Chapter - 4

Anisotropic expansion and acceleration of the universe driven by tachyonic matter

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4. Anisotropic expansion and acceleration of the universe driven by tachyonic matter

4.1 Introduction

Though the present day universe appears to be isotropic, there is no evidence that the early universe was also of the same type. There is a cosmological view that the universe might have been anisotropic and also inhomogeneous in the very early era and that in the course of its evolution these characteristics have been wiped out under the action of some process or mechanism, and finally an isotropic and homogeneous universe had resulted. It would therefore be of some interest to study different types of anisotropic cosmological models [179-191]. It is well known that the relativistic cosmological models for spatially homogeneous and anisotropic space-time belong either to the Bianchi types or Kantowski - Sachs cosmological models. There is also fairly good evidence from the BOOMERANG observations of CMB (Cosmic Microwave Background) that the universe underwent a period of acceleration, so called Primordial Inflation at early times. Recent astrophysical data obtained from high red shift surveys of Supernovae (SnIa) [9, 171] indicate that the present universe is also passing through a phase of accelerated expansion. Friedmann equation can be consistent with such an accelerated expansion only if the universe is populated by some medium with negative pressure [24, 25, 26, 37, 80, 109, 113, 153, 173]. One of the possible sources which could provide such a negative pressure is a scalar field \( \phi \) with the Lagrangian of the type \( L = -V(\phi) \sqrt{1 - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} \). Recently it has been suggested that the tachyonic condensate in a class of string theories can be described by an effective scalar field with a Lagrangian of the above form [66]. The evolution of this condensate can have worth
exploring cosmological significance. Here the basic idea is that the usual open string vacuum is unstable but there exists a stable vacuum with zero energy density which is stable. This state is associated with the condensation of electric flux tubes of closed strings which can be described successfully by using an effective Born-Infeld action.

Attempts have been made to construct viable cosmological model using rolling tachyon field as a suitable candidate for inflation, dark matter and dark energy. For this the tachyon potential field is considered as a viable model of dark energy [85, 177].

In this chapter, we study the role of tachyonic matter in Bianchi type I, Kantowski Sachs, Bianchi type III anisotropic cosmological models of the universe. The chapter is organized as follows. In section 2, the Einstein field equations for tachyon in case of four dimensions are written down. In section 3, the solutions of the field equations for the three types of cosmologies are derived. In section 4, the field equations and solutions for higher dimensional Bianchi type I cosmologies are obtained. We conclude this chapter with a brief physical interpretation of the solutions in section 5.

4.2 Field equations:

The action for the tachyon scalar $\varphi$ is given by Born-Infeld like action

$$s = \int \sqrt{-g} \, dx^4 \left( \frac{R}{16\pi G} - V(\varphi) \sqrt{1 - g_{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi} \right)$$

(1)

where $V(\varphi)$ is the tachyon potential.

Taking variation of $s$ w.r.t. $g_{\mu\nu}$ we get the field equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}$$

(2)

The energy-momentum tensor $T_{\mu\nu}$ for tachyonic matter is
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\[ \mathcal{T}_{\mu\nu} = V(\varphi) \left[ \frac{\partial_{\mu} \varphi \partial_{\nu} \varphi}{\sqrt{1 + g^a_{a\beta} \partial_{a} \varphi \partial_{\beta} \varphi}} - g_{\mu\nu} \sqrt{1 + g^a_{a\beta} \partial_{a} \varphi \partial_{\beta} \varphi} \right] \]  \hspace{1cm} (3)

The general (3+1) metric for axially symmetric Bianchi-I, Bianchi-III and Kantowski-Sachs space-time can be described by \[61\].

\[ ds^2 = dt^2 - a_1^2(t) dx_1^2 - a_2^2(t) d\Omega_k^2 \]  \hspace{1cm} (4)

where \( d\Omega_k^2 = dx_2^2 + dx_3^2 \) for \( k = 0 \) (Bianchi-I Model),

\[ d\Omega_k^2 = d\theta^2 + \sin^2 \theta d\phi^2 \]  \hspace{1cm} for \( k = 1 \) (Kantowski-Sachs Model),

\[ d\Omega_k^2 = d\theta^2 + \sin^2 \theta d\phi^2 \]  \hspace{1cm} for \( k = -1 \) (Bianchi-III Model),

