechanism of excited nuclei and a possible liquid gas phase transition taking place in the nuclear matter has been a subject of hot debate during the last two decades or so [2-4]. From the presence of the power law in fragment mass distribution and from the observation of the values of the exponents of various charge moments close to those of ordinary fluid, coupled with the strong similarity of the nuclear and Van-der-Waals potentials, it was inferred that multifragmentation of nuclei might be analogous to a continuous phase transition from liquid to a gas [5-7].

Campi [8-9] and Bauer et al. [10-11] showed that the methods of estimation of various moments of cluster size as used in percolation can be applied to analyze multifragmentation data to realize the possible association of criticality with such processes. Subsequently a large number of workers [8-23] have applied the technique of cluster approximation in analyzing projectile fragmentation data for various systems at different energies. From all such studies a number of important information about the possible liquid-gas phase transition in nuclear matter could be gathered. A few of these are: tentative estimation of critical temperature, estimation of various exponents and their scaling behaviour [16-18, 24], system size dependence of the position and height of critical singularity [18], role of Coulomb and surface energies [25] on MF process etc.

Finite size effect [26-33] has been found to have considerable influence on MF mechanism. This is due to the fact that the hot piece of nuclear matter produced in any nuclear collision has at most a few hundreds of nucleons and so is not adequately described by the properties of infinite nuclear matter. As stated earlier surface and Coulomb effects can play a significant role in finite nuclear system. These effects have been evaluated and lead to a sizable reduction of the critical temperature [28]. Finite size effects have been found to reduce the temperature by 2-6 MeV depending on the size of nuclei while the Coulomb force is responsible for
a further reduction of 1-3 MeV. However large reductions due to small sizes are associated with small reductions from Coulomb effect. Consequently, in the range A=50-400 a total reductions of about 7 MeV is calculated leading to a “critical” temperature of about 10 MeV for nuclei or hot pieces of nuclear matter produced in collisions between very heavy nuclei. The authors of reference [28] indicate that, due to some approximation the derived values can be regarded as upper limits. Finally we can recall that in infinite nuclear matter, the binding energy per particle is 16 MeV whereas it is about 8 MeV in a finite nucleus. A finite system like a nucleus with its limited number of constituents does not exhibit, in the neighborhood of critical point, a sharp singularity in the traditionally accepted signatures of critical behavior. It rather shows a rounded finite peak [18] at a temperature $T = T'$ where $T'$ is a temperature dependent on the system size. This temperature $T'$ is regarded as the critical temperature of the finite system at and above which the distinction between the phases vanishes; $T'$ corresponds to $T_c$, the critical temperature for an infinite system, as the system size approaches infinity. The effect of finite size thus not only lowers the peak value, but also lowers the critical point [34-35] thereby influencing even the order of phase transition.

EOS collaboration [18-19, 27] has analyzed the fragments mass distribution resulting from the interactions of 1 AGeV Au, La and Kr on carbon and observed that while for larger systems like Au and La the fragment yield distribution follows a power law with exponent values greater than 2; for Kr, the mass yield distribution is exponential with exponent value 1.88. They reported that there is a systematic variation in the peak value of the reduced variance, $\sigma^2$ for Au, La and Kr. For Au and La, the peak values of $\gamma_2$ are reported to be greater than 2 and for Kr, the smallest of the three, the value is found to be less than 2. Such variation in the peak value of $\gamma_2$ was attributed to system size effect ruling out the possible phase transition in Kr to be a continuous one. It has been argued by EOS group that a change in the size of the fragmenting nuclei changes the Coulomb energy [25] of the system which in turn shifts the critical point. In contrast to percolation [36], Ising [37] or microcanonical Monte Carlo (MMMC) [38] model predictions, they reported a decrease in the critical temperature with the increase of system size. Such
observation was explained in the light of statistical multifragmentation model (SMM) [39] and attributed to the dominant role played by the Coulomb energy over surface energy in the multifragmentation mechanism. In other studies, it was found that finite size effect and Coulomb force lead to a considerable reduction in the critical temperature [17,19,28] and the critical temperature decreases with the decrease of system size. Thus there exists clear contradictions, both in theory and observation, about the roles played by charge and finite size of the nuclei in nuclear multifragmentation processes and till date it is not very clear which of these two effects plays the more dominant role in MF mechanism.

In chapter IV, to realize the signature of phase transition, the data of this work on Mg-Em interaction at 4.5 AGeV has been analyzed using scaled factorial moment technique and a clear evidence of non thermal liquid gas phase transition is observed for the charge distribution of the projectile fragments. In this chapter an attempt has been made, by using cluster approximation technique, to examine the influence of system size on the critical behavior of nuclear matter in the fragmentation of Mg nuclei at 4.5 AGeV.

