Chapter 3

A Method For Voltage Stability Analysis Of A Power System

3.1 Introduction

Power flow Jacobian matrix is extensively used by many authors to investigate the voltage stability problem in a power system. It is established that the voltage stability condition of a power system gets disturbed when the power flow Jacobian matrix approaches to singularity. Tramuchit and Thomas [13] have proposed the minimum singular value of the Newton-Raphson power flow Jacobian matrix as a global voltage stability index. They also presented an optimal algorithm which could yield the largest minimum value of corresponding power flow Jacobian matrix. Instead of performing SVD for every change of system operating condition, they established incremental linear relationship between parameters and increments of the minimum singular value. Lof et. al. [8, 9] have also calculated the minimum singular value and the corresponding (left and right) singular vector. The minimum singular value indicates the proximity to the steady state stability limit. Further, they have discussed the use of static voltage stability indices based on SVD of the power flow Jacobian matrix and matrices derived from Jacobian matrix.
A two parameter continuation technique has been proposed for evaluating branch outage contingencies introducing new branch parameter [34]. A bifurcation point computation based on continuation power flow method has been proposed by the prediction of Q-limit breaking point [35].

A voltage stability index VMPI (voltage margin proximity index) is proposed considering voltage limits, especially lower voltage limits [36]. A simple, computationally very fast local voltage-stability index has been proposed using Tellegen's theorem [37]. It is easy to implement in the wide-area monitoring and control center or locally in a numerical relay. A new node voltage stability index called the equivalent node voltage collapse index (ENVCI), which is based on ESM and uses only local voltage phasors, is presented [38]. Well-known ZIP model has been used to represent loads having components with different power to voltage sensitivities [39] and also, the choice of voltage stability index in the context of load modeling has been suggested.

In this chapter, a new method has been proposed to analyze voltage instability problem of multi-bus interconnected power system. The power flow Jacobian matrix of a multi-bus interconnected is transformed into a two by two matrix using sensitivity relation of the buses. This lumps all other buses to a source bus keeping the target load bus as the only load bus of the system, whose voltage stability condition has to be investigated. As such, the singularity of this matrix will be the indication of system collapse due to voltage instability problem. The elements of the transformed two by two matrix and the bus voltage of the target bus are used to develop an algorithm to determine load margin of a target bus of the system. Simulation has been carried out on IEEE 30 bus and IEEE 118 bus systems to verify the validity of the proposed method.
3.2 Analysis of Voltage Stability Problem For a Multi-Bus Inter Connected Power System

It is established that at the point of voltage collapse in a power system, the determinant of load flow Jacobian matrix becomes zero. To quantify the condition of singularity of load flow Jacobian matrix of a multibus system at the point of voltage collapse, it is transformed into a two by two matrix relating $\Delta \delta_k$, $\Delta V_k$, $\Delta P_k$ and $\Delta Q_k$ for a target $kth$ bus. The change in voltage magnitudes and voltage phase angles of a multi-bus inter connected power system are related to change in real and reactive power injection at the buses as follows:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
H & N \\
M & L
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix}
\]  

(3.1)

where H, N, M, and L are the elements of Jacobian matrix used for the load flow analysis of Newton Raphson method. Equation (3.1) can be expressed as:

\[
\begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix} =
\begin{bmatrix}
H & N \\
M & L
\end{bmatrix}^{-1}
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
W & X \\
Y & Z
\end{bmatrix}
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]  

(3.2)

Without loss of generality, let bus number 1 be the slack bus and the system has $npv$ number of PV buses from bus 1 to bus $npv$ (including slack bus), then using the elements of $W$, $X$, $Y$ and $Z$ of equation (3.2) the change in voltage phase angle and voltage magnitude of a $kth$ target bus can be expressed as:

\[
\Delta \delta_k = w_{k,2} \Delta P_2 + w_{k,3} \Delta P_3 + ... + w_{k,k} \Delta P_k + ... + w_{k,N} \Delta P_N
\]
The right hand side of the equation (3.3) and (3.4) contain change in real and reactive power terms for all buses including the \( k \)th target load bus. To relate \( \Delta \delta_k \) and \( \Delta V_k \) of equation (3.3) and (3.4) to only \( \Delta P_k \) and \( \Delta Q_k \), it is required to represent change in real and reactive power terms for the other buses in terms of \( \Delta P_k \) and \( \Delta Q_k \).