For the line element (4) using (2) we get,

\[ 2 \frac{a_1}{a_2} \frac{a_2}{a_2})^2 + \frac{2}{a_2^2} \frac{a_2}{a_2})^2 \frac{a_1}{a_2} \frac{a_2}{a_2})^2 = 8\pi G \left( \frac{V(\varphi)}{1 - \varphi^2} \right) = 8\pi G \rho \]  \hspace{1cm} (5)

\[ \frac{a_1}{a_2} + \frac{a_2}{a_2} + \frac{a_1}{a_2} \frac{a_2}{a_2})^2 = 8\pi GV(\varphi) \sqrt{1 - \varphi^2} = -8\pi G p \]  \hspace{1cm} (6)

\[ 2 \frac{a_2}{a_2} \frac{a_2}{a_2})^2 + \frac{2}{a_2^2} \frac{a_2}{a_2})^2 \frac{a_1}{a_2} \frac{a_2}{a_2})^2 = 8\pi GV(\varphi) \sqrt{1 - \varphi^2} = -8\pi G p \]  \hspace{1cm} (7)

Here \( \rho = \left( \frac{V(\varphi)}{1 - \varphi^2} \right) \) and \( p = -V(\varphi) \sqrt{1 - \varphi^2} \) are the density and pressure of the tachyonic matter.

The equation of state for tachyonic matter is

\[ p = \omega \rho \]  \hspace{1cm} (8)

The scalar field equation of motion is

\[ \frac{\dot{\varphi}}{\sqrt{1 - \varphi^2}} + \left( \frac{a_1}{a_2} + \frac{2}{a_2} \right) \dot{\varphi} + \frac{1}{V d\varphi} = 0 \]  \hspace{1cm} (9)

Here we have two independent equations having four unknowns \( a_1, a_2, \rho, p \). Two more equations are required to find an exact solution.

4.3 Solutions:

As majority of cosmological models belong to either power law form or exponential form, we presently considering a power law relation between scale factors and time co-
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ordinates as,

\[ a_1 = a_{01} t^n \quad \text{and} \quad a_2 = a_{02} t^m \]  \hspace{1cm} (10)

Now since the right hand sides of (6) and (7) are identical, we have

\[ \frac{a_1}{a_1} + \frac{a_2}{a_2} + \frac{a_1 a_2}{a_1 a_2} = 2 \frac{a_3}{a_2} + \frac{a_3}{a_2} + \frac{k}{a_2^2} \]  \hspace{1cm} (11)

which gives \[ n = m \text{ or } n + 2m = 1 \] for \( k = 0 \) and \( m = 1, n^2 - 1 = \frac{k}{a_{02}^2} \text{ for } k \neq 0. \)

4.3.A Considering \( k = 0 \) and \( n = m \) from (10) we have

\[ a_1 = a_{01} t^n \quad \text{and} \quad a_2 = a_{02} t^n \]  \hspace{1cm} (12)

Using these conditions we get from (5), (6) and (7)

\[ \varphi(t) = \left( \frac{2}{3n} \right)^{\frac{1}{2}} t + \varphi_0 \]  \hspace{1cm} (13)

\[ V(\varphi) = \frac{3n^2}{8 \pi G} \left( 1 - \frac{2}{3n} \right)^{\frac{1}{2}} t^{-2} \]  \hspace{1cm} (14)

where \( n > \frac{2}{3} \)

Combining (13) and (14) we get,

\[ V(\varphi) = \frac{n}{4 \pi G} \left( 1 - \frac{2}{3n} \right)^{\frac{1}{2}} (\varphi - \varphi_0)^{-2} \]  \hspace{1cm} (15)

For such a potential, it is possible to have arbitrarily rapid expansion with large \( n \).

Using equation (13), (14) and (15) we have

\[ \rho_\varphi = \frac{n}{4 \pi G} (\varphi - \varphi_0)^{-2} = \frac{3n}{8 \pi G} t^{-2} \]  \hspace{1cm} (16)

From equation (8) we get,

\[ p_\varphi = (\varphi^2 - 1) \frac{n}{4 \pi G} (\varphi - \varphi_0)^{-2} = \frac{2 - 3n}{8 \pi G} t^{-2} \]  \hspace{1cm} (17)

The deceleration parameter is,

\[ q = -1 - \frac{H}{\dot{H}} = -1 + \frac{1}{n} \]  \hspace{1cm} (18)

Now for positive \( \rho_\varphi \) we must have \( n > 0 \) and also for accelerating universe we must have \( n > 1 \). So for \( n > 1 \), we have the positive \( \rho_\varphi \) and negative \( p_\varphi \) of the accelerating universe.
The CMBR and Supernovae observation suggest $0.85 \leq H_0 t_0 \leq 1.13$ that gives $0.85 \leq n \leq 1.13$. So the result agrees with the present observational data of the universe.

