Recent cosmological observations, including the observations of distant Type-la supernovae, the Large Scale Structure, Wilkinson Microwave Anisotropy Probe in the Cosmic Microwave Background Radiation, the mass-energy density estimates from galaxy clusters, weak lensing etc. suggest that ordinary matter and energy constitutes only about 4% of the universe and the remaining 96% of the universe is yet unknown. Out of these 96%, about 23% is some form of hitherto unknown matter called dark matter and about 73% is dubbed as dark energy. The present observed accelerated expansion of the universe has been attributed to this exotic component, dark energy, with negative pressure which can induce repulsive gravity causing the accelerated expansion of the universe. The question of the actual nature and composition of the dark matter and dark energy of the universe today is one of the most exciting and challenging problems.

This Ph. D. thesis on a study of dark matter and dark energy consists of seven chapters.

In Chapter 1, a brief introduction to dark matter and dark energy is given.

In Chapter 2, we have considered Raychaudhuri equation. Though A. K. Raychaudhuri formulated his equation without the knowledge of accelerated expansion of the universe, we have shown that dark energy can be incorporated into Raychaudhuri equation through enlarged law of gravitation. In this chapter we have considered the dynamical cosmological term, quintessence, k-essence, tachyon and phantom scalar field as the candidates of dark energy.
In Chapter 3, we have obtained analytical solutions of the evolution of mass of black holes and worm holes immersed in a generalised Chaplygin gas (GCG) model and calculated the evolution of the mass of black hole and worm hole embedded in a universe filled with GCG. The GCG model represents the unification of dark matter and dark energy of the universe. For the equation of state (EOS) $\omega = \frac{p}{\rho} = -1$ of the dark component in GCG model it is found that the mass of the black hole increases and the mass of the worm hole decreases as the universe expands and both the masses become constant when the dark energy component of the GCG model becomes dominant in the universe.

In Chapter 4, we have studied the anisotropic expansion and acceleration of the universe driven by tachyonic matter. Though the present day universe appears to be isotropic, there is no evidence that the early universe was also of the same type. There is a cosmological view that the universe might have been anisotropic and also inhomogeneous in the very early era and that in the course of its evolution these characteristics have been wiped out under the action of some process or mechanism, and finally an isotropic and homogeneous universe had resulted. Hence the expansion and acceleration of the universe driven by tachyonic matter are studied by taking the anisotropic models of the universe Bianchi type I, Kantowski Sachs and Bianchi type III metrics in four dimensions and also Bianchi type I metric in higher dimensions. A set of solutions is obtained and their physical implications are discussed.

In Chapter 5, we have studied a homogeneous and anisotropic universe filled with matter and holographic dark energy components. Assuming deceleration parameter to be a constant, an exact solution to Einstein’s field equations in axially symmetric Bianchi type-I line element is obtained. A correspondence between the holographic dark energy models with the quintessence dark energy models is also established.
Quintessence potential and the dynamics of the quintessence scalar field are reconstructed, which describe accelerated expansion of the universe.

In Chapter 6, a homogeneous and anisotropic universe filled with new agegraphic dark energy (NADE) and cold dark matter components is studied. Here also, assuming deceleration parameter to be a constant, we obtained some exact solutions to Einstein's field equations in axially symmetric Bianchi type-I line element. We established a correspondence between the interacting NADE models with the tachyon and bulk viscous dark energy models. Tachyon potential, the dynamics of the tachyonic scalar field and bulk viscous coefficient are reconstructed, which describe accelerated expansion of the universe.

In the last Chapter, viz. Chapter 7, we have considered the dark energy model with the equation of state \( p_{DE} = -\rho_{DE} - A\rho_{DE}^\alpha \) which leads to four finite life time future singularities of the universe for different values of the parameters \( A \) and \( \alpha \). Since from the matter dominated era to the dark energy dominated era the ratio of the dark energy density to the matter energy density increases as the universe expands for these future singularities, the universe passes through a significant time when the dark energy density and the matter energy density are nearly comparable. Considering \( \frac{1}{r_0} < r = \frac{p_{DE}}{\rho_M} < r_0 \), where \( r_0 \) is any fixed ratio, we have calculated the fraction of total life time of the universe when the universe passes through the coincidental stage for these singularities. It has been found that the fractional time varies as \( \alpha \) varies within the range for which these finite life time future singularities occur and the fraction is smaller for smaller values of \( r_0 \). Importance of the fractional time and observational limits onto the values of the parameter \( A \) and \( \alpha \) has also been discussed.