5.2 Mathematical formalism

The distance $\varepsilon$ of a given event from the critical point is generally measured by taking a difference between the total charged fragment multiplicity, $m$ and the multiplicity at the critical point, $m_c$. Thus:

$$\varepsilon = |m_c - m|$$

(5.1)

The total charged fragment multiplicity $m$ as defined in ref. [8-10, 17], is:

$$m = N_f + N_\alpha + N_{prot}$$

(5.2)

where $N_f$, $N_\alpha$ and $N_{prot}$ denote the number of heavy PFs with charge $Z_{PF} \geq 3$, alpha particles with $Z_{PF} = 2$ and the number of emitted protons with $Z_{PF} = 1$ respectively. Here $N_{prot}$ is determined by using charge balance of the PFs, $m$ is a parameter which is considered to be linearly related with the temperature $T$ of the system.
Thus, $s = mc - m$ gives a measure of the distance of a given event from the critical point [16, 41-43].

5.3 Charge moments and Conditional moments

In order to extract relevant information from experimental data we need exclusive experiments in which the sizes (generally the charges) of almost all fragments are measured event by event. The method of single event moments is used here, following campi.

For a single event, Campi [8-10] has defined the $k^{\text{th}}$ moment of charge distribution as:

$$M_k(\varepsilon) = \sum n_{z_{pf}}(\varepsilon)Z_{pf}^k \quad (5.3)$$

and for a collection of data, $<M_k(\varepsilon)>$ in the small bins of multiplicity $m$ as:

$$<M_k(\varepsilon)> = \frac{1}{N} \sum \sum' n_{z_{pf}}(\varepsilon)Z_{pf}^k$$

$$\quad (5.4)$$

Here $n_{z_{pf}}$ is the normalized charge distribution and is defined as $n_{z_{pf}} = N_{z_{pf}} / Q_{pf}$, $Q_{pf}$ is the sum of charges of all the projectile spectator protons, fast alpha particles and heavy projectile fragments with charges $Z_{pf} \geq 3$. $N$ denotes the total number of events in a given small range of $\varepsilon$, and $M_k^{(i)}$ is the $k^{\text{th}}$ order charge distribution moment for $i^{\text{th}}$ event.

These moments are related to the basic physical quantities. For example, $M_0^{(i)}$ is the number of fragments (minus one) present in the event $i$. $M_i^{(i)}$ is the mass (or charge) of these $M_0^{(i)}$ fragments,

$$M_i^{(i)} = Z_0 - Z_{\text{max}}(i) \quad (5.5)$$

and $M_i^{(i)} / M_0^{(i)}$ is the mean size of these fragments. Moments with $k < 0$ are mainly sensitive to the distribution of very light fragments.
The variance of the distribution is given by:

$$\sigma^{(t)^2} = \frac{M_2^{(t)}}{M_0^{(t)}} - \left(\frac{M_1^{(t)}}{M_0^{(t)}}\right)^2$$  \hspace{1cm} (5.6)

Which is related to a quantity $\gamma_2$ defined as:

$$\gamma_2^{(t)} = \frac{\sigma^{(t)^2}}{<Z^{(t)}>^2} + 1 = \frac{M_2^{(t)}M_0^{(t)}}{M_1^{(t)^2}}$$  \hspace{1cm} (5.7)

This quantity is known as reduced variance.

5.4 Results and discussion

5.4.1 Frequency distribution of charged projectile fragments

Frequency distribution of various charged projectile fragments with $Z_{PF} \geq 1$ emitted from Mg-Em interactions is plotted in Fig. 1(a). Similar distribution fitted with a power law has also been reported in projectile fragmentation of $^{238}\text{U}$ at 0.96 AGeV, $^{84}\text{Kr}$ at 1.25 AGeV, $^{131}\text{Xe}$ at 1.22 AGeV and $^{84}\text{Kr}$ at 0.95 AGeV in nuclear emulsion [17,20-23,40]. For the experimental data of the present investigation, considering total number of system constituents as 12, yields of the fragments charge distribution, lying between 1-6, have been re-plotted in log-log scale in Fig.1(b) and compared with the results of GU [17] works on Kr-Em interactions at 0.95 AGeV. A straight line fit using least square approach to the respective data points gives the values of the exponent $\tau$, $2.54 \pm 0.417$ and $2.12 \pm 0.15$ respectively for Mg-Em and Kr-Em [17] interactions.
Fig. 5.1(a) Frequency distribution of various charged projectile fragments with $Z_{PF} \geq 1$ for Mg-Em interactions at 4.5 AGeV.
5.4.2 Size effects on fluctuation in $Z_{\text{max}}$

It is known that a system exhibits significant fluctuations in the neighborhood of the critical point in a small range of the control parameter and appears at increasingly large scale as $\varepsilon \to 0$. Elliott et al. [18] have pointed out that in the study of the critical behavior of the nuclear system using cluster approximation technique; the most readily observed fluctuations in the cluster distribution are those in the size of the largest cluster.

In Fig. 5.2, the standard deviations of $Z_{\text{max}}$ normalized with respect to the charge of the projectile are shown as a function of multiplicity for both Mg and Kr-Em interactions. While for Kr large fluctuations in the multiplicity range between 11-19 are readily seen from this plot with a peak at $m = 17 \pm 1$, for Mg, such fluctuation is observed in the range of 4-9 with a peak at $m = 5 \pm 1$. The
distinct differences in the heights and positions of the two peaks as seen in Fig. 2 is believed to be due to different system size of the fragmenting nuclei [17-19,26,40].

Fig. 5.2 Standard deviation of normalized $Z_{\text{max}}$ as a function of multiplicity $m$ for Mg-Em and Kr-Em interactions.

5.4.3 Fluctuation in $\gamma_2$

The $\gamma_2$ values have been calculated event by event as a function of total charge multiplicity for Mg and compared the results with the results of the earlier works on Kr-Em [17] and are presented in Fig. 5.3(a). The error estimations are made considering these to be independent statistical errors only. It can readily be seen from this plot that a considerable change in the height and position of $\gamma_2$ values take place as one goes from Kr to Mg projectile system. While the peak height is almost 5 times more in case of Kr than that of Mg, the position of the peak is 4.5 times less in Mg than that of the other one. In Fig. 5.3(b), the results of the present investigation have been compared with the result of EOS [26-27] works. It is
interesting to see that both the height and position of the peak values of $\gamma_2$ vary systematically as one varies the system size from Au down to Mg via La and Kr. In the table 5.1, the values of these quantities have been listed for different systems as obtained by earlier workers of GU group [17] and EOS collaboration independently.

Fig. 5.3(a) Variation of $\gamma_2$ with total charged fragment multiplicity $m$ for Mg-Em and Kr-Em interactions.
Fig. 5.3(b) Variation of $\gamma_2$ with total charged fragment multiplicity $m$ for Mg-Em and Au, La and Kr on C.

To cross check the size effect, the variation of $\gamma_2$ is plotted against $m / Z_{proj}$ in Fig. 5.3(c). From this plot it is readily evident that the transition is taking place almost at same normalized multiplicity (temperature) for various systems under consideration. Collapsing of transition temperature (multiplicity) to a particular value, when normalized by respective beam size (charge) rather confirms size effect only.
Fig. 5.3(c) Variation of $\gamma_2$ against $m/Z_{proj}$.

Table 5.1 Variation of peak height and peak position in $\gamma_2$ with the system size.

<table>
<thead>
<tr>
<th>Systems compared</th>
<th>Peak Height of $\gamma_2$</th>
<th>Peak Position of $\gamma_2$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Au-C at 1 AGeV</td>
<td>3.24</td>
<td>28 ± 3</td>
<td>EOS [26,27]</td>
</tr>
<tr>
<td>La-C at 1 AGeV</td>
<td>2.43</td>
<td>24 ± 3</td>
<td>EOS [26,27]</td>
</tr>
<tr>
<td>Kr-C at 1 AGeV</td>
<td>1.93</td>
<td>18 ± 2</td>
<td>EOS [26,27]</td>
</tr>
<tr>
<td>Kr-Em at 0.95 AGeV</td>
<td>1.5</td>
<td>18 ± 1</td>
<td>GU group [17]</td>
</tr>
<tr>
<td>Mg-Em at 4.5 AGeV</td>
<td>0.39</td>
<td>5 ± 1</td>
<td>Present work</td>
</tr>
</tbody>
</table>
Thus, unlike EOS findings of ref.[25], the emulsion data of the present work for a smaller system like Mg confirms percolation, Ising and MMMC models prediction that the critical temperature should decrease with the decrease of size of the fragmenting nuclei [19,26-27].

5.4.4 Fluctuation in $M_2$

Another traditional signature, often used in cluster approximation technique to realize critical behaviour, is the exhibition of a peaking behaviour in the mean value of second moment of charge distribution $<M_2>$ in a small bin of the control parameter. In the present investigation $M_2$ is calculated excluding the largest fragment which for the present work is considered to be 9. While excluding the largest fragment generally half of the size is considered. This is mainly because to exclude the contribution of fission fragments. Since in present case fission is not a possibility, it is therefore considered to be $Z_{PF} = 9$.

In Fig. 5.4(a), $<M_2>$ for Mg-Em data is plotted against m and compared with the results of Kr-Em interactions [17]. Fig. 5.4(b) represents the same plot comparing present results on Mg-Em and earlier GU result on Kr-Em with EOS result on Au-C interactions. Clear evidence of size effect could be seen again with significant differences in the position and heights of the peaks.
Fig. 5.4(a) Average of second charge moments as a function of multiplicity $m$ for Mg-Em and Kr-Em interactions.
Fig. 5.4(b) Average of second charge moments as a function of multiplicity m for Mg-Em, Kr-Em and Au-C.
References