Also when $0 < n < 1$ then $\rho_\varphi$ and $q$ is positive which gives the past deceleration of the universe.

This also agrees with the FRW result.

**4.3.B** We now consider $k = 0$ and $n + 2m = 1$. For this relation we have from (10)

$$a_1 = a_{01} t^n \quad \text{and} \quad a_2 = a_{02} t^{\frac{1-n}{2}}$$

Using these conditions from (5), (6), and (7) we get

$$\varphi(t) = \varphi_0$$

$$V(\varphi) = \frac{(1-n)(3n+1)}{32\pi G} t^{-2}$$

Using (20) and (21) we have

$$\rho_\varphi = \frac{(1-n)(3n+1)}{32\pi G} t^{-2}$$

$$p_\varphi = (\dot{\varphi}^2 - 1) \frac{(1-n)(3n+1)}{32\pi G} t^{-2} = -\frac{(1-n)(3n+1)}{4} t^{-2}$$

The deceleration parameter is,

$$q = -1 - \frac{\dot{H}}{H^2} = 2$$

Also we have $H_0 t_0 = \frac{1}{3}$.

For positive $\rho_\varphi$ we must have $n < 1$. This gives the negative value of the pressure of the tachyon scalar field and the positive value of the deceleration parameter. The potential function is positive and decreases w.r.t. time and $\varphi$ is constant. So in this case it is not possible to construct a present accelerating model of the Universe.

**4.3.C** Lastly for $k \neq 0, m = 1, n^2 - 1 = \frac{k}{a_{02}^2}$ we have the scale factors in the form

$$a_1 = a_{01} t^n \quad \text{and} \quad a_2 = a_{02} t$$

(25)
Using these conditions from (5), (6) and (7) we get

\[ \varphi(t) = \sqrt{\frac{2}{n+2}} t + \varphi_0 \]  

(26)

\[ V(\varphi) = \frac{n\sqrt{n(n+2)}}{8\pi G} t^{-2} \]  

(27)

where \( n(n+2) > 0 \)

Combining (26) and (27) we get,

\[ V(\varphi) = \frac{3}{n^2 \sqrt{n+2}} (\varphi - \varphi_0)^{-2} \]  

(28)

Using (26) and (27) we get,

\[ \rho_\varphi = \frac{n(n+2)}{8\pi G} t^{-2} \]  

(29)

From equation (8) we get,

\[ p_\varphi = -\frac{n^2}{8\pi G} t^{-2} \]  

(30)

The deceleration parameter is,

\[ q = -1 - \frac{H}{H^2} = -1 + \frac{3}{n+2} \]  

(31)

Now for positive \( \rho_\varphi \) we must have \( n > 0 \) and also for accelerating universe we must have \( n > 1 \). So for \( n > 1 \), we have the positive \( \rho_\varphi \) and negative \( p_\varphi \) of the accelerating universe.

The CMBR and Supernovae observation suggest \( 0.85 \leq H_0 t_0 \leq 1.13 \) that gives \( 0.55 \leq n \leq 1.39 \). So the result agrees with the present observational data of the universe.

### 4.4 Bianchi type I metric in higher dimensions:

The Bianchi type-I metric in case of \((n+1)\)-dimensions is

\[ ds^2 = dt^2 - \sum_{i=1}^{n} a_i^2(t)(dx_i)^2 \]  

(32)

where \( a_i \)'s are functions of \( t \) only. The Einstein equations for the metric (32) are
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\[ \frac{a_1}{a_1} \sum_{i=2}^{n} \frac{a_i}{a_i} + \frac{a_2}{a_2} \sum_{i=3}^{n} \frac{a_i}{a_i} + \ldots + \frac{a_{n-1}}{a_{n-1}} \frac{a_n}{a_n} = 8\pi G \rho \]  
\[ \text{(33)} \]

\[ \sum_{i=1}^{n} \frac{a_i}{a_i} - \frac{a_1}{a_1} + \frac{a_2}{a_2} \sum_{i=2}^{n} \frac{a_i}{a_i} + \frac{a_3}{a_3} \sum_{i=3}^{n} \frac{a_i}{a_i} + \ldots + \frac{a_{n-1}}{a_{n-1}} \frac{a_n}{a_n} = 8\pi G \rho \]  
\[ \text{(34)} \]

We consider the case of \((n + 1)\)-dimensions with

\[ a_1 = a_2 = a_3 = a \text{ and } a_4 = a_5 = \ldots = a_n = \dot{a} \text{ where } a = a_0 t^k \text{ and } \dot{a} = a_0 t^k. \]

We get

\[ \rho = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} = \frac{1}{c^2} \left[ 3k^2 + 3(n - 3)k \dot{k} + \frac{(n-3)(n-4)}{2} \dot{k}^2 \right] \]  
\[ \text{(35)} \]

\[ p = -V(\phi)\sqrt{1 - \dot{\phi}^2} = \frac{1}{c^2} \left[ 6k^2 - 3k + 3(n - 4)k \dot{k} - (n - 4) \dot{k}^2 + \frac{(n-3)(n-4)}{2} \dot{k}^2 \right] \]  
\[ \text{(36)} \]

The relation between \(k\) and \(\dot{k}\) is given by the equality of the above equations as

\[ 3k^2 - k + (n - 6)k \dot{k} + k - (n - 3) \dot{k}^2 = 0 \]  
\[ \text{(37)} \]

The isotropic three-space will expand as \(t\) increases if \(k > 0\) and the extra dimensions contract as \(t\) increases if \(k < 0\).

For \(n = 4\), from (35) and (36) we get
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\[ \rho_\phi = \left( \frac{v(\phi)}{\sqrt{1 - \dot{\phi}^2}} \right) = \frac{3}{t^2} k(k + \dot{k}) \]  
(38)

\[ p_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2} = \frac{3}{t^2} k(2k - 1) \]  
(39)

And from (37) we get

\[ 3k^2 - k - 2k\dot{k} + k - \dot{k}^2 = 0 \]  
(40)

which gives \( k = \dot{k} \) or \( 3k + \dot{k} = 1 \)

For \( k = \dot{k} \) we get

\[ \rho_\phi = \left( \frac{v(\phi)}{\sqrt{1 - \dot{\phi}^2}} \right) = \frac{6}{t^2} k^2 \]

\[ p_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2} = \frac{3}{t^2} k(2k - 1) \]

For \( k > 1 \) both the dark energy density and pressure are positive. So in this case it is not possible to construct present accelerating model of the universe.

And for \( 3k + \dot{k} = 1 \) we get

\[ \rho_\phi = \left( \frac{v(\phi)}{\sqrt{1 - \dot{\phi}^2}} \right) = \frac{3}{t^2} k(-2k + 1) \]

\[ p_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2} = -\frac{3}{t^2} k(-2k + 1) \]

which gives

\[ \phi(t) = \phi_0(\text{constant}) \]

and \( V(\phi) = \frac{3}{t^2} k(-2k + 1) \)

In this case also for positive \( \rho_\phi \), we have \( k < \frac{1}{2} \)

which gives the past deceleration of the universe.

4.5 Discussion

Since the exponential form of the scale factors give the uniform values of the parameters which do not satisfy the present observational data of the universe, so
considering the scale factors in the power law form of time we obtain the solutions of the field equations.

Firstly, we have found the solution in axially symmetric Bianchi type I cosmology for the case $n = m$ in four dimensions. Positive density of the tachyonic matter indicates that $n > 0$. It is found that as the universe expands the tachyon rolls down to the minimum of its potential. When $n > 1$ we get accelerated expanding cosmological model of the universe. In this case the tachyon can be considered as an alternative candidate for dark energy with negative pressure. Secondly for the case $n + 2m = 1$ in axially symmetric Bianchi type I cosmology, we get the deceleration parameter positive which shows the past deceleration of the universe. So this model in this case failed to explain the present acceleration of the universe. For both Kantowski–Sachs and Bianchi type III metrics, we get the present accelerating model of the universe for $n > 1$. In Bianchi type I with higher dimensions we get the past deceleration of the universe and do not get the present accelerating model of the universe.