Recognizing the fact that the change in bus voltage angle is primarily influenced by the change in real power injection at the buses\(^{[40]}\) and taking \( k \)th bus as reference bus, the change in bus real power injections to the change in voltage phase angles can be expressed as:

\[
\begin{bmatrix}
\Delta P_1 \\
\vdots \\
\Delta P_{k-1} \\
\Delta P_k \\
\vdots \\
\Delta P_N
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial P_1}{\partial \delta_1} & \frac{\partial P_1}{\partial \delta_{k-1}} & \cdots & \frac{\partial P_1}{\partial \delta_N} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial P_{k-1}}{\partial \delta_1} & \frac{\partial P_{k-1}}{\partial \delta_{k-1}} & \cdots & \frac{\partial P_{k-1}}{\partial \delta_N} \\
\frac{\partial P_k}{\partial \delta_1} & \frac{\partial P_k}{\partial \delta_{k-1}} & \cdots & \frac{\partial P_k}{\partial \delta_N} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial P_N}{\partial \delta_1} & \frac{\partial P_N}{\partial \delta_{k-1}} & \cdots & \frac{\partial P_N}{\partial \delta_N}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_1 \\
\vdots \\
\Delta \delta_{k-1} \\
\Delta \delta_k \\
\vdots \\
\Delta \delta_N
\end{bmatrix}
\tag{3.5}
\]

The change in real power injection at target \( k \)th bus can be expressed as:

\[
\Delta P_k = \sum_{j=1}^{N} \frac{\partial P_k}{\partial \delta_j} \Delta \delta_j
\]
Now, applying equation (3.5) in equation (3.6), we have

\[
\Delta P_k = \left[ \frac{\partial P_k}{\partial \delta_1} \cdot \frac{\partial P_k}{\partial \delta_{k-1}} \cdot \frac{\partial P_k}{\partial \delta_{k+1}} \cdot \frac{\partial P_k}{\partial \delta_N} \right] \left[ \begin{array}{c}
\Delta \delta_1 \\
\Delta \delta_{k-1} \\
\Delta \delta_{k+1} \\
\Delta \delta_N
\end{array} \right] (3.6)
\]

\[
= \gamma_1 \Delta P_1 + \cdots + \gamma_{k-1} \Delta P_{k-1} + \gamma_{k+1} \Delta P_{k+1} + \cdots + \gamma_N \Delta P_N (3.7)
\]

But, equations (3.3) and (3.4) do not have term \( \Delta P_1 \). Therefore, term \( \Delta P_1 \) of equation (3.7) is made zero and it becomes:

\[
\Delta P_k = \gamma_2 \Delta P_2 \cdots + \gamma_{k-1} \Delta P_{k-1} + \gamma_{k+1} \Delta P_{k+1} + \cdots + \gamma_N \Delta P_N
\]

\[
= \left[ \gamma \right] \left[ \Delta P \right] (3.8)
\]

\( \left[ \gamma \right] \) is a row matrix, therefore, \( \left[ \Delta P \right] \) values are to be calculated using pseudo-inverse technique i.e.,

\[
\left[ \Delta P \right] = \left[ \gamma \right]^T \left( \left[ \gamma \right] \left[ \gamma \right]^T \right)^{-1} \Delta P_k
\]

(3.9)

Solving the above equation the value of \( \Delta P_1 \) can be written as;
\[ \Delta P_i = \frac{\gamma_i}{N} \Delta P_k = \beta_i \Delta P_k \text{ for } i = 2 \ldots N, \ i \neq k \quad (3.10) \]

Similarly, the change in bus voltage is primarily influenced by the change in reactive power injection at the buses[40] and taking the \( k \text{th} \) bus as the reference bus, the change in bus reactive power injections to the change in voltage magnitudes can be expressed as:

\[
\begin{bmatrix}
\Delta Q_1 \\
\vdots \\
\Delta Q_{k-1} \\
\Delta Q_k \\
\vdots \\
\Delta Q_N
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial Q_2}{\partial V_1} & \frac{\partial Q_2}{\partial V_{k-1}} & \frac{\partial Q_2}{\partial V_{k+1}} & \frac{\partial Q_2}{\partial V_N} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{\partial Q_{k-1}}{\partial V_1} & \frac{\partial Q_{k-1}}{\partial V_{k-1}} & \frac{\partial Q_{k-1}}{\partial V_{k+1}} & \frac{\partial Q_{k-1}}{\partial V_N} \\
\frac{\partial Q_k}{\partial V_1} & \frac{\partial Q_k}{\partial V_{k-1}} & \frac{\partial Q_k}{\partial V_{k+1}} & \frac{\partial Q_k}{\partial V_N} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{\partial Q_N}{\partial V_1} & \frac{\partial Q_N}{\partial V_{k-1}} & \frac{\partial Q_N}{\partial V_{k+1}} & \frac{\partial Q_N}{\partial V_N}
\end{bmatrix}
\begin{bmatrix}
\Delta V_1 \\
\vdots \\
\Delta V_{k-1} \\
\Delta V_k \\
\vdots \\
\Delta V_N
\end{bmatrix}
\quad (3.11)
\]

The change in reactive power injection at target \( k \text{th} \) bus can be expressed as:

\[
\Delta Q_k = \sum_{j=1}^{N} \frac{\partial Q_k}{\partial V_j} \Delta V_j
\]

\[
= \begin{bmatrix}
\frac{\partial Q_k}{\partial V_1} & \frac{\partial Q_k}{\partial V_{k-1}} & \frac{\partial Q_k}{\partial V_{k+1}} & \frac{\partial Q_k}{\partial V_N}
\end{bmatrix}
\begin{bmatrix}
\Delta V_1 \\
\Delta V_{k-1} \\
\Delta V_{k+1} \\
\Delta V_N
\end{bmatrix}
\quad (3.12)
\]

