Chapter 3

Electromagnetic Radiation - from Partons and Hadrons

3.1 Significance of Electromagnetic Radiations

The main objective of Relativistic Heavy Ion Collisions is to study the transient phase, i.e. Quark Gluon Plasma (QGP) which is believed to permeate the early universe a few micro-seconds after the Big Bang. Collision between nuclei at ultra relativistic energies produce charged particles either in hadronic or in partonic state depending on the collision energy. Interaction of these charged particles produce electromagnetic (EM) radiation. However, hadrons being strongly interacting objects give snapshot of evolution only from the freeze-out surface. So they have hardly any information about the interior of the plasma. Whereas, EM radiation, e.g. the thermal photons and dileptons, from such collision are expected to provide an accurate information about the initial condition and the history of evolution of the plasma while it cools and hadronizes. This is possible since photons interact only through the EM interaction. The EM interaction strength
is small compared to that of strong interaction ($\alpha \ll \alpha_s$) and thus dominates the dynamics of nuclear collision processes. Therefore, its mean free path ($\lambda = 1/n\sigma$) is larger than the size of the system. Because of their negligible final-state interactions with the hadronic environment, once produced it brings the electromagnetic particles about to escape unscathed carrying the clean information of all stages of the collision (described in Sec. 1.3). The EM radiations produce all stages of collision process contribute to the measured photon spectra, in principle a careful analysis may be useful to uncover the whole space-time history of nuclear collision. Hence EM radiations - real and the virtual photons (dilepton), are considered as efficient probes to study dynamical evolution of the matter formed in relativistic heavy ion collision [39, 40, 41, 42, 43, 44, 45, 65]. However, as they are emitted continuously, they sense in fact the entire space-time history of the reaction. This expectation has led to an intense and concerted efforts toward the identification of various sources of such radiations. While initially this signals ware treated as thermometer of the dense medium created, but later on recent investigations and calculations suggest it might serve qualitatively as chronometer [63] and flow-meter [64] of HIC.

### 3.2 Various Sources of EM Radiations

As argued previously that EM radiations emerge out copiously from all stages of collision, so in order to proceed, it is useful to identify various sources of photons and dileptons produced in the HIC. So the “inclusive” photon spectrum coming from such collision in usual sense can be defined as: the unbiased photon spectrum observed in pp, pA or AA collision. This spectrum is built up from a cocktail of various components (discussed below). Depending on their origin, there are two different types of sources which is schematically presented in Fig. 3.1, i.e. “direct photons” and “photons from decay
of hadrons”. The term “direct photons” meant for those photons and dileptons which produce directly from collision between the particles. One can subdivide this broad category of “direct photons” into “prompt photons”, “pre-equilibrium photons” and “thermal photons” depending on their origin. On the other hand, the decay photons don’t come directly from the collision, rather from the decay of hadrons.

3.2.1 (A) Transverse Momentum ($p_T$) Dependence of EM Radiations:

The EM spectra provided by the experimentalist are mingled with various sources of photons and dileptons and it is difficult to distinguish different sources experimentally. However, real interest lies in the thermal photons and dileptons since it is expected to render an information about the initial condition and the history of evolution of the plasma while it cools and hadronizes. Thus, theoretical models are used with great advantage to identify these sources of photons and their relative importance and characteristics in the spectrum. As indicated in Fig. 3.2, the high $p_T$ part of the spectra is
strongly dominated by prompt contributions and low $p_T$ domain is populated by EM radiations from decay and the thermal photons and dileptons originate from the intermediate $p_T$. So subtracting out the prompt and decay contributions from the measured inclusive spectra of photons and dileptons, one can get pure contribution coming from the thermalized matter. Theoretically photons and dileptons emerging from QGP and hadronic phase can be calculated separately (will be discussed later). And the calculations based on theory infer that the photons and lepton pairs form hadronic matter dominate the spectrum at lower $p_T$ ($\sim 1-2$ GeV) whereas photons and dileptons form QGP dominate in the intermediate $p_T$ range, i.e. $p_T \sim 2-3$ GeV (depending on the models) [66].

- **Prompt Photons and Dileptons:**

  The *prompt* photons and dileptons are produced from the hard collisions between the partons inside the nucleons of the incoming nuclei in early collision stage before the
system thermalizes. It is the best understood part of the photons and dileptons production as can be regulated by perturbative QCD technique. The associated spectrum has power law kind of behaviour and dominates at large transverse momentum region (as shown in Fig. 3.2). Large momentum transfer results in small coupling constant which justifies the use of perturbative techniques. However, hadrons take part in experiments rather than partons. In non-perturbative regime, the theoretical calculation of momentum distribution of partons inside hadrons is beyond ones’ ability. Inevitably, one must find the platform where the void between what can be measured experimentally and what can be calculated perturbatively can be interconnected. So factorization method is the technique where one can interlink the short-distance (perturbative) part with the long-distance (non-perturbative) and expresses as follows [12]:

$$d\sigma = F(\mu, \Lambda_{QCD}) \otimes d\hat{\sigma}(Q, \mu)$$  \hspace{1cm} (3.1)

where the $\hat{\sigma}$ can be calculated perturbatively as a function of $\alpha_s$ treating the scattering process of parton interaction. The other factor, $F(\mu, \Lambda_{QCD})$, contains all long distance effects. Although $F(\mu, \Lambda_{QCD})$ depends on $\alpha_s$ in this case become large enough resulting non-perturbative situation and must, therefore, be obtained from data of various type of hard scattering process. The factorization scale $\mu$ is an arbitrary parameter. It can be thought of as a scale which separates the long and short-distance physics. Thus a parton emitted with a small transverse momentum, less than the scale $\mu$, is considered part of hadron structure and is absorbed into the parton distribution function.

The prompt photon contributions basically come from (i) Compton scattering ($qg \rightarrow g\gamma$), (ii) quark anti-quark annihilation process ($q\bar{q} \rightarrow g\gamma$) and quark fragmentation ($q \rightarrow q\gamma$) of the partons of the nucleons in colliding nuclei (shown in Fig. 3.3) and can be well described by the techniques of pQCD [67].

The invariant cross section of the reaction ($A + B \rightarrow \gamma + anything$) can be written in
Figure 3.3: The inclusive photon production in collision of particles A and B in partonic level by the direct partonic subprocess and the fragmentation of partons is shown in (a) and (b) respectively.

