CHAPTER - 9

OPTIMAL TIME INTERVAL BETWEEN SCREENING TESTS FOR
PROMOTIONS IN MANPOWER PLANNING

9.1. INTRODUCTION

In the previous three chapters, cost analysis for recruitments and promotions are discussed. In this chapter, the optimum time interval between screening tests for promotions is obtained so as to minimize the expected total cost incurred in the process. More specifically, an organization with three grades is considered where the vacancies are filled either by promotions in the form of screening test (or) by direct recruitments. A mathematical model is constructed and the optimal time interval between successive screening tests is obtained so as to minimize the expected total cost. The results are illustrated by numerical examples assuming specific distribution.

Recruitments of the required human resource play a vital part to achieve the goals of an organization. The use of compartment models in manpower planning for promotions in quite common. Cardenas, Matis [1975], Parde [1975], and Thakur and Rescigno [1978] have discussed compartment models in manpower system. Agrafiotis [1991], has discussed a general time dependent stochastic model for the analysis of two compartment reversible system with non-homogeneous Poisson inputs and arbitrary residence times in each of compartments. He has derived various results in terms of the distribution of the residence times only and expressed these results in the form of convolution
integrals, which are readily evaluated by means of known computational algorithms. He has applied his theoretical results to examples from the literature on manpower planning and considered some practical implications for the personnel and labour fields. Mariappan and Srinivasan (2002) have obtained the optimum time interval between screening tests for promotions in an organization having two compartments. In the present chapter, the optimum cost analysis is made for an organization having three compartments under a different set of conditions.

Consider an organization which has three compartments (grade) $B_1$, $B_2$ and $B_3$ where the sizes of $B_1$ and $B_2$ are fixed as $n_1$ and $n_2$ respectively. The compartment $B_2$($B_1$) may be thought of as one consisting of personnel with greater skills, efficiency and administrative capabilities than those in $B_1$($B_2$). Transition of personnel from $B_1$ to $B_2$ and $B_2$ to $B_3$ is allowed and in between there is a screening test to evaluate the competence of individuals to get into $B_2$ from $B_1$ and into $B_3$ from $B_2$. The personnel in $B_1$ and $B_2$ are first recruited and kept in reserve. Assuming that they are given some training to improve their capabilities, keeping these personnel in $B_1$, $B_2$ and training them involves a maintenance cost (or) reserve cost. Conducting the test but with no persons getting entry to $B_2$ and $B_3$ involves some cost namely screening test cost which is a total loss. In case no person gets selected and enters into $B_2$ and $B_3$, the vacancies in $B_2$ and $B_3$ remain unfilled and each such unfilled vacancy give rise to some shortage cost in terms of loss of productivity. To offset the loss,
recruitment of personnel from outside to $B_2$ and $B_1$ is made on an emergency basis. The longer the time interval between the screening tests, greater will be the cost of maintenance of personnel in $B_1$ and $B_2$, which in turn increases the cost of shortages in $B_2$ and $B_1$, respectively. Frequent screening test results in higher test costs.

In this chapter, two mathematical models are constructed. In Model -I, the screening test for promotions from $B_1$ to $B_2$, $B_2$ to $B_1$ are not conducted simultaneously. In this case $T_1$ is the time between two consecutive screening tests for promotions from $B_1$ to $B_2$ and $T_2 \ (T_2 \neq T_1)$ is the time between two consecutive screening tests for promotions from $B_2$ to $B_1$. In Model-II, the screening test for above cited promotions are conducted simultaneously. In this case, $T$ is the time between two consecutive screening tests. In fact Model -II is a special case of Model -I when $T_1 = T_2 = T$.

The main objective of this chapter is to obtain the optimal time interval between successive screening tests to minimize the expected total cost. The rest of this chapter is organized as follows: In section 9.2, description of both the models is given. Section-9.3 deals with the cost analysis for the two models and section 9.4 deals with a special case by assuming the time between successive screening tests follow exponential distribution. In section 9.5, numerical illustrations are provided. In section 9.6, the conclusions are stated.
9.2. DESCRIPTION OF THE MODELS

In this section, the assumptions and notations of both the models are stated below.

