CHAPTER 6

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In this chapter we attempt to highlight the future scope of our work. There are many interesting problems that can be taken up in future. We list some of the problems.

1. Graded extending modules and their characteristics:

The notion of extending modules can be studied in graded case using the graded concept of essentiality and complements. The notion of graded continuous module, graded quasi continuous module can be developed and related results can be generalized to graded version. Various characteristics of above type of graded modules can be investigated.

2. Graded supplemented modules and their characteristics:

The notion of graded small submodules of a graded module can be developed and the concepts like graded hollow modules, supplemented modules can be studied. This will lead us to the study of dual Goldie dimension in graded case. Many aspects of discrete and lifting modules can be developed.

3. Graded modules with finiteness conditions:

The study of Goldie dimension and singularity can be continued by modifying the notion of singularity and rank function. One can investigate various aspects of Graded module with finiteness conditions like ACC on graded essential submodules, ACC on graded supplemented submodules, chain conditions on graded small submodules etc. Moreover, ACC on graded non essential submodules may lead to Artinian or noetherian character of the graded module. DCC on graded non essential submodules may exhibit finiteness condition of the substructures of such graded module. The study of graded modules with finiteness conditions will lead to the finiteness conditions of graded socle, graded radical of the graded module and various related results can be studied in graded
case. The study of graded modules with finiteness conditions would throw light on understanding the theory in a unified and systematic manner and also would provide avenue for looking at their applications.

4. **Topological aspects of Graded modules:**

Study of different types of ideals plays a vital role in the topological aspects of ring theory. A topology introduced on the set of prime ideals, known as Zariski topology is a topic of interest. A topology on the set of graded weakly prime ideals can be introduced and various properties can be investigated. In the similar way a topology on the set of graded weakly prime submodules can be introduced and related properties can be investigated. Imposing chain conditions on the graded ring or module one can investigate the nature of the corresponding topological space.

Moreover one can consider a topological graded module and study the Goldie dimension and krull dimension of such modules,

5. **Fuzzy aspects of graded modules:**

Fuzzy set theory permits the gradual assessment of the membership of the elements in the set, described with the aid of a membership function valued in the real unit interval [0; 1]. In 1965, Zadeh introduced this concept of fuzzy set, and it was a new episode towards the development of science and engineering. Fuzzy algebraic structures represent a natural extension of classical algebraic structures and it is introduced in 1971 by Rosenfeld by defining fuzzy group. Since then, hundreds of papers and several books have been written on fuzzy algebraic structures. The notion of graded modules and rings can be studied in fuzzy setting. The concepts like graded multiplication modules, graded singularity, graded modules with finiteness conditions can be fuzzified and it will be an interesting topic of research.