Now, applying equation (3.11) in equation (3.12), we have
\[
\Delta Q_k = \begin{bmatrix}
\frac{\partial Q_k}{\partial V_1} & \frac{\partial Q_k}{\partial V_{k-1}} & \frac{\partial Q_k}{\partial V_{k+1}} & \frac{\partial Q_k}{\partial V_N} \\
\end{bmatrix}
\begin{bmatrix}
\frac{\partial Q_2}{\partial V_1} & \frac{\partial Q_2}{\partial V_{k-1}} & \frac{\partial Q_2}{\partial V_{k+1}} & \frac{\partial Q_2}{\partial V_N} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{\partial Q_{k-1}}{\partial V_1} & \frac{\partial Q_{k-1}}{\partial V_{k-1}} & \frac{\partial Q_{k-1}}{\partial V_{k+1}} & \frac{\partial Q_{k-1}}{\partial V_N} \\
\frac{\partial Q_{k+1}}{\partial V_1} & \frac{\partial Q_{k+1}}{\partial V_{k-1}} & \frac{\partial Q_{k+1}}{\partial V_{k+1}} & \frac{\partial Q_{k+1}}{\partial V_N} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{\partial Q_N}{\partial V_1} & \frac{\partial Q_N}{\partial V_{k-1}} & \frac{\partial Q_N}{\partial V_{k+1}} & \frac{\partial Q_N}{\partial V_N} \\
\end{bmatrix}^{-1}
\begin{bmatrix}
\Delta Q_1 \\
\vdots \\
\Delta Q_{k-1} \\
\Delta Q_{k+1} \\
\vdots \\
\Delta Q_N \\
\end{bmatrix}
\]

\[
= \alpha_1 \Delta Q_1 + \cdots + \alpha_{k-1} \Delta Q_{k-1} + \alpha_{k+1} \Delta Q_{k+1} + \cdots + \alpha_N \Delta Q_N
\]

But, equations (3.3) and (3.4) do not have terms \(\Delta Q_1\) ... \(\Delta Q_{npv}\). Therefore, terms \(\Delta Q_1\) ... \(\Delta Q_{npv}\) of equation (3.13) are made zero and it becomes:

\[
\Delta Q_k = \alpha_{npv+1} \Delta Q_{npv+1} + \cdots + \alpha_{k-1} \Delta Q_{k-1} + \alpha_{k+1} \Delta Q_{k+1} + \cdots + \alpha_N \Delta Q_N
\]

\[
= [\alpha][\Delta Q]
\]

(3.14)

[\alpha] is a row matrix, therefore, \([\Delta Q]\) values are to be calculated using pseudoinverse technique i.e.,

\[
[\Delta Q] = [\alpha]^T \left( [\alpha] [\alpha]^T \right)^{-1} [\Delta Q]
\]

(3.15)

Solving the above equation the value of \(\Delta Q_i\) can be written as;

\[
\Delta Q_i = \frac{\alpha_i}{\sum_{j=npv+1}^{N} \alpha_j} \Delta Q_k = \lambda_i \Delta Q_k \text{ for } i = npv + 1...N, \ i \neq k
\]

(3.16)

Substituting values of \(\Delta P_i\) and \(\Delta Q_i\) from equations (3.10) and (3.16) in equation (3.3) and (3.4) we have

\[
\begin{bmatrix}
\Delta \delta_k \\
\Delta V_k
\end{bmatrix}
= \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
\Delta P_k \\
\Delta Q_k
\end{bmatrix}
\]

(3.17)
Where

\[
A = w_{kk} + \sum_{j=2}^{N} w_{kj}\beta_j \\
B = x_{kk} + \sum_{j=n_{pv}+1}^{N} x_{kj}\lambda_j \\
C = y_{kk} + \sum_{j=2}^{N} y_{kj}\beta_j \\
D = z_{kk} + \sum_{j=n_{pv}+1}^{N} z_{kj}\lambda_j
\]  

Therefore,

\[
\begin{bmatrix}
\Delta P_k \\
\Delta Q_k
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}^{-1}
\begin{bmatrix}
\Delta \delta_k \\
\Delta V_k
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial P_k}{\partial \delta_k} & \frac{\partial P_k}{\partial V_k} \\
\frac{\partial Q_k}{\partial \delta_k} & \frac{\partial Q_k}{\partial V_k}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_k \\
\Delta V_k
\end{bmatrix}
\]  

At the point of voltage collapse

\[
J_D = \det \left\{ \begin{bmatrix}
\frac{\partial P_k}{\partial \delta_k} & \frac{\partial P_k}{\partial V_k} \\
\frac{\partial Q_k}{\partial \delta_k} & \frac{\partial Q_k}{\partial V_k}
\end{bmatrix} \right\} = 0
\]  