The factorized form as follows [68]:

\[
E_\gamma \frac{d\sigma}{d^3 p_\gamma} = \sum_{a,b,c} \int [dx_a dx_b F_1^a(x_a, \mu) F_2^b(x_b, \mu) \times \{E_\gamma \frac{d\hat{\sigma}}{d^3 p_\gamma} (a + b \rightarrow \gamma) \\
+ \int dz_c E_\gamma \frac{d\hat{\sigma}}{d^3 p_\gamma} (a + b \rightarrow c) D_3^c(z_c, \mu)]],
\]  

(3.2)

where a, b and c stands for the partons, \( F_{1,2}(x, \mu) \) is the parton distribution functions and \( D_3(z, \mu) \) is the fragmentation function. In the Eq. 3.2, the leading order cross-sections are considered for the total contribution from the photon productions and has been written in terms of two different terms. The first one expresses the direct partonic process \( (ab \rightarrow \gamma \text{ is illustrated in Fig 3.3 (a)}) \), Compton scattering and annihilation processes of quark and anti-quark and the second term represents quark fragmentation process \( (ab \rightarrow c \text{ is shown in Fig 3.3 (b)}) \).

The process of high-mass lepton pair emerging from \( q\bar{q} \) annihilation in a proton-proton collision is described by Drell Yan process [69] (illustrated in Fig. 3.4) and is the best understood part of production of dilepton.

In the naive parton model, the invariant cross-section for producing lepton pair \( l^+l^- \)
Figure 3.4: The Drell Yan process: $q\bar{q} \rightarrow l^+l^-$

with large invariant mass-squared, $M^2 = (p_{l^+} + p_{l^-})^2 \gg 1\text{GeV}^2$, in the collision of beam A and target B is simply obtained by simply weighing the subprocess invariant cross-section for $q\bar{q} \rightarrow \gamma^* \rightarrow l^+l^-$ with parton distribution functions $f_q(x, M^2)$ and $f_{\bar{q}}(x, M^2)$ extracted from deep inelastic scattering and summing over all quark anti-quark combination in beam and target.

$$E \frac{d\sigma_{AB}}{d^3p_\gamma} = \sum_q \int dx_1 dx_2 f_q(x_1, M^2) f_{\bar{q}}(x_2, M^2) E \frac{d\hat{\sigma}_{DY}}{d^3p} \quad (3.3)$$

where the partonic invariant cross-section for $q + \bar{q} \rightarrow l^+l^-$ is calculated using pQCD [68].

The prompt photons and dileptons from Drell-Yan processes can be estimated from pQCD and the experimental results from pp collisions (at same $\sqrt{s_{NN}}$) may be used to check the validation of the calculation. The production of high $p_T$ photon in A-A collision may be expressed in terms of p-p yield by using the following relation,

$$\frac{dN^{AA}}{d^2p_Tdy} = \frac{N_{coll}(b)}{\sigma_{in}^{pp}} \frac{d\sigma^{NN}}{d^2p_Tdy} = T_{AA}(b) \frac{d\hat{\sigma}_{NN}}{d^2p_Tdy} \quad (3.4)$$

where, $T_{AA}(b)$ is thickness function, $N_{coll}(b)$ is the number of inelastic nucleon-nucleon collision and $\sigma_{in}^{NN}$ is the inelastic cross-section of nucleon-nucleon (calculated using pQCD). The $T_{AA}(b)$ and $N_{coll}(b)$ can be calculated using Glauber model [53].

- **Pre-equilibrium Photons and Dileptons:**

  The *pre-equilibrium photons and dileptons* are produced in the pre-equilibrium stage
where \( \tau \leq \tau_i \) (described in Section 1.3), where \( \tau_i \) is the thermalization time, i.e., before the thermalization sets in the system. In the present work thermalization time scale at RHIC (\( \tau_i = 0.6 \text{ fm/c} \)) and LHC (\( \tau_i = 0.1 \text{ fm/c} \)) energies are taken to be very small. In such scenario the contribution from pre-equilibrium stage will be very small and hence neglected.

- **Thermal Photons and Dileptons**:

  At \( \tau \geq \tau_i \), the system is produced in QGP phase and with expansion it reverts to hot hadronic gas at a temperature \( T \sim T_c \). Thermal equilibrium may be maintained in the hadronic phase until the mean free path remains comparable to the system size. The EM radiations emerge from these thermalized matter (color shaded portion of Fig. 1.8), i.e. from both quark matter (QM) above \( T_c \) and hadronic matter (HM) when \( T \leq T_c \) is known as *thermal* photons or dileptons.

  Thermal photons from the QM arise mainly due to annihilation \( (q\bar{q} \rightarrow g\gamma) \) and Compton \( (q(\bar{q})g \rightarrow q(\bar{q})\gamma) \) processes [66, 70, 71]. Later, it was shown that photons from the processes [72]: \( gg \rightarrow gg\gamma, qq \rightarrow qq\gamma, qq \rightarrow q\gamma \) and \( gg\bar{q} \rightarrow g\gamma \) contribute in the same order \( O(\alpha_s^2) \) as Compton and annihilation processes. The relevant reactions and decays for photon production from HM are: (i) \( \pi\pi \rightarrow \rho\gamma \), (ii) \( \pi\rho \rightarrow \pi\gamma \) (with all possible mesons in the intermediate state [73]), (iii) \( \pi\pi \rightarrow \eta\gamma \) and (iv) \( \pi\eta \rightarrow \pi\gamma \), \( \rho \rightarrow \pi\pi\gamma \) and \( \omega \rightarrow \pi\gamma \). [44, 46, 73, 74, 75]. The reactions involving strange mesons: \( \pi K^* \rightarrow K\gamma, \pi K \rightarrow K^*\gamma, \rho K \rightarrow K\gamma \) and \( K K^* \rightarrow \pi\gamma \) [46] are also responsible for the production of thermal photons from hot hadron gas. Contributions from other decays, such as \( K^*(892) \rightarrow K\gamma, \phi \rightarrow \eta\gamma, b_1(1235) \rightarrow \pi\gamma, a_2(1320) \rightarrow \pi\gamma \) and \( K_1(1270) \rightarrow \pi\gamma \) have been found to be small [76] for \( p_T > 1 \text{ GeV} \). Like photons, the production of lepton pairs from hot QGP is dominated by \( q\bar{q} \rightarrow \gamma^* \rightarrow l^+l^- \) [77, 78, 79]
and in the hadronic matter the dominant processes are the decay of light vector mesons \((\rho, \omega\) and \(\phi\)); i.e. \(\rho \to l^+l^-, \omega \to l^+l^-\) and \(\phi \to l^+l^-\). \([45, 42, 80, 81, 82]\)

The schematic representation in Fig. 3.2 shows an exponential damping of the thermal photons spectrum at large energy. As the photons from hadronic phase dominate the spectra in low \(p_T\) (< 1 GeV), so there is small window around \(p_T \sim 2 - 3\)GeV for the detection of contribution from QGP. Disentangling the thermal photons coming out only from QGP phase is not trivial.

