Optimized Discretization Schemes For Brain Images

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Abstract:

In medical image processing active contour method is the important technique in segmenting human organs. Geometric deformable curves known as levelsets are widely used in segmenting medical images. In this modeling, evolution of the curve is described by the basic lagrange pde expressed as a function of space and time. This pde can be solved either using continuous functions or discrete numerical methods. This paper deals with the application of numerical methods like finite difference and TVd-RK methods for brain scans. The stability and accuracy of these methods are also discussed. This paper also deals with the more accurate higher order non-linear interpolation techniques like ENO and WENO in reconstructing the brain scans like CT, MRI, PET and SPECT is considered.

Keywords: Active contour, Geometric Deformable model, Levelset, Pde, ENO, WENO, Medical Scans.

1. Introduction

Image segmentation has played an important role in medical imaging. Segmented images are used in different applications such as quantification of tissue volumes, diagnosis and localization of pathology, study of anatomical structures, treatment planning, partial volume correction of functional imaging data and computer integrated surgery. Medical Image segmentation still remains a difficult task due to tremendous variability of object shapes and the variation of the image quality. In particular medical images are often corrupted by noise and sampling artifacts which can cause considerable difficulties when conventional techniques such as edge detection and thresholding applied to clinical segmentation. To address these difficulties deformable models are extensively used in medical imaging. Deformable models can be divided into either parametric or geometric methods[1]. Geometric deformable models based on curve evolution and levelsets are extensively used implicit technique. In this paper optimum discretization and interpolation schemes of levelset curves are considered. The performance of these functions are clearly discussed.

In deformable model the object to be segmented is initially represented with a curve of any shape like rectangle, circle or polygon. This curve will move in a direction normal to itself with a known speed.
function $F$. The objective is to track the motion of this interface as it evolves. The speed function $F$ depends on many factors such as geometric properties (local) like normal, curvature, global properties like shape and position of the front and independent properties like underlying fluid velocity that passively transports the front. The motion of the front can be characterised by the initial value lagrange partial difference equation (PDE) which is similar to heat equation is represented as follows.

$$\Phi_t + F \nabla \Phi = 0$$

Where $F$ is the desired velocity of the interface, $\phi$ is the levelset function and $\nabla$ is the gradient operator. In general normal component of the velocity has contribution in shape modification, whereas the tangential component has no contribution in shape modification. Generally $F_N$ depends on $X(x,y,-2D)$, $t$ and $\nabla \Phi$.

The Eq(1) in terms of normal velocity component is represented as

$$\Phi_t + F_N |\nabla \Phi| = 0$$

Where $F_N$, $N$, $k$ represents normal velocity field component, normal to the curve and mean curvature respectively. They are defined as follows.

$$F_N = F \frac{\nabla \Phi}{|\nabla \Phi|} = F \cdot N$$

$$N = \frac{\nabla \Phi}{|\nabla \Phi|}$$

The mean curvature is the divergence of the normal

$$k = -\nabla \frac{\nabla \Phi}{|\nabla \Phi|} = -\nabla \cdot N$$

The curve represented in Eq (1) represents the segmentation of the object includes both time and space derivatives. The evolution can be represented by using either continuous techniques or discrete numerical methods. When curve evolves with a constant speed becomes Hamilton-Jacobi can form corners i.e. curve is not differentiable and hence weak solution must be constructed to continue the solution. This can be obtained by means of an entropy condition. Which is a unique viscosity solution. The appearance of these singularities in the solution needs another kind of solution. Hence numerical solution on cartesian grids is used. The key methods are monotonicity, upwind differencing, Essentially NonOscilatory(ENO) and Weighted Essentially NonOscilatory(WENO) schemes. In the cartesian grid the spatial derivatives in eqn (1) are approximated using first order forward, backward and second order central difference techniques and time discretization is performed with Total Variation diminishing Runge-Kutta (TVD-RK) method.

2. Method

The evolution of the curve is considered in terms of discretization of time and spatial coordinates. A closed parametric curve $C_0(s) = (x(s), y(s)), s \in [0,1]$ is placed around image parts of interest. Then this curve evolves under internal and external energy. The evolution [10,11] is given by Eq (1). The process is shown in Fig.1. The modeling of the moving front comes from discretizing the lagrangian form of Eq (1). In this approach the parameterization $s$ is discretized into a set of marker particles whose position at any time are used to reconstruct the front. Divide parameterization interval $[0,s]$ into $M$ equal intervals of size $\Delta s$ yielding $M+1$ mesh points $s_i = i\Delta s, i = 0,1,2,..M$. As shown in Fig. Divide the time into equal intervals $\Delta t$. The image of each mesh point $i\Delta s$ at each time step $n\Delta t$ is a marker point $\left(x_i^n, y_i^n\right)$ on the moving front. The goal of this numerical algorithm is to produce new values $\left(x_i^{n+1}, y_i^{n+1}\right)$ from previous positions.
2.1. **Time Discretization**

Once $\phi$ and $F$ are defined at every grid point, we can apply numerical methods to evolve $\phi$ forward in the moving interface across the grid. At some point, say $t^n$ say $\Phi^n = \Phi(t^n)$ respect current value of $\Phi$.

Updating $\Phi$ in time $\Phi^{n+1} = \Phi(t^{n+1})$ where $t^{n+1} = t^n + \Delta t$.

First order accurate time discretization of eqn (1) in the forward Euler method given by

$$\frac{\Phi^{n+1} - \Phi^n}{\Delta t} + F^n \nabla \Phi^n = 0 \quad \text{(6)}$$

2.2. **Spatial Discretization**

In the cartesian grid, the spatial derivatives $\nabla \Phi$ in Eq(1) need to be approximated using finite difference techniques. The first order forward and backwards difference and second order central difference is defined as follows. $\{\Phi_x\}$ denotes the spatial derivatives of $\phi$ at the point $x_i$.

- **Forward difference**
  \[
  D^+ \Phi_i = \frac{\partial \Phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_i}{\Delta x} \quad \text{(7)}
  \]

- **Backward difference**
  \[
  D^- \Phi_i = \frac{\partial \Phi}{\partial x} \approx \frac{\phi_i - \phi_{i-1}}{\Delta x} \quad \text{(8)}
  \]

- **Central difference**
  \[
  D^0 \Phi_i = \frac{\partial \Phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x} \quad \text{(9)}
  \]

A second order finite difference

$$\frac{\partial^2 \phi}{\partial x^2} \approx \frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{\Delta x^2} \quad \text{(10)}$$

The combination of forward Euler time discretization with the upwind difference scheme is convergent in discretizing the curve in time and space respectively. I.e. This approximation is consistent and stable. Stability guarantees that small errors in the approximation are not amplified as the solution marched forward in time. Stability[12] can be forced using CFL (Courat-Friedeichs-Levy) condition which asserts that numerical wave should propagate at least as fast as the physical wave, i.e. numerical wave speed of $-\Delta x/\Delta t$ must be as at least as fast as the physical wave speed $F$. 

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**Fig. 1. Curve evolution on the grid**
This leads to the CFL time step restriction of $\Delta t \leq \frac{\Delta x}{\max |F|}$

In 2-D $\Delta t \max \left\{ \left| \frac{F_x}{\Delta x} \right| + \left| \frac{F_y}{\Delta y} \right| \right\} = \alpha$ where $0 < \alpha < 1$.

2.3HJ-ENO & HJ-WENO

The upwind method can also be improved by using more accurate approximation for $\phi$. Harten et al. introduced the idea of ENO interpolation of data for numerical solution. The basic idea was to compute numerical flux functions using the smoothest possible polynomial interpolants. Osher & Sethian implemented this Hamilton-Jacobi ENO (HJENO). This method allows one to extend first order accurate upwind differencing to higher order spatial accuracy by providing better numerical approximation to $\phi$. The goal of ENO scheme is to choose the single approximation with the least error by choosing the smoothest possible polynomial interpolation $\phi$. ENO approximates $\phi$ through first, second and third order derivatives i.e. ENO1, ENO2 and ENO3. It uses any one of the derivative hence information will be lost in smooth regions. Liu et al. proposed weighted ENO (WENO). This method uses the convex combination of all third orders to minimize the resulting errors. It is further extended HJWENO and obtained fifth order $\phi$ accuracy.

3. Results & Conclusions

3.1. Results

In this section results of different accuracy schemes like upwinding with various smoothing non-linear interpolation techniques is applied on CT, MRI, PET scans of brain is provided. The scheme is analysed for accuracy and stability by varying the number of iterations. The accuracy of the schemes is also observed through noisy images also. These techniques applied on brain scan CT with gaussian noise (row2), MRI with speckle noise (row1) and PET (row3) with brain tumors is shown in Fig.2.

![Fig.2. CT, MRI and PET scans for implementing ENO, WENO](image_url)

(a) input images MRI-t2, CT, PET
(b) WENO images MRI-t2, CT, PET
(c) ENO images MRI-t2, CT, PET
The accuracy is also observed for brain CT scan by varying no of iteration in case of ENO and WENO schemes. The computation time is measured in each case. The result is shown in Fig. 3. and quantitatively in table 1.

<table>
<thead>
<tr>
<th>No. of iterations</th>
<th>ENO1</th>
<th>ENO2</th>
<th>ENO3</th>
<th>WENO</th>
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<tr>
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<td>0.140000</td>
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<td>0.187000</td>
<td>0.188000</td>
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<td>50</td>
<td>0.672000</td>
<td>0.688000</td>
<td>0.671000</td>
<td>0.703000</td>
</tr>
</tbody>
</table>

Table.1. Computation time of ENO and WENO schemes for No.of iterations 10, 25, 50

Fig.3. Accuracy of ENO and WENO schemes for CT scan

3.2. Conclusions

HJ-ENO and HJ-WENO allow us to discretize the spatial terms of levelset equation upto third and fifth order respectively, while the forward Euler time discretization is only first order accurate in time. This is acceptable since practical experience suggests that levelset methods are sensitive to spatial accuracy. Hence higher order accuracy is required. HJ-WENO is very useful than ENO since it takes the convex combination of three ENOS which minimizes the error and increasing accuracy. It is also observed that time discretization is TVD where as spatial discretization is total variation bounded (TVB). It is observed that accuracy is improved in case of WENO compared to ENO since it uses all the ENO combinations, hence there is no data loss which is shown in figs 2 and 3 col2 and row3 respectively. It is also observed that computational time increases in WENO compared to ENO since it is fifth order whereas ENO is third order.

References