3.3 Determination Of Load Margin

In this section, a procedure for determination of load margin of the target kth bus has been developed using the elements of the modified two by two Jacobian matrix given by the equation (3.22) and the bus voltage of the target bus. Equation(3.22) can be represented as two bus system with the target kth load bus, Y-Bus elements and an equivalent source as depicted in figure - 3.1.
Now, for this equivalent two bus system, the expression for real and reactive power injections at the target $k$th bus are as follows:

\[ P_k = G_{kk}V_k^2 + V_GV_k(G_{kg} \cos(\delta_k) + B_{kg} \sin(\delta_k)) \]  \hfill (3.24)
\[ Q_k = -B_{kk}V_k^2 + V_GV_k(G_{kg} \sin(\delta_k) - B_{kg} \cos(\delta_k)) \]  \hfill (3.25)

Where, $G_{kk}$, $B_{kk}$, $G_{kg}$ and $B_{kg}$ are the elements of admittance matrix $[Y]$ for the equivalent two bus system.

Under power balanced condition,

\[ P_k = -P_{Dk} \]  \hfill (3.26)
\[ Q_k = -Q_{Dk} \]  \hfill (3.27)

The elements of Jacobian matrix for the equivalent two bus system can be expressed as

\[ \frac{\partial P_k}{\partial \delta_k} = V_GV_k(-G_{kg} \sin(\delta_k) + B_{kg} \cos(\delta_k)) \]  \hfill (3.28)
\[ \frac{\partial P_k}{\partial V_k} = 2G_{kk}V_k + V_G(G_{kg} \cos(\delta_k) + B_{kg} \sin(\delta_k)) \]  \hfill (3.29)
\[
\frac{\partial Q_k}{\partial \delta_k} = V_k(G_k \cos(\delta_k) + B_k \sin(\delta_k)) \quad (3.30)
\]
\[
\frac{\partial Q_k}{\partial V_k} = -2B_k V_k + V_k(G_k \sin(\delta_k) - B_k \cos(\delta_k)) \quad (3.31)
\]

At the point of voltage collapse the determinant of the Jacobian matrix becomes zero, i.e.,

\[
\text{Det}[J] = J_D = \frac{\partial P_k}{\partial \delta_k} \frac{\partial Q_k}{\partial V_k} - \frac{\partial P_k}{\partial V_k} \frac{\partial Q_k}{\partial \delta_k} = 0 \quad (3.32)
\]

The change in determinant value of the two bus equivalent system of the multibus system with respect to change in \(V_k\) and \(\delta_k\) can be expressed as:

\[
\Delta J_D = \frac{\partial J_D}{\partial \delta_k} \Delta \delta_k + \frac{\partial J_D}{\partial V_k} \Delta V_k
\]

\[
= \begin{bmatrix}
\frac{\partial J_D}{\partial \delta_k} & \frac{\partial J_D}{\partial V_k}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_k \\
\Delta V_k
\end{bmatrix}
\quad (3.33)
\]

Again, the expression for \(J_D\) in terms of \(\frac{\partial P_k}{\partial \delta_k}, \frac{\partial Q_k}{\partial \delta_k}, \frac{\partial P_k}{\partial V_k}\) and \(\frac{\partial Q_k}{\partial V_k}\) is as follows:

\[
J_D = \frac{\partial P_k}{\partial \delta_k} \frac{\partial Q_k}{\partial V_k} - \frac{\partial P_k}{\partial V_k} \frac{\partial Q_k}{\partial \delta_k} \quad (3.34)
\]

Therefore, \(\frac{\partial J_D}{\partial \delta_k}\) can be expressed as:

\[
\frac{\partial J_D}{\partial \delta_k} = \left[ \frac{\partial P_k}{\partial \delta_k} \frac{\partial Q_k}{\partial V_k} + \frac{\partial Q_k}{\partial \delta_k} \frac{\partial P_k}{\partial \delta_k} \right] - \left[ \frac{\partial P_k}{\partial \delta_k} \frac{\partial Q_k}{\partial \delta_k} + \frac{\partial Q_k}{\partial V_k} \frac{\partial P_k}{\partial \delta_k} \right] \quad (3.35)
\]

The terms \(\frac{\partial Q_k}{\partial \delta_k}, \frac{\partial P_k}{\partial \delta_k}, \frac{\partial Q_k}{\partial \delta_k}\) and \(\frac{\partial P_k}{\partial \delta_k}\) are derived utilizing equations from (3.28) to (3.31).