- **Photons and Dileptons from Decay:**

The thermal photons are emitted from the hot hadron gas until the freeze out temperature is reached. After the freeze-out of the fireball, photons and dileptons are also produced from the decays of long lived (compared to strong interaction time scale) hadrons and known as “photons from decay”. For example, photons are produced from the decays like \(\pi^0 \to \gamma\gamma\) and \(\eta^0 \to \gamma\gamma\) etc. Similarly dileptons are produced from the process \(\pi^0 \to \gamma e^+e^-, \eta^0 \to \gamma e^+e^-\) and \(\omega \to \gamma e^+e^-\) etc. are commonly known as Dalitz decays. The experimentally measured photon spectra are highly contaminated by the huge background from the decays. This makes the disentanglement of the thermal photon a more challenging task.

WA98 collaboration follows the subtraction method using invariant mass analysis \([83]\) for all photons for each pair \(p_T\) bin. The photon-pair combinatorial background is estimated by event mixing and then the decay photon from the real pair spectra is subtracted out. The yield in the \(\pi^0\) mass peak is extracted to obtain the raw neutral pion \(p_T\) spectra. These are then corrected for conversions, for \(\pi^0\) identification. In addition, \(\eta_S\) are extracted in a limited transverse momentum range with an analogous
procedure. The final inclusive photon spectra are to check for a possible photon excess beyond that form long-lived radiative decays. The background calculations are based on $\pi^0$ spectra and measured $\eta/\pi^0$-ratio. The spectral shapes of other hadrons having radiative decays are calculated assuming $m_T$ scaling with yield relative to $\pi^0$'s taken from the measurement. It should be noted that the measured contribution (from $\pi^0, \eta$) amounts to $\approx 97\%$ of the total photon background.

The major problem while performing invariant mass analysis arises from the accidental (false) photon pairs giving rise to pion mass and it is not possible to distinguish them from the correlated pairs in this method [65]. To overcome this problem, a mixed event analysis [84] procedure has been used successfully. The basic idea of mixed event technique is to compare particle spectrum from one event to the result for particle combinations from different events, which are a priori not correlated. As a first step, properly normalized mixed events are constructed by randomly sampling photons from different events. The difference of the invariant mass spectra of the real event and the mixed event then gives the pion and $\eta$ distributions. Once again the decay photon spectrum is subtracted from the inclusive photon spectrum to get the direct photons.

An alternative approach of separating direct photons from decay background is by measuring the “quasi-real” virtual photons which appear as low mass electron-positron pair. It is assumed that any source of real photons also produces low mass virtual photons which decay into $e^+e^-$ pair. This method is known as internal conversion method [85, 86]. The key advantage of this method is the greatly improved signal to background ratio which is achieved by elimination of the contribution of Dalitz ($\pi^0$) decay. The experimentally measured quantity is the ratio of $e^+e^-$ pairs in a particular invariant mass bin and the direct photon spectrum is obtained by multiplying $\gamma_{\text{dir}}^*/\gamma_{\text{incl}}^*$ to the measured inclusive photon spectrum. Tagging of decay photons is another very
useful method used by experimentalists for the subtraction of decay background [87].

**Characteristics of Invariant Momentum Distribution and Effect of Flow on It:**

The invariant momentum distribution of photons and dileptons produce from a thermal source depends on the temperature \( T \) of the source through the thermal phase space distributions of the participants of the reaction that produces the photons and dileptons [53]. As a result the \( p_T \) spectra of thermal photons and dileptons reflects the temperature of the source through the phase space factor \( e^{-E/T} \). Hence ideally the photons with intermediate \( p_T \) values \( (\sim 2 - 3 \text{ GeV}, \text{depending on the value of initial temperature}) \) reflect the properties of QGP (realized when \( T > T_c \), \( T_c \) is the transition temperature). Therefore, one should look into the \( p_T \) spectra for these values of \( p_T \) for the detection of QGP. However, for an expanding system the situation is far more complex. The thermal phase space factor changes by flow \( e.g. \) the transverse kick received by low \( p_T \) photons due to flow originating from the low temperature hadronic phase (realized when \( T < T_c \)) populates the high \( p_T \) part of the spectra [88]. As a consequence the intermediate or the high \( p_T \) part of the spectra contains contributions from both QGP and hadrons. Thus it is not easy task to disentangle the photons coming from pure partonic phase. However, the \( p_T \) integrated invariant mass spectra of dilepton may be useful to extract properties of QGP.

### 3.2.2 (B)Invariant Mass(M) Dependence of EM Radiations:

Being massive, dileptons make situation different from photons. They have two kinematic variables - \( p_T \) and M. Out of these two, the \( p_T \) spectra is affected by the flow, however, the \( p_T \) integrated \( M \) spectra remain unaltered by the flow in the system. It
should be mentioned here that for $M$ below $\rho$ peak and above $\phi$ peak dileptons from QGP dominates over its hadronic counterpart (assuming the contributions from hadronic cocktails are subtracted out) if the medium effect of spectral function of the low mass vector mesons are not taken into account. However, the spectral function of low mass vector mesons (mainly $\rho$) may shift toward lower invariant mass region due to non-zero temperature and density effects. As a consequence the contributions from the decays of $\rho$ mesons to lepton pairs could populate the low $M$ window and may dominate over the contributions from the QGP phase [45, 42, 89]. All these suggests that the invariant mass distribution of dilepton can be used as a clock for HIC and a judicious choice of $p_T$ and $M$ windows will be very useful to characterize the flow in QGP and hadronic phase.

Obtaining the dilepton invariant mass distributions from experimental data is technically very challenging because of the small dilepton-decay branching ratios of $\rho$, $\omega$, and $\phi$ mesons as there are many other hadronic sources available those produce leptons. The detector therefore must have an excellent lepton identification capability to detect the dileptons. It must also provide a means to successfully isolate the combinatorial background, where the background caused by an $l^+$ being erroneously paired up with an $l^-$ from other origin (e.g., a $e^+$ from $\pi^0 \rightarrow \gamma e^+e^-$ paired up with an $e^-$ from $\gamma^* \rightarrow e^+e^-$ occurring in the same event) [90]. This combinatorial background is found to be very large, specially in the high energy heavy ion collision experiments. The PHENIX experiment (for $e^+e^-$) at RHIC has reported a signal to background ratio of about 1/100 for minimum bias Au+Au collisions at 200A GeV [90, 91]. To subtract this combinatorial background, methods like event mixing, like-sign pair subtraction are quite useful [92]. However, a broad continuous background due to Dalitz decays still populate the dilepton invariant mass spectrum. The measured distribution is compared with the hadronic cocktail (which contains all known sources of $l^+l^-$ pairs produced in
the detector acceptance) in order to extract the vector-meson contributions [90].

The measured dilepton spectra can be divided into several phases. Depending on the invariant mass of the emitted dileptons, it can be classified into three distinct regimes (discussed below [42]) and a schematic diagram of dilepton mass distribution is shown in Fig 3.5.