Assumptions

1. The size of $B_1$ and $B_2$ are fixed.

2. For Model-I, the screening tests are not conducted simultaneously and so transition from $B_2$ to $B_3$ is not permitted. For Model-II, transition from $B_1$ to $B_2$ and $B_2$ to $B_3$ is permitted on the basis of screening test conducted simultaneously.

3. If there are $k$ vacancies in $B_2$, $r$ out of $k$ are selected from $B_1$ with constant probability $p_1$ and $(k-r)$ are selected outside $B_1$ with probability $q_1$ and $p_1 + q_1$.

4. If $m$ vacancies exist in $B_1$, $j$ out of $m$ are selected from $B_2$ with constant probability $p_2$ and $(m-j)$ are selected outside $B_2$ with probability $q_2$ and $p_2 + q_2$. Here $p_1 > p_2$.

Notations

$n_1$ : the size of $B_1$

$n_2$ : the size of $B_2$

$C_{r_1}$ : the cost of retention of each person in $B_1$ per unit of time

$C_{r_2}$ : the cost of retention of each person in $B_2$ per unit of time
\( C_{11} \) : the cost of screening test for transition form \( B_1 \) to \( B_2 \) per unit of time

\( C_{12} \) : cost of screening test for transition form \( B_2 \) to \( B_3 \) per unit of time

\( C_{21} \) : the cost of each unfilled vacancy in \( B_2 \) per unit of time

\( C_{22} \) : the cost of each unfilled vacancy in \( B_3 \) per unit of time

\( T_1 \) : time between successive screening tests for the promotions from \( B_1 \) to \( B_2 \)

\( T_2 \) : time between successive screening tests for the promotions from \( B_2 \) to \( B_3 \)

\( F() \) : cumulative distribution of \( T_1 \) and \( f() \), its density function

\( G() \) : cumulative distribution of \( T_2 \) and \( g() \), its density function

\( F_k() \) : \( k \) fold convolution of \( F() \)

\( G_k() \) : \( k \) fold convolution of \( G() \)

9.3. COST ANALYSIS

Model I

\( T_1 \neq T_2 \)

The expected total cost for this model is found to be

\[
E[C(T_1, T_2)] = [(1 - F(T_1))C_{11} + (1 - G(T_2))C_{22}] + (T_1C_{11} + T_2C_{22})
\]

\[
\left\{ \sum_{k=1}^{\infty} [F_k(T_1) - F_{k+1}(T_1)] k c, p^i, q^{k-r} (k-r)C_{11} + \sum_{m=1}^{\infty} [G_m(T_2) - G_{m+1}(T_2)] mc, p^j, q^{m-j} (m-j)C_{22} \right\}
\]

... (9.3.1)
In (9.3.1), the first term denotes the cost when no screening test takes place in \((0, T_i)\) for promotions from \(B_i\) to \(B_j\) and no screening tests in \((0, T_j)\) for promotions from \(B_j\) to \(B_k\); the second term is the cost when some screening test are conducted in \((0, T_i)\) for promotions from \(B_i\) to \(B_j\) and in \((0, T_j)\) for promotions from \(B_j\) to \(B_k\), but no one has been selected and the last two terms are the cost for conducting \(k\) screening tests for promotions from \(B_i\) to \(B_j\), out of which only \(r\) has been selected and \(m\) screening tests for promotions from \(B_j\) to \(B_k\), out of which only \(j\) has been selected.

From (9.3.1), one can obtain the optimal values \(T_i^*\) and \(T_k^*\) when \(F\) and \(G\) are known.

**Model - II**

\(T_i = T_j = T\), In this case \(F() = G()\)

The expected total cost for this model is found to be

\[
E[C(T)] = \left(1 - F(T)\right)\left(C_{i1} + C_{i2}\right) + T\left(C_{i1} + C_{i2}\right) + \sum_{k=0}^{\infty} \left[F_k(T) - F_{k+1}(T)\right]kC_{r1}p_i^kq_i^{k-r}(k-r)C_{r1}
\]

\[
+ \sum_{m=1}^{\infty} \left[F_m(T) - F_{m+1}(T)\right]mC_{r2}p_j^m q_j^{m-j}(m-j)C_{r2}
\]

\[
... (9.3.2)
\]
In (9.3.2) the first term denotes the cost when no screening test takes place in \((0,T)\) for promotions from \(B_1\) to \(B_2\) and from \(B_2\) to \(B_3\), the second term is the cost when there are some screening tests in \((0,T)\) for promotions from \(B_1\) to \(B_2\) and from \(B_2\) to \(B_3\), but no one has been selected and the last two terms are the cost for conducting \(k\) screening tests for promotions from \(B_1\) to \(B_2\), out of which only \(r\) has been selected and \(m\) screening tests for promotions from \(B_2\) to \(B_3\), out of which only \(j\) has been selected.

From (9.3.1), one can obtain the optimal value \(T^*\) when \(F\) is known.

9.4. SPECIAL CASE

Model -I

Assume that \(F\) and \(G\) are exponential distribution with parameters \(\lambda\) and \(\mu\) respectively.

Using (9.3.1),

\[
E[C(T_1, T_2)] = e^{-\lambda T_1} C_{r_1} + e^{-\mu T_2} C_{r_2} + T_1 C_{i_1} + T_2 C_{i_2} + \sum_{k=1}^{\infty} e^{-\alpha T_1} \frac{(\lambda T_1)^k}{k!} k q_1 C_{i_1} \\
+ \sum_{m=1}^{\infty} e^{-\mu T_2} \frac{(\mu T_2)^m}{m!} m q_2 C_{i_2}
\]

\[
= e^{-\lambda T_1} C_{r_1} + e^{-\lambda T_1} C_{r_2} + T_1 C_{i_1} + T_2 C_{i_2} + \lambda T_1 q_1 C_{i_1} + \mu T_2 q_2 C_{i_2} \quad \ldots (9.4.1)
\]
It is known that

1. \( E[C(T_1, T_2)] \) attains its maxima or minima if

\[
\left( \frac{\partial^2}{\partial T_1^2} E[C(T_1, T_2)] \right) - \left( \frac{\partial^2}{\partial T_2^2} E[C(T_1, T_2)] \right)^2 > 0
\]

2. \( E[C(T_1, T_2)] \) is minimum if

\[
\left( \frac{\partial^2}{\partial T_1^2} E[C(T_1, T_2)] \right) > 0 \quad \text{and} \quad \left( \frac{\partial^2}{\partial T_2^2} E[C(T_1, T_2)] \right) > 0
\]

3. The optimum values \( T_1^* \) and \( T_2^* \) for which \( E[C(T_1^*, T_2^*)] \) is minimum are obtained by solving

\[
\left( \frac{\partial}{\partial T_1} E[C(T_1, T_2)] \right) = 0 \quad \text{and} \quad \left( \frac{\partial}{\partial T_2} E[C(T_1, T_2)] \right) = 0
\]

From (9.4.1),

\[
\left( \frac{\partial}{\partial T_1} E[C(T_1, T_2)] \right) = -\lambda e^{-\lambda T_1} C_{r1} + C_{r1} + \lambda q_1 C_{s1}
\]

\[
\left( \frac{\partial}{\partial T_1} E[C(T_1, T_2)] \right) = 0 \quad \text{gives}
\]

\[
\lambda e^{-\lambda T_1} C_{r1} = C_{r1} + \lambda q_1 C_{s1}
\]

\[
e^{-\lambda T_1} = \frac{C_{r1} + \lambda q_1 C_{s1}}{\lambda C_{r1}}
\]

\[
T_1^* = \frac{1}{\lambda} \log \left( \frac{\lambda C_{r1}}{C_{r1} + \lambda q_1 C_{s1}} \right)
\]

... (9.4.2)
\[
\left( \frac{\partial^2}{\partial T_1^2} E[C(T_1, T_2)] \right)_{T=\bar{T}_1} = \left[ \lambda^2 e^{-\lambda T} C_{r1} \right]_{T=\bar{T}}^\prime \\
= \lambda^2 C_{r1} \left( \frac{C_{r1} + \lambda q_1 C_{r1}}{\lambda C_{r1}} \right) \\
= \lambda (C_{r1} + \lambda q_1 C_{r1})
\]

which is positive.

In a similar way,

\[
T_2^* = \frac{1}{\mu} \log \left( \frac{\mu C_{r2}}{C_{r2} + \mu q_2 C_{r2}} \right)
\]

... (9.4.3)

\[
\left( \frac{\partial^2}{\partial T_2^2} E[C(T_1, T_2)] \right)_{T=\bar{T}_2} \text{ is positive and } \left( \frac{\partial^2}{\partial T_1 T_2} E[C(T_1, T_2)] \right)_{(T_1^*, T_2^*)} = 0
\]

\[
\therefore \left( \frac{\partial^2}{\partial T_1^2} E[C(T_1, T_2)] \right)_{T=\bar{T}_1} \left( \frac{\partial^2}{\partial T_2^2} E[C(T_1, T_2)] \right)_{T=\bar{T}_2} - \left( \frac{\partial^2}{\partial T_1 T_2} E[C(T_1, T_2)] \right)_{(T_1^*, T_2^*)} > 0
\]

Thus \(E[C(T_1^*, T_2^*)]\) is minimum at \((T_1^*, T_2^*)\) where \(T_1^*\) and \(T_2^*\) are given by (9.4.2) and (9.4.3).

**Model-II**

Assume that the distribution \(F(x)\) is exponential with parameter \(\lambda\).

Using (9.3.2),

\[
E[C(T)] = e^{\lambda T} (C_{r1} + C_{r2}) + T(C_{r1} + C_{r2}) + \sum_{k=1}^{\infty} e^{-\lambda T} \frac{(\lambda T)^k}{k!} kq_1 C_{r1} + \sum_{m=1}^{\infty} e^{-\lambda T} \frac{(\lambda T)^m}{m!} mq_2 C_{r2}
\]

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\[ e^{-\lambda t} (C_{r1} + C_{r2}) + T(C_{r1} + C_{r2}) + (\lambda T) e^{-\lambda T} (q_1 C_{s1} + q_2 C_{s2}) \sum_{k=1}^{\infty} \frac{(\lambda T)^{k-1}}{(k-1)!} \]
\[ + (\lambda T) e^{-\lambda T} q_3 C_{s2} \sum_{m=1}^{\infty} \frac{(\lambda T)^{m-1}}{(m-1)!} \]

\[ = e^{-\lambda T} (C_{r1} + C_{r2}) + T(C_{r1} + C_{r2}) + (\lambda T) e^{-\lambda T} (q_1 C_{s1} e^{\lambda T} + q_2 C_{s2} e^{\lambda T}) \]

i.e. \[ E[C(T)] = e^{-\lambda T} (C_{r1} + C_{r2}) + T(C_{r1} + C_{r2}) + (\lambda T)(q_1 C_{s1} + q_2 C_{s2}) \]

... (9.4.4)

Differentiating (9.4.4) with respect to \( T \) and equating to zero,

\[-\lambda e^{-\lambda T} (C_{r1} + C_{r2}) + (C_{r1} + C_{r2}) + \lambda (q_1 C_{s1} + q_2 C_{s2}) = 0 \]

i.e. \[ \lambda e^{-\lambda T} (C_{r1} + C_{r2}) = (C_{r1} + C_{r2}) + \lambda (q_1 C_{s1} + q_2 C_{s2}) \]

\[ e^{-\lambda T} = \frac{C_{r1} + C_{r2} + \lambda (q_1 C_{s1} + q_2 C_{s2})}{\lambda (C_{r1} + C_{r2})} \]

i.e. \[ T' = \frac{1}{\lambda} \log \left[ \frac{\lambda (C_{r1} + C_{r2})}{(C_{r1} + C_{r2}) + \lambda (q_1 C_{s1} + q_2 C_{s2})} \right] \]

... (9.4.5)

and

\[ \left[ \frac{d^2}{dT^2} E[C(T)] \right]_{T=T'} = \left[ \lambda^2 e^{-\lambda T} (C_{r1} + C_{r2}) \right]_{T=T'} \]

\[ = \lambda \left[ (C_{r1} + C_{r2}) + \lambda (q_1 C_{s1} + q_2 C_{s2}) \right] \]

which is positive.

Hence \( T' \) given by (9.4.5) is optimal and \( E[C(T')] \) is minimum.
9.5. NUMERICAL ILLUSTRATION

Model-I

Case 1

Taking \( C_{t_1} = 500, \ C_{t_2} = 700, \ C_{r_1} = 1000, \ C_{r_2} = 800, \ C_{s_1} = 400, \ C_{s_2} = 300 \)

\[ \lambda = \mu = 2, \quad q_1 = 0.4, \quad q_2 = 0.5 \text{ in (9.4.4) and (9.4.5)}, \]

the optimal values of \( T_1 \) and \( T_2 \) are \( T_1^* = 0.1936 \), \( T_2^* = 0.1021 \) units of time.

Further, \( E[C(T_1^*, T_2^*)] = 1170.85 \).

Case 2

Taking \( C_{t_1} = 500, \ C_{t_2} = 700, \ C_{r_1} = 1000, \ C_{r_2} = 800, \ C_{s_1} = 400, \ C_{s_2} = 300 \)

\[ \lambda = 2, \mu = 3, \quad q_1 = 0.4, \quad q_2 = 0.5 \text{ in (9.4.4) and (9.4.5)}, \]

the optimal values of \( T_1 \) and \( T_2 \) are \( T_1^* = 0.1936 \), \( T_2^* = 0.1065 \) units of time.

Further \( E[C(T_1^*, T_2^*)] = 1074.56 \).

Case 3

Taking \( C_{t_1} = 500, \ C_{t_2} = 700, \ C_{r_1} = 1000, \ C_{r_2} = 800, \ C_{s_1} = 400, \ C_{s_2} = 300 \)

\[ \lambda = 3, \mu = 2, \quad q_1 = 0.4, \quad q_2 = 0.5 \text{ in (9.4.4) and (9.4.5)}, \]
the optimal values of \( T \) and \( T \) are \( T^* = 0.16197 \) \( T^* = 0.1021 \) units of time.

Further, \( E[\mathcal{C}(T^*, T^*)] = 1087.50 \).

**Model-II**

Fixing \( C_{i1} = 500, C_{i2} = 700, C_{r1} = 1000, C_{r2} = 800, C_{r1} = 400, C_{r2} = 300, \)

\( \lambda = 2, q_1 = 0.4, q_2 = 0.5 \)

From (9.4.2), the optimal value of \( T \) is \( T^* = 0.148 \) units of time.

And \( E[\mathcal{C}(T^*)] = 1268.16 \).

**9.6. CONCLUSION**

If the average number of screening tests for promotions from \( B_1 \) to \( B_2 \), is equal to the average number of screening tests for promotions from \( B_2 \) to \( B_3 \), then the minimum cost is 1170.85. If the average number of screening test for promotions from \( B_1 \) to \( B_2 \), is less than the average number of screening test for promotions from \( B_2 \) to \( B_1 \), then the minimum cost is 1074.56. If the average number of screening test for promotions from \( B_1 \) to \( B_2 \), is greater than the average number of screening test for promotions from \( B_2 \) to \( B_1 \), then the minimum cost is 1087.50. This shows that the organization can minimize the total cost by planning the number of occurrences of the two screening tests from \( B_1 \) to \( B_2 \), and from \( B_2 \) to \( B_3 \).