It is to be ensured that the terms \(\frac{\partial Q_k}{\partial \delta_k}, \frac{\partial P_k}{\partial \delta_k}, \frac{\partial Q_k}{\partial \delta_k}\) and \(\frac{\partial P_k}{\partial \delta_k}\) contain only the elements of the modified two by two Jacobian matrix and the bus voltage of the target bus. Because, they are the known values for the transformed two bus system.
\[
\frac{\partial \delta Q_k}{\partial \delta_k} = V_G (G_k \cos(\delta_k) + B_k \sin(\delta_k)) = \frac{\partial Q_k}{V_k} \\
\frac{\partial \delta P_k}{\partial \delta_k} = -V_G V_k (G_k \cos(\delta_k) + B_k \sin(\delta_k)) = -\frac{\partial Q_k}{\partial \delta_k} \\
\frac{\partial \delta Q_k}{\partial \delta_k} = V_G V_k (-G_k \sin(\delta_k) + B_k \cos(\delta_k)) = \frac{\partial P_k}{\partial \delta_k} \\
\frac{\partial \delta P_k}{\partial \delta_k} = V_G (-G_k \sin(\delta_k) + B_k \cos(\delta_k)) = \frac{\partial Q_k}{V_k}
\]

Now, applying values of \(\frac{\partial \delta Q_k}{\partial \delta_k}, \frac{\partial \delta P_k}{\partial \delta_k}, \frac{\partial \delta Q_k}{\partial \delta_k}\) and \(\frac{\partial \delta P_k}{\partial \delta_k}\) of equations (3.36), (3.37), (3.38) and (3.39) in equation (3.35) we have,

\[
\frac{\partial J_D}{\partial \delta_k} = \frac{\partial Q_k}{V_k} \left[ \frac{1}{V_k} \frac{\partial P_k}{\partial \delta_k} - \frac{\partial Q_k}{\partial \delta_k} \right] - \frac{\partial P_k}{V_k} \left[ \frac{1}{V_k} \frac{\partial Q_k}{\partial \delta_k} + \frac{\partial P_k}{\partial \delta_k} \right]
\]

Similarly, the term \(\frac{\partial J_D}{\partial V_k}\) can be expressed as:

\[
\frac{\partial J_D}{\partial V_k} = \left[ \frac{\partial P_k}{V_k} \frac{\partial \delta Q_k}{\partial V_k} + \frac{\partial Q_k}{V_k} \frac{\partial \delta P_k}{\partial \delta_k} \right] - \left[ \frac{\partial P_k}{V_k} \frac{\partial \delta Q_k}{\partial V_k} + \frac{\partial Q_k}{V_k} \frac{\partial \delta P_k}{\partial \delta_k} \right]
\]

Again, utilizing equations from (3.28) to (3.31), we have

\[
\frac{\partial \delta Q_k}{\partial V_k} = -2B_{kk} \\
\frac{\partial \delta P_k}{\partial V_k} = V_G (-G_k \sin(\delta_k) + B_k \cos(\delta_k)) = \frac{\partial P_k}{V_k} \\
\frac{\partial \delta Q_k}{\partial V_k} = V_G (G_k \cos(\delta_k) + B_k \sin(\delta_k)) = \frac{\partial Q_k}{V_k} \\
\frac{\partial \delta P_k}{\partial V_k} = 2G_{kk}
\]

Processing equation (3.29) and equation (3.30), \(G_{kk}\) can be expressed as:
\[
G_{kk} = \frac{1}{2V_k} \left[ \frac{\partial P_k}{\partial V_k} - \frac{1}{V_k} \frac{\partial Q_k}{\partial \delta_k} \right] 
\]

Processing equation (3.28) and equation (3.31), \( B_{kk} \) can be expressed as:

\[
B_{kk} = -\frac{1}{2V_k} \left[ \frac{\partial Q_k}{\partial V_k} + \frac{1}{V_k} \frac{\partial P_k}{\partial \delta_k} \right] 
\]  

(3.47)

Now, applying values of \( \frac{\partial P_k}{\partial V_k}, \frac{\partial Q_k}{\partial V_k}, \frac{\partial P_k}{\partial \delta_k} \) and \( \frac{\partial Q_k}{\partial \delta_k} \) of equations (3.42), (3.43), (3.44), (3.45), (3.46) and (3.47) in equation (3.41) we have,

\[
\frac{\partial J_D}{\partial V_k} = \frac{\partial P_k}{\partial \delta_k} \left[ \frac{1}{V_k} \left[ \frac{\partial Q_k}{\partial V_k} + \frac{1}{V_k} \frac{\partial P_k}{\partial \delta_k} \right] + \frac{1}{V_k} \frac{\partial Q_k}{\partial V_k} \right] - \frac{\partial Q_k}{\partial \delta_k} \left[ \frac{1}{V_k} \frac{\partial P_k}{\partial V_k} + \frac{1}{V_k} \frac{\partial P_k}{\partial \delta_k} \right] 
\]

(3.48)

The terms \( \frac{\partial P_k}{\partial \delta_k}, \frac{\partial Q_k}{\partial V_k}, \frac{\partial Q_k}{\partial V_k} \) and \( \frac{\partial Q_k}{\partial \delta_k} \) of equations (3.40) and (3.48) are the elements of the modified Jacobian matrix of a multibus system represented by equation (3.22) and \( V_k \) is the bus voltage of the \( k \)th target bus. Now, to relate change in real power injection at target \( k \)th bus to the change in determinant of the modified Jacobian matrix of a multibus system, variables \( \Delta \delta_k \) and \( \Delta V_k \) of equation (3.33) are replaced by \( \Delta P_k \) and \( \Delta Q_k \) as follows:

\[
\Delta J_D = \left[ \frac{\partial P_k}{\partial \delta_k} \frac{\partial Q_k}{\partial V_k} \right]^{-1} \left[ \frac{\partial P_k}{\partial \delta_k} \frac{\partial Q_k}{\partial V_k} \right]^{-1} \left[ \frac{\partial P_k}{\partial \delta_k} \frac{\partial Q_k}{\partial V_k} \right] \left[ \frac{\partial P_k}{\partial \delta_k} \frac{\partial Q_k}{\partial V_k} \right] \left[ \frac{\partial P_k}{\partial \delta_k} \frac{\partial Q_k}{\partial V_k} \right]^-1 = \left[ \begin{array}{cc} A & B \\ C & D \end{array} \right] 
\]

(3.49)

The system collapse occurs when \( J_D \) becomes zero. Now, if \( J_D^0 \) is the determinant value corresponding to the defined operating condition of the system, then the required
change in $\Delta J_D$ of equation (3.49) to force $J_D$ zero is $\Delta J_D = 0 - J_D^0 = -J_D^0$ and corresponding change in injection at $kth$ bus will be

$$\Delta P_k = -\frac{J_D^0}{f}$$

(3.50)

Therefore, load margin at $kth$ load bus, i.e., the additional load that can be supplied to the $kth$ load bus to push it to the proximity of voltage collapse is:

$$P^\text{margin}_{Dk} = -\Delta P_k = \frac{J_D^0}{f}$$

(3.51)

Therefore, predicted critical load for $kth$ load bus at the point of voltage collapse is:

$$P^\text{pred}_D = P_D + P^\text{margin}_{Dk}$$

(3.52)

But, $\Delta P^\text{margin}_{Dk}$ is determined using linear relation between $\Delta J_D$ and $\Delta P_k$. As such, load margin for the $kth$ load bus ($P^\text{margin}_{Dk}$) would be more than the actual load margin of the bus. Further, for wide change in $\Delta J_D$, the predicted load value will be considerably high compared to that of actual critical load value at the point of voltage collapse. Slight over prediction of load margin could mislead system planner and operator in the decision making related to allowing more load in the load buses which are some what close to voltage stability limit. Therefore, it is required to confirm the actual load margin or critical load of a load bus through an iterative load flow analysis using the load margin determined by the equation (3.51). To ensure convergence of the load flow analysis in the proposed iterative procedure, it is required to normalize the predicted critical load value $P^\text{pred}_D$ in such a way that the modified predicted critical load value remains below the actual critical load of the bus.

For this purpose, a reduction factor ($RF$) and a distance factor ($DF$) are used to normalize the predicted critical load value for the $kth$ load bus as follows:

$$P^\text{crit}_{Dk} = P_D + RF DF P^\text{margin}_{Dk}$$

(3.53)
The distance factor is expressed as

$$DF = \left( \frac{P_{Dk} + P_{Dk}^{margin}}{P_{Dk}} \right)$$  \hspace{1cm} (3.54)$$

The distance factor ($DF$) will be higher when prediction is done for wide change of $\Delta J_D$, i.e., far from the proximity of voltage collapse limit. Thus, it will normalize the effect of over prediction due to wide change of $\Delta J_D$. Therefore, as the iterative procedure approaches the proximity of voltage collapse limit $P_{Dk}^{margin}$ will become very small, thus, the term $\frac{P_{Dk} + P_{Dk}^{margin}}{P_{Dk}}$ will also approach to 1. When the value of $J_D$ becomes negative the iterative procedure has to be terminated.

### 3.4 Procedure for Distribution of Normalized Predicted Load Margin to the Generators

Equation (3.8) represents sensitivity relation for all the buses with respect to the target $k$th bus. It is justified to supply the normalized predicted load margin ($\begin{bmatrix} RF & P_{Dk}^{margin} \end{bmatrix}$) of the $k$th target bus from the generators having higher sensitivity factor values. Therefore, to distribute the the normalized predicted load margin of $k$th target bus, the sensitivity relation for generator buses with respect to target $k$th bus are utilized. Equation (3.8) can be expressed for generator buses only as follows;

$$\Delta P_k = \begin{bmatrix} RF & P_{Dk}^{margin} \end{bmatrix} = \sum_{j=1}^{NG} \gamma_{Gj} \Delta P_{Gj}$$

$$= [\gamma_{G}] [\Delta P_{G}]$$  \hspace{1cm} (3.55)$$

Where, $\gamma_{Gj}$ is the sensitivity of the $j$th generation bus with respect to the $k$th target bus. To determine the additional contributions from generation buses to meet the normalized predicted load margin, pseudo-inverse technique has to be applied to equation (3.55).
\[
\Delta P_G = \left[ \gamma_G \right]^T \left( \left[ \gamma_G \right] \left[ \gamma_G \right]^T \right)^{-1} \left[ \frac{RF}{D_F} P_{Dk}^{\text{margin}} \right]
\] (3.56)

Solving the above equation, the value of \( \Delta P_G \) can be expressed as;

\[
\Delta P_{Gi} = \frac{\gamma_{Gi}}{N} \sum_{j \neq i} \gamma_{Gj} \left\{ \frac{RF}{D_F} P_{Dk}^{\text{margin}} \right\} \text{ for } i = 1...NG
\] (3.57)

Therefore, modified generation values are

\[
P_{Gi} = P_{Gi} + \Delta P_{Gi} \text{ for } i = 1...NG
\] (3.58)

subjected to the limiting constraints

\[
P_{Gi} \leq P_{Gi}^{\text{max}} \text{ for } i = 1...NG
\] (3.59)

Where, \( P_{Gi}^{\text{max}} \) is the maximum limit on generation for \( ith \) generating station.

### 3.5 Algorithm Of The Iterative Procedure

Reduction factor (RF) is used to normalize the over prediction of load margin (which is determined using linear relation between \( \Delta P_{Dk}^{\text{margin}} \) and \( \Delta J_D \) governed by equation(3.49)) with the objective of ensuring convergence of load flow analysis used in the proposed iterative procedure. As such, load at a bus must be always less than actual critical load of the bus, i.e. to say that \( P_{j}^{\text{crit}} \) determined using equation(3.53) must be less than actual critical load of the bus. In case, \( P_{j}^{\text{crit}} \) determined using equation(3.53) becomes slightly more than actual critical load of the bus (due to improper selection of Reduction Factor (RF)), the load flow analysis of the iterative procedure will not converge. To take into account of such a situation, the proposed algorithm is equipped
with a step after the load flow analysis. This step reloads the loads and generations of the system of the previous iteration values and reduces the reduction factor \( RF \) as \( RF = \frac{RF}{1.5} \) and load flow analysis is carried out again before proceeding to the other portion of the algorithm. This step helps in changing the value of reduction factor \( RF \) to ensure proper normalization (by Reduction Factor \( RF \)) of critical load governed by equation (3.49). The algorithm of the proposed iterative procedure is as follows:

1. Initialize the load flow data for the system and set iteration count \( K = 0, RF = (0.3 \text{ to } 0.5) \).

2. Initialize the bus voltages \([V]\) and angles \([\delta]\) and conduct load flow analysis of the system.

3. Check for load flow convergence criteria. If load flow has converged, then go to step-4.

   Otherwise, reduce reduction factor \( RF = \frac{RF}{1.5} \) and reload \( P^{K+1}_Dk = P^K_Dk, Q^{K+1}_Dk = Q^K_Dk \) also the generating stations output with their \( Kth \) iteration values and go to step-2.

4. Determine \( P^r_Dk \) using equation (3.53) and assign \( P^{K+1}_Dk = P^r_Dk, Q^{K+1}_Dk = \tan\phi_k P^{K+1}_Dk \).

5. Distribute the additional load \( \frac{RF \cdot P^{\text{margin}}}{DF} \) among the generating stations based on defined criteria subjected to generation limits of the generating stations as described in section-3. In case of reactive power limit violation, a PV bus has to be changed into a PQ bus by assigning reactive power at its limit.
6. Check for $J_D < 0.0$, if not, $K = K + 1$ and go to step-2.

7. Stop.

3.6 Simulation, Results And Discussions

To verify the validity and applicability of the proposed method, simulations were carried out on IEEE 30 bus and IEEE 118 bus systems. The aim of the simulations were to examine the nature of change of $J_D$ for different load bus of IEEE 30 and IEEE 118 bus system with respect to change in load at the target bus. For this purpose, load at a target bus is increased gradually manually using an interactive load flow program and the change in $J_D$ values were recorded. It is found that the $J_D$ reduces parabolically with increase in load and at the point of voltage collapse it becomes zero. Figures- 3.2 and 3.3 illustrate the variation of $J_D$ value of two bus equivalent system of the IEEE 30 bus system with respect to change in load at load buses 21 and 29 for load power factors 0.8 and 0.9. Figures- 3.4 and 3.5 illustrate the variation of $J_D$ value of two bus equivalent system of the IEEE 118 bus system with respect to change in load at load buses 88 and 118 for load power factors 0.8 and 0.9.

It is observed that $J_D$ value of two bus equivalent system of IEEE 30 and 118 bus systems have different initial values for different target buses, as such, $J_D$ value for a bus reflects the voltage stability characteristic of the bus.

The proposed algorithm is used to determine critical load of a bus with different reduction factors ($RF$). It has been found that $RF$ values between 0.1 to 0.5 ensures convergence of load flow analysis in the iterative process for IEEE 30 and IEEE 118 bus system for any target bus of the system. But, with $RF = 0.3$ to 0.5, the iterative procedure terminates with less number of iterations. During iteration, the additional
load assigned on the target \( (k) \) bus was distributed among all the generating stations based on their load contribution (subjected to their limits) as described in section-4. Continuation power flow analysis technique is also used to determine the critical load at the target buses with step size \( \sigma = 0.001 \). However, in this case predicted normalized load is not distributed among the generators. Therefore, the proposed algorithm is also used to determine the critical load at the target buses without distributing predicted normalized load among the generators.

Table-3.1 and table - 3.2 represent the simulation results for some of the buses of
Figure 3.4: Variation of $J_D$ for load change at load bus 88 of IEEE 118 bus system for load power factors pf=0.8 & 0.9.

Figure 3.5: Variation of $J_D$ for load change at load bus 118 of IEEE 118 bus system for load power factors pf=0.8 & 0.9.
IEEE 30 bus system with $RF = 0.4$ with power factor 0.9 and 0.8 respectively. Table-3.3 and table -3.4 represent the simulation results for some of buses of IEEE 118 bus system with $RF = 0.4$ with power factor 0.9 and 0.8 respectively. Tables represent the target bus selected for determination of critical load with respect to its voltage collapse limit, initial value of two by two Jacobian matrix for the target bus, initial load at the target bus, final value of two by two Jacobian matrix when its become negative, critical load for the target bus and remarks about the methods. The abbreviation used in the remarks column of the tables stand for:

- **PMWGD** = Proposed method with generation distribution.
- **PMWOGD** = Proposed method without generation distribution.
- **CLFWOGD** = Continuation power flow without generation distribution.

It has been observed that the distance factor $DF = \left( \frac{P_{Dk} + P_{margin}}{P_{Dk}} \right)$ becomes very very close to 1, when the iterative process terminates. But, at the beginning of the iterative process it appears to be high depending upon change of $\Delta J_D$ used for the prediction of load margin for the load bus. It normalizes the effect of over prediction of load margin due to wide change of $\Delta J_D$ and ensures convergence of the load flow analysis of the system during the iterative process.

The programs were executed on a P C with Pentium-4 processor having processor speed of 1.5 GHz, and LINUX operating system. The simulation results show that the proposed method requires considerably less CPU time compared to that of continuation power flow analysis. Continuation power flow analysis requires on an average 20-30 times more CPU time than that of proposed method. However, the critical load determined for target buses by proposed method without generation distribution are slightly more (after third decimal place) than that of values determined by continuation power flow without generation distribution. It is due to the fact that the step size $\sigma$ for continuation power flow analysis is taken as 0.001. If $\sigma$ is taken as 0.0001, the critical load determined for
Table 3.1: Critical load of some of the buses (k) of IEEE 30 bus system for load pf 0.9.
Table 3.2: Critical load of some of the buses(k) of IEEE 30 bus system for load pf 0.8.
### Table 3.3: Critical load of some of the buses (k) of IEEE 118 bus system for load pf 0.9.

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<th>$P_{Dk}$</th>
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</tr>
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Table 3.4: Critical load of some of the buses($k$) of IEEE 118 bus system for load pf 0.8.
target buses by proposed method without generation distribution and by continuation power flow without generation distribution become same. But, CPU time for continuation power flow increases by another 20-30 times. It is to be noted that sensitivity based distribution of normalized predicted load margin of a target bus among generators ensures higher critical load of a target bus in comparison to that of continuation power flow analysis without distribution of normalized predicted load margin among generators.

3.7 Conclusion

This chapter proposed an algorithm for voltage stability analysis of a target/selected load bus using the singularity condition of the load flow Jacobian matrix. The load flow Jacobian matrix of an inter connected multi-bus power system is transformed into a two by two elements Jacobian matrix with respect to a target/selected load bus by incorporating the effect of all the other buses of the system to the target bus. To incorporate the effect of other buses to the target bus, sensitivity relations have been formulated to relate change in injection at the target/selected bus with respect to the change in injections in the other buses of the system. Simulations on IEEE 30 and 118 bus system indicated that the determinant of two bus equivalent system of a multibus system reduces with increase in load at the target bus and it becomes zero (almost near to zero) when load flow solution does not converge i.e., voltage collapse takes place. Therefore, it could be concluded that when the modified two by two matrix become singular, actual load flow Jacobian matrix also becomes singular. But, it is observed that determinant value of two bus equivalent system of IEEE 30 and 118 bus systems have different initial values for different target buses, as such, it shows that the transformed two by two elements Jacobian matrix reflects the property/quality of the target bus.

The algorithm proposed for the determination of critical load of a bus with respect to its voltage collapse limit of a power system works for all buses of IEEE 30 and IEEE 118 bus system, as such, it could be used for any inter connected power system. The
use of reduction factor ($RF$) and distance factor ($DF$) ensures convergence of the load flow analysis of the system during the proposed iterative procedure. These two factors effectively normalize the prediction of load margin, which is carried out using the linear relation between $\Delta J_D$ and $\Delta P_k$ governed by equation (3.51). The simulation results show that the proposed method requires considerably less CPU time compared to that of continuation power flow analysis. Moreover, the sensitivity based distribution of normalized predicted load margin of a target bus among generators ensures higher critical load for a target bus.