![Dilepton Mass Distribution Diagram](image)

Figure 3.5: Expected different sources of dilepton production in heavy ion collision as function of invariant mass [42].

- **High Mass Region (HMR):** \( M \geq M_{J/\Psi} (= 3.1 GeV), p_T \sim 3 - 5 GeV \)

The HMR region corresponds to early pre-equilibrium phase \((\tau < \tau_i)\), where the lepton pairs are produced with large invariant mass \((M > 3 \text{ GeV})\) and the dominant contributions are from the hard scattering between the partons, like Drell Yan annihilation [69]. The final abundance of the heavy quarkonia \((J/\Psi, \Upsilon)\) and
their contribution to the spectrum is suppressed due to the Debye screening and as a result the bound states are dissolved.

- **Intermediate Mass Region (IMR):** \((M_\phi \leq M \leq M_{J/\psi}, \ p_T \sim 1 - 3\, \text{GeV})\)

  Thermalization is achieved in the system after a time scale \((\tau_i)\). In this domain, the dileptons from the QGP are produced from via quark-antiquark annihilation dominates. In this regime, due to higher temperature the continuum radiation from QGP dominates the dilepton mass spectrum and thus this region is important for the detection of QGP. The decays of “open charm” mesons, i.e, pairwise produced \(D\bar{D}\) mesons \([93]\) followed by semileptonic decays contribute a large in this domain of M. Although an enhanced charm production is interesting in itself - probably related to the very early collision states - it may easily mask the thermal plasma signal. To some what lesser extent, this also hold true for the lower-mass tail of Drell-Yan production \([69]\). As the heavy quarks produced in HIC do not get thermalized so their contribution may be estimated from pp collision data with the inclusion of nuclear effects like shadowing etc.. Hence they do not become part of the flowing QGP, then the lepton pairs which originate from the decays of heavy flavors will not contribute to flow \([94]\). Thus, the lepton pairs produced from the decays of heavy flavors and Drell Yann have been ignored in the present work.

- **Low Mass Region (LMR):** \((M \leq M_\phi (= 1.02\, \text{GeV}), \ p_T < 1\, \text{GeV})\)

  With subsequent expansion and cooling, the QGP converts into a hot hadron gas at the transition temperature, \(T_c\). At later stages, the dileptons are preferentially radiated from hot hadron gas from the decay of (light) vector meson, such as the \(\rho, \omega \) and \(\phi\). The low M domain of the lepton pairs are dominated by the decays of \(\rho\). Medium modification of \(\rho\) will change the yield in this domain of M. The change of \(\rho\) spectral function is connected with the chiral symmetry in the bath, therefore the measurement of low M lepton pairs has great importance to study the chiral
symmetry restoration [95] at high temperature and density. Thus the invariant mass of the lepton pair directly reflects the mass distribution of the light vector mesons. This explains the distinguished role that vector mesons in conjunction with their in-medium modifications play for dilepton measurements in HIC.

So far, we have discussed the different sources of photons and dileptons. Usually, the decay contribution is subtracted out from the measured inclusive spectra of photons and dileptons and the hard contribution is controlled by pQCD. As QGP is expected to form in the HIC experiments, so the basic intention of the present study to study the properties of QGP. Therefore, we have emphasized more on the study of thermal photons and dileptons in this dissertation. The detailed study of the emission of thermal photons and dileptons coming from HIC has been carried out in the subsequent sections.

3.3 Formulation of Thermal Emission Rate of EM Radiations

The importance of the electromagnetic probes for the study of thermodynamic state of the evolving matter was first proposed by Feinberg in 1976 [96]. Feinberg showed that the emission rates can be related to the electromagnetic current-current correlation function in a thermalized system. Generally the production of a particle which interacts weakly with the constituents of the thermal bath (the constituents may interact strongly among themselves) can always be expressed in terms of the discontinuities or imaginary parts of the self energies of that particle [97]. In this section, therefore, there is a discussion on how the electromagnetic emission rates (real and virtual photons) is related to the photon spectral function (which is connected with the discontinuities in the interacting propagators) in a thermal system [41], which in turn is connected to
the hadronic electromagnetic current-current correlation function [39] through Maxwell equations. It will be shown that the photon emission rate can be obtained from the dilepton emission rate by appropriate modifications.

We begin our discussion with the dilepton production rate which is given by [39, 44, 45]

\[
\frac{dN_{l^+l^-}}{d^4x d^4p} \equiv \frac{dR}{d^4p} = L^{\mu\nu}(p) W_{\mu\nu}(p) ,
\] (3.5)

where \(L^{\mu\nu}(p)\) is lepton tensor and obtained as

\[
L^{\mu\nu}(p) = \frac{(4\pi\alpha)^2}{M^4} \int \frac{d^3p_1}{(2\pi)^3 2p_1^0} \frac{d^3q_2}{(2\pi)^3 2q_2^0} Tr\left[ (\not{p}_1 - m)\gamma^\mu (\not{p}_2 + m)\gamma^\nu \right] \delta^{(4)}(p - p_1 - p_2)
= - \frac{\alpha^2}{6\pi^2 M^2} \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{M^2} \right) L(M^2) \] (3.6)

with \(p_{1,2}^0 = (m_{l^\pm}^2 + q_{1,2}^2)^{1/2}\) and the factor \(L(M^2) = (1 + 2m_l^2/M^2) \sqrt{1 - 4m_l^2/M^2}\) arises from the Dirac spinors (lepton pair) in the final state. \(L(M^2) = 1\) by neglecting the rest mass of the leptons (\(m_{l^\pm}\) is considered as \(m_{e^\pm}\)) as compared to their individual 3-momenta \(|\vec{p}_1|, |\vec{p}_2|\). \(M^2 = (p_1 + p_2)^2\) is the total four-momentum square of the pair in the heat bath. The effect of the partonic and hadronic medium is encoded in the tensor \(W_{\mu\nu}(p)\). It is obtained from the (thermal) average of the electromagnetic current-current correlation function as

\[
W_{\mu\nu}(p) = \int d^4x e^{-ipx} \langle j_{\mu}^e(x) j_{\nu}^e(0) \rangle
\] (3.7)

\(W_{\mu\nu}(p)\) contains the effect of strong interactions and is related to the imaginary part of the retarded current-current correlation function through the following relation;

\[
W_{\mu\nu} = (-2) f_{BE}(p_0, T) \text{Im}\Pi_{\mu\nu}^e
\] (3.8)

Inserting Eqs. 3.6 and 3.8 into 3.5, and exploiting gauge invariance, \(p^\mu \Pi_{\mu\nu}^e = 0\), one obtains the general result

\[
\frac{dR_{l^+l^-}}{d^4p} = - \frac{\alpha^2}{3\pi^3 M^2} f_{BE}(p_0; T) \text{Im}\Pi_{\mu\nu}^e(p_0, \vec{p}) \] (3.9)
with $f_{BE}(p_0, T) = 1/(e^{p_0/T} - 1)$ the Bose distribution function and the imaginary part of the EM current-current correlator is related to EM spectral function.

To obtain the real photon emission rate per unit volume ($dR$) from a system in thermal equilibrium we note that the dilepton emission rate differs from the photon emission rate in the following way. The factor $e^2 L_{\mu\nu}/p^4$ which is the product of the electromagnetic vertex $\gamma^* \rightarrow l^+ l^-$, the leptonic current involving Dirac spinors and the square of the photon propagator should be replaced by the factor $\sum \epsilon_{\mu} \epsilon_{\nu}^* (= -g_{\mu\nu})$ for the real (on-shell) photon. Finally the phase space factor $d^3 q_1/[(2\pi)^3 E_1] d^3 q_2/[(2\pi)^3 E_2]$ should be replaced by $d^3 p/[(2\pi)^3 p_0]$ to obtain

$$dR = -\frac{e^{-\beta p_0}}{2(2\pi)^3} g^{\mu\nu} W_{\mu\nu} \frac{d^3 p}{p_0}.$$  \hfill (3.10)

As in the case of dileptons this expression can be reduced to

$$\frac{dR}{p_0 d^3 p} = \frac{\alpha}{2\pi^3} g^{\mu\nu} f_{BE}(p_0; T) \text{Im}\Pi_{\mu\nu}^{em}.$$  \hfill (3.11)

The emission rate given above is correct up to order $e^2$ in electromagnetic interaction but exact, in principle, to all order in strong interaction. However, for all practical purposes one is able to evaluate up to a finite order of loop expansion. Now it is clear from the above results that to evaluate photon and dilepton emission rate from a thermal system we need to evaluate the imaginary part of the photon self energy. The Cutkosky rules at finite temperature or the thermal cutting rules [98, 99, 100] give a systematic procedure to calculate the imaginary part of a Feynman diagram. The Cutkosky rule expresses the imaginary part of the $n$-loop amplitude in terms of physical amplitude of lower order $(n - 1$ loop or lower). This is shown schematically in Fig. (3.6). When the imaginary part of the self energy is calculated up to and including $L$ order loops where $L$ satisfies $x + y < L + 1$, then one obtains the photon emission rate for the reaction $x$ particles $\rightarrow y$ particles $+ \gamma$ and the above formalism becomes equivalent to the relativistic kinetic theory formalism [40]. For a reaction $1 + 2 \rightarrow 3 + \gamma$ the photon (of energy $E$) emission
rate is given by [73]

\[
E \frac{dR}{d^3p} = \frac{\mathcal{N}}{16(2\pi)^7 E} \int_{(m_1+m_2)^2}^{\infty} ds \int_{t_{\text{min}}}^{t_{\text{max}}} dt |\mathcal{M}|^2 \int dE_1 \\
\times \int dE_2 \frac{f(E_1)f(E_2)[1 + f(E_3)]}{\sqrt{aE_2^2 + 2bE_2 + c}},
\]

(3.12)

where

\[
a = -(s + t - m_2^2 - m_3^2)^2
\]

\[
b = E_1(s + t - m_2^2 - m_3^2)(m_2^2 - t) + E[(s + t - m_2^2 - m_3^2)(s - m_1^2 - m_2^2) - 2m_1^2(m_2^2 - t)]
\]

\[
c = -E_1^2(m_2^2 - t)^2 - 2E_1 E[2m_2^2(s + t - m_2^2 - m_3^2) - (m_2^2 - t)(s - m_1^2 - m_2^2)]
\]

\[
+E_1^2[(s - m_1^2 - m_2^2)^2 - 4m_1^2m_2^2](s + t - m_2^2 - m_3^2)(m_2^2 - t)
\]

\[
\times(s - m_1^2 - m_2^2) + m_2^2(s + t - m_2^2 - m_3^2)^2 + m_1^2(m_2^2 - t)^2
\]

\[
E_{1\text{min}} = \frac{(s + t - m_2^2 - m_3^2)}{4E} + \frac{Em_1^2}{s + t - m_2^2 - m_3^2}
\]

\[
E_{2\text{min}} = \frac{Em_2^2}{m_2^2 - t} + \frac{m_2^2 - t}{4E}
\]

\[
E_{2\text{max}} = -\frac{b}{a} + \frac{\sqrt{b^2 - ac}}{a}
\]

\(\mathcal{N}\) is the overall degeneracy of the particles 1 and 2, \(\mathcal{M}\) is the invariant amplitude of the reaction (summed over final states and averaged over initial states), \(f\) denotes the thermal distribution functions and \(s, t, u\) are the usual Mandelstam variables.

In a similar way the dilepton emission rate for a reaction \(a \bar{a} \rightarrow l^+ l^-\) can be obtained
as

\[
\frac{dR}{d^4p} = \int \frac{d^3p_a}{2E_a(2\pi)^3} f(p_a) \int \frac{d^3p_{\bar{a}}}{2E_{\bar{a}}(2\pi)^3} f(p_{\bar{a}}) \int \frac{d^3p_1}{2E_1(2\pi)^3} \int \frac{d^3p_2}{2E_2(2\pi)^3} | \mathcal{M} |_{a\bar{a} \rightarrow l+ l-}^2 (2\pi)^4 \delta^{(4)}(p_a + p_{\bar{a}} - p_1 - p_2) \delta^{(4)}(p - p_a - p_{\bar{a}}). \]

(3.13)

where \( f(p_a) \) is the appropriate occupation probability for bosons or fermions.

### 3.4 Emission of Thermal Photons from Heavy Ion Collision

The *thermal* photons emerge just after the system thermalizes (\( \tau > \tau_i \)) from both QGP due to partonic interactions and hot hadrons (see Fig. 1.8) due to interactions among the hadrons. Now with the formalism given above production of thermal photons from QGP and hot hadronic gas is discussed in the section 3.4.1 and 3.4.2 respectively.

#### 3.4.1 Photons Emission from Quark Gluon Plasma

The contribution from QGP to the spectrum of thermal photons due to annihilation \((q\bar{q} \rightarrow g\gamma)\) and Compton \((q(\bar{q})g \rightarrow q(\bar{q})\gamma)\) processes has been calculated in Refs. [66, 70] using hard thermal loop (HTL) approximation [71]. The rate of hard photon emission is then obtained as [66]

\[
E \frac{dR_{\gamma}^{\text{QGP}}}{d^3q} = \sum_f e_f \frac{\alpha_s T^2}{2\pi^2} e^{-E/T} \ln(2.912E/g_s^2T). \]

(3.14)

where \( \alpha_s \) is the strong coupling constant. Later, it was shown that photons from the processes [72]: \( gg \rightarrow gg\gamma, qq \rightarrow qq\gamma, q\bar{q} \rightarrow q\gamma \) and \( gq \bar{q} \rightarrow g\gamma \) contribute in the same order \( O(\alpha_s) \) as Compton and annihilation processes (shown in Fig. 3.7). The complete calculation of emission rate from QGP to order \( \alpha_s \) has been performed by
resuming ladder diagrams in the effective theory [101]. In the present work this rate has been used. The temperature dependence of the strong coupling, $\alpha_s$, has been taken from [102].

### 3.4.2 Photons Emission from Hot Hadronic Gas

For the photon spectra from hadronic phase we consider an exhaustive set of hadronic reactions and the radiative decay of higher resonance states [73, 74, 75].

To evaluate the photon emission rate from a hadronic gas we model the system as consisting of $\pi$, $\rho$, $\omega$ and $\eta$. The relevant vertices for the reactions $\pi \pi \rightarrow \rho \gamma$ and $\pi \rho \rightarrow \pi \gamma$ and the decay $\rho \rightarrow \pi \pi \gamma$ are obtained from the following Lagrangian [74] (see Fig. 3.8):

$$\mathcal{L} = -g_{\rho\pi\pi}\vec{\rho}^\mu \cdot (\vec{\pi} \times \partial_\mu \vec{\pi}) - eJ^\mu A_\mu + \frac{e}{2} F_{\mu\nu} (\vec{\rho}_\mu \times \vec{\rho}_\nu)_3, \quad (3.15)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, is the Maxwell field tensor and $J^\mu$ is the hadronic part of
the electromagnetic current given by

\[ J^\mu = (\vec{\rho}_\nu \times \vec{B}^{\mu\nu})_3 + (\vec{\pi} \times (\partial^\mu \vec{\pi} + g_{\rho\pi\pi} \vec{\rho} \times \vec{\rho}^\mu))_3 \]  

(3.16)

with \( \vec{B}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu - g_{\rho\pi\pi}(\vec{\rho}_\mu \times \vec{\rho}_\nu) \).

For the sake of completeness we have also considered the photon production due to the reactions \( \pi \eta \rightarrow \pi \gamma, \pi \pi \rightarrow \eta \gamma \) and the decay \( \omega \rightarrow \pi \gamma \) using the following interaction:

\[ \mathcal{L} = \frac{g_{\rho\pi\eta}}{m_\eta} \epsilon_{\mu\nu\alpha\beta} \partial^\mu \rho^\nu \rho^\alpha \rho^\beta \eta + \frac{g_{\omega\rho\pi}}{m_\pi} \epsilon_{\mu\nu\alpha\beta} \partial^\mu \omega^\nu \rho^\alpha \rho^\beta \pi + \frac{em_\rho^2}{g_{\rho\pi\pi}} A_\mu \rho^\mu \]  

(3.17)

The last term in the above Lagrangian is written down on the basis of Vector Meson Dominance (VMD) [103, 104]. To evaluate the photon spectra, we have taken the relevant amplitudes for the above mentioned interactions from Ref. [73, 74]. The effects of hadronic form factors [46] have also been incorporated in the present calculation. The reactions involving strange mesons: \( \pi K^* \rightarrow K \gamma, \pi K \rightarrow K^{*\gamma}, \rho K \rightarrow K \gamma \) and \( KK^* \rightarrow \pi \gamma \) [46, 76] have also been incorporated in the present work. Contributions from other decays, such as \( K^*(892) \rightarrow K \gamma, \phi \rightarrow \eta \gamma, b_1(1235) \rightarrow \pi \gamma, a_2(1320) \rightarrow \pi \gamma \) and \( K_1(1270) \rightarrow \pi \gamma \) have been found to be small [76] for \( p_T > 1 \text{ GeV} \).
With all photon producing hadronic reaction, the static thermal emission rate of photons for hadronic phase have been evaluated [46, 66, 73, 74, 101] and shown in Fig. 3.9 for $T = 200$ MeV. The reaction involving $\rho$ mesons has dominant contribution. The rate at low photon energy is dominated by reaction with $\rho$ in final state, because these reactions are endothermic with most of the available energy going into rho mass. At high photon energy reactions with the $\rho$ in initial state are dominant because these reactions are exothermic; most of the rho mass is available for the production of high energy photons. Similar remarks can be made concerning reactions involving $\eta$ mesons, but as the value of $g_{\rho\rho\eta}$ is smaller thus so are the rates. All the isospin combinations for the above processes have properly been implemented.

**Emission of Photon from QM vs. HM**

In Sec. 3.4.1 and 3.4.2, the static thermal emission rates of high energy photons producing from QGP and hadronic gas have been discussed. In Fig. 3.10, the thermal rates of QGP and hadron is compared at $T = 200$ MeV. However, the results indicate that the
thermal rate of production of photons from QGP and hadron gas with energy $\approx 1 - 3$ GeV are similar. Not only in the shape of production curve but also the overall magnitude is same. The hadron gas shine as brightly as QGP. The conclusion is that high energy photons make a good “thermometer” for hot hadronic matter created in HIC. The thermal production rate only depend on temperature, so any temperature deduced from the thermally produced photons is nearly independent of the assumption about the phase of matter.

\subsection{3.4.3 Total Invariant Momentum Spectra of Thermal Photons:}

In this section we evaluate photon spectrum from a dynamically evolving system. The evolution of the system is governed by relativistic hydrodynamic. The photon production from an expanding system can be calculated by convoluting the static thermal emission rate with the expansion dynamics, which can be expressed as follows:

$$\frac{dN_\gamma}{d^2p_T dy} = \sum_i \int \left[ \frac{dR_\gamma}{d^2p_T dy}(E^*, T) \right]_i d^4x \quad (3.18)$$

Figure 3.10: Comparison of photon spectrum produce from QGP and hot hadron gas at $T = 200$ MeV
where the \( d^4x \) is the four volume. The energy, \( E^* \) appearing in Eq. 3.18 should be replaced by \( u^\mu p_\mu \) for a system expanding with space-time dependent four velocity \( u^\mu \). Under the assumption of cylindrical symmetry and longitudinal boost invariance, \( u^\mu \) can be written as:

\[
\begin{align*}
    u &= \gamma_T(\tau, r)(t/\tau, v_r(\tau, r), z/\tau) \\
    &= \gamma_T(M_T \cosh \eta, u_x, u_y, M_T \sinh \eta) \\
    &= \gamma_T(M_T \cosh \eta, v_r \cos \phi, v_r \sin \phi, M_T \sinh \eta) \\
\end{align*}
\] (3.19)

where \( v_r(\tau, r) \) is the radial velocity, \( \gamma_r(\tau, r) = (1 - v_r(\tau, r))^{-1/2} \) and therefore, for the present calculations,

\[
u^\mu p_\mu = \gamma_r(M_T \cosh(y - \eta) - v_r p_T \cos \phi)
\] (3.20)

For massless photon the factor \( u^\mu p_\mu \) can be obtained by replacing \( M_T \) in Eq. 3.20 by \( p_T \).

For the system produced in QGP phase reverts to hot hadronic gas at a temperature \( T \sim T_c \). Thermal equilibrium may be maintained in the hadronic phase until the mean free path remains comparable to the system size. The term “\( (dR/d^2p_Tdy)_i = [(\ldots)f_{BE}] \)” is the static rate of photon production \(^1\), where \( i \) stands for quark matter (QM), mixed phase (M) (in a 1st order phase transition scenario) and hadronic matter (HM) respectively. The \( p_T \) dependence of the photon and dilepton spectra originating from an expanding system is predominantly determined by the thermal factor \( f_{BE} \). The total momentum distribution can be obtained by summing the contribution from QM and HM, where the distribution for both the phases can be obtained by choosing the phase space appropriately.

The \( d^4x \) integration has been performed by using relativistic hydrodynamics with longitudinal boost invariance \([24]\) and cylindrical symmetry \([58]\) along with the inputs

\(^1\)By conversion of variables \( dp_x dp_y dp_z = J dp_T dy d\phi \), where \( J = E_{p_T}, \Rightarrow d^3p/E = 2\pi p_T dp_T = d^2p_T dy \).
Table 3.1: The values of various parameters - thermalization time ($\tau_i$), initial temperature ($T_i$) and hadronic multiplicity $dN/dy$ (the value of $dN/dy$ for various beam energies and centralities are calculated from the Eq. 2.16) - used in the present calculations.

<table>
<thead>
<tr>
<th>$\sqrt{s_{NN}}$</th>
<th>centrality</th>
<th>$dN/dy$</th>
<th>$\tau_i$ (fm)</th>
<th>$T_i$(MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.3 GeV</td>
<td>0-06%</td>
<td>700</td>
<td>1.0</td>
<td>200</td>
</tr>
<tr>
<td>200 GeV</td>
<td>0-20%</td>
<td>496</td>
<td>0.6</td>
<td>227</td>
</tr>
<tr>
<td></td>
<td>20-40%</td>
<td>226</td>
<td>0.6</td>
<td>203</td>
</tr>
<tr>
<td></td>
<td>min. bias</td>
<td>184</td>
<td>0.6</td>
<td>200</td>
</tr>
</tbody>
</table>

(given in the Table 3.1) as the initial conditions (described in Section 2.3.1) for SPS and RHIC energies.

To estimate $dN/dy$ for RHIC, we have taken $dn_{pp}/dy = 2.43$ and $x = 0.1$ at $\sqrt{s_{NN}} = 200$ GeV. It should be mentioned here that the values of $dN/dy$ (through $N_{\text{part}}$ and $N_{\text{coll}}$ in Eq. 2.16) and hence the $T_i$ (through $dN/dy$ in Eq. 2.15) depend on the centrality of the collisions. For SPS, $dN/dy$ is taken from experimental data [83]. We use the EoS obtained from the lattice QCD calculations by the MILC collaboration [105]. We consider kinetic freeze out temperature, $T_f=140$ MeV (here $T_f$ is treated as a parameter) for all the hadrons. The ratios of various hadrons measured experimentally at different $\sqrt{s_{NN}}$ indicate that the system formed in heavy ion collisions chemically decouple at $T_{ch}$ which is higher than $T_f$ which can be determined by the transverse spectra of hadrons [38](here the $T_f$ is treated as a parameter). Therefore, the system remains out of chemical equilibrium from $T_{ch}$ to $T_f$. The deviation of the system from the chemical equilibrium is taken in to account by introducing chemical potential for each hadronic species. The chemical non-equilibration affects the yields through the phase space factors of the hadrons which in turn affects the productions of the EM probes. The value of the chemical potential has been taken in to account following Ref. [106].
In the subsequent sections, we study the $p_T$ distribution of photons and dileptons. As mentioned before, the relativistic hydrodynamics has been used to describe the space time evolution of the matter formed in HIC. The initial conditions of the hydrodynamics and the static rates are discussed earlier are constrained to reproduce the experimental data available from SPS and RHIC energies. Subsequently, the ratio of $p_T$ spectra of photons and dileptons are used to extract the radial flow velocity (will be discussed in the next chapter).

### 3.4.4 Results and Discussion on $p_T$ Distributions of Photons

For comparison with direct photon spectra as extracted from HIC two further ingredients are required. With all the ingredients we have reproduced the $p_T$ spectra of direct photon for both SPS and RHIC energies. The prompt photons are normally estimated by using perturbative QCD. However, to minimize the theoretical model dependence here we use the available experimental data from p-p collisions to estimate the hard photon and normalized it to A-A data with $T_{AA}(b)$ for different centrality, i.e. the photon production from A-A collision and p-p collision are related through the following relation,

$$
\frac{dN^{AA}}{d^2p_Tdy} = \frac{N_{coll}(b)}{\sigma_{pp}^{in}} \frac{d\sigma^{NN}}{d^2p_Tdy} = T_{AA}(b) \frac{d\sigma^{NN}}{d^2p_Tdy}
$$

(3.21)

where $N_{coll}(b)$ is taken for the corresponding experiments and the the typical $\sigma_{pp}^{in}$ ($\sigma_{pp}^{in}$ 41mb for RHIC and 30mb for SPS).

**Photon Spectrum for WA98 Collaboration :**

The WA98 photon spectra from Pb+Pb collisions is measured at $\sqrt{s_{NN}} = 17.3$ GeV.
However, no data at this collision energy is available for pp interactions. Therefore, prompt photons for p+p collision at $\sqrt{s_{NN}} = 19.4$ GeV has been used [108] to estimate the hard contributions for nuclear collisions at $\sqrt{s_{NN}} = 17.3$ GeV. Appropriate scaling [83] has been used to obtain the results at $\sqrt{s_{NN}} = 17.3$ GeV. For the Pb+Pb collisions the result has been appropriately scaled by the number of collisions at this energy (this is shown in Fig. 3.11 as prompt photons). The high $p_T$ part of the WA98 data is reproduced by the prompt contributions reasonably well. At low $p_T$ the hard contributions under estimate the data indicating the presence of a thermal source. The thermal photons with initial temperature $= 200$ MeV along with the prompt contributions explain the WA98 data well (Fig. 3.11), with the inclusion of non-zero chemical potentials for all hadronic species considered [95, 106, 109]. In some of the previous works [110, 111, 112, 113, 114, 115] the effect of chemical freeze-out is ignored. As a result either a higher value of $T_i$ or a substantial reduction of hadronic masses in the medium was required [110]. In the present work, the data has been reproduced without any such effects.
Photon Spectrum for PHENIX Collaboration:

In Fig. 3.12, transverse momentum spectra of photons at RHIC energy for Au-Au collision for three different centralities (0-20%, 20-40% and min. bias.) at mid-rapidity shown, where the red tangles are the direct photon data measured by PHENIX collaboration [85] from Au-Au collision at $\sqrt{s_{NN}} = 200$ GeV, blue dashed line is the contribution of the prompt photons and the black solid line is thermal + prompt photons. For the prompt photon contribution at $\sqrt{s_{NN}} = 200$ GeV, we have used the available experimental data from pp collision and normalized it to Au-Au data with $T_{AA}(b)$ for different centrality [116] (using Eq. 3.21). At low $p_T$ the prompt photons underestimate the data indicating the presence of a possible thermal source. The thermal photons along with the prompt contributions explain the data [85] from Au-Au collisions at $\sqrt{s_{NN}} = 200$
GeV reasonably well. The reproduction of data is satisfactory (Fig. 3.12) for all the centralities with the initial temperature shown in Table 3.1 [117].

3.5 Emission of Thermal Dileptons from Heavy Ion Collision

Unlike real photon, dilepton are massive. Thus dilepton has two kinematic variables, invariant mass (M) and transverse momentum ($p_T$). Again, the $p_T$ spectra is affected due to flow, whereas the $p_T$ integrated M-spectra remain unaltered by flow. By tuning this two parameters, different stages of expanding fireball can be understood. Dileptons having large M and high $p_T$ are emitted early from the hot zone of the system. On the other hand, those having lower M and $p_T$ produced at later stage of the fireball when the temperature is low. Because of an additional variable, the invariant pair mass $M$, dileptons have the advantage over real photons [118].

The production of thermal dileptons from QGP (Sec. 3.5.1) and hot hadronic gas (Sec. 3.5.2) is described below.

3.5.1 Dileptons Emission from QGP

Above a critical temperature $T \geq T_c$, the production of lepton pairs from thermal QM is dominated by the annihilation process of $q\bar{q} \rightarrow l^+l^-$. The static thermal emission rate of dilepton from QM is given by $(q\bar{q} \rightarrow \gamma^* \rightarrow l^+l^-)$ [79] (also [77, 78]),

$$\frac{dR_{l^+l^-}}{d^4p} = -\frac{\alpha^2}{12\pi^4}L(M^2)f_{BE}\left(N_c\sum_fe_f^2\left[1 + \frac{2T}{\bar{p}}\ln\left(\frac{n_+}{n_-}\right)\right]\right)$$

(3.22)
where $N_c (= 3)$ is the number of colors, $e_f$ is the charge of the quark and $n_\pm = 1 / \left( e^{(p_0 \pm |\vec{p}|/2T) + 1} \right)$

### 3.5.2 Dileptons Emission from Hot Hadronic Gas

Below $T_c$, the appropriate degrees of freedom to describe strongly interacting matter are hadrons. In this regime, low $M$ dileptons are produced from the hadronic interactions. In the HM, the hadronic current may be decomposed as

$$
j_H^\mu = \frac{1}{2} (\bar{u} \gamma^\mu u - \bar{d} \gamma^\mu d) + \frac{1}{6} (\bar{u} \gamma^\mu u + \bar{d} \gamma^\mu d) - \frac{1}{3} \bar{s} \gamma^\mu s
$$

where vector currents are named by lowest mass hadron $\rho^0$, $\omega$ and $\phi$ in the corresponding channel. And in analogy with the Eq. 3.7, $W_{\mu\nu}$ can be written as, $W_{\mu\nu} = W^\rho_{\mu\nu} + W^\omega_{\mu\nu}/9 + W^\phi_{\mu\nu}/9$, where $W^V_{\mu\nu} = K_V p^V_{\mu\nu}(p_0, \vec{p})$, “$V$” stands for light vector mesons ($\rho^0$, $\omega$ and $\phi$), $p^V_{\mu\nu}(p_0, \vec{p})$ is the spectral function which is related to imaginary part of the propagator ($D^V_{\mu\nu}$)and $K_V = F^2_V m^2_V$, $F_V$ is obtained from the partial decay widths into $e^+e^-$ ($F_V=0.156$ GeV, $0.046$ GeV and $0.079$ GeV for $\rho^0$, $\omega$ and $\phi$ respectively). For HM, the standard rate for lepton pair production (Eq. 3.9) from decays of light vector mesons $\rho, \omega$ and $\phi$ has been considered in [45, 42, 80, 81, 82]. In addition, the spectral function of $\rho$ and $\omega$ has been augmented with a continuum contribution given by

$$
\frac{dR_{l^+l^-}}{d^4 p} = \frac{\alpha^2}{\pi^3} \int_{BE} \sum V=\rho,\omega A^\text{cont}_V
$$

where the continuum part of the vector mesons spectral functions constrained by experimental data [80, 119] have been included here in the following parametrized form

$$
A^\text{cont}_V = \frac{1}{8\pi} \left( 1 + \frac{\alpha_s}{\pi} \right) \frac{1}{1 + exp((w_0 - M)/\delta)}
$$
with $\omega_0=1.3 \ (1.1)$ for $\rho \ (\omega)$ GeV and $\delta = 0.2$ for both $\rho$ and $\omega$. The continuum contribution of the $\omega$ contains an additional factor of $1/9$. Since the continuum part of the vector meson spectral functions are included in the current work the processes like four pions annihilations [120] are excluded to avoid double counting.

### 3.5.3 Invariant Mass and Momentum Spectra of Dileptons:

The dileptons can be used as an efficient probe for QGP diagnostics, provided one can subtract out contributions from Drell-Yan process, decays of vector mesons within the life time of the fire ball and hadronic decays occurring after the freeze-out. Like hard photons, lepton pairs from Drell-Yan processes can be estimated by pQCD. The $p_T$ spectra of thermal lepton pair suffer from the problem of indistinguishableness between QGP and hadronic sources unlike the usual invariant mass ($M$) spectra which shows characteristic resonance peaks in the low $M$ region. The invariant transverse momentum distribution of thermal dileptons ($l^+l^-$) is given by:

$$
\frac{d^2N_{l^+l^-}}{dp_T dy} = \sum_{i=Q,M,H} \int_i \left( \frac{dR_{l^+l^-}}{dp_T dy dM^2} \right)_i M^2 dM d^4x. \quad (3.26)
$$

The invariant transverse mass distribution of thermal dileptons ($l^+l^-$) is given by:

$$
\frac{d^2N_{l^+l^-}}{2MdM dy} = \sum_{i=Q,M,H} \int_i \left( \frac{dR_{l^+l^-}}{dp_T dy dM^2} \right)_i p_T dp_T d^4x. \quad (3.27)
$$

The limits for integration over $p_T$ and $M$ can be fixed judiciously to detect contributions either from quark matter or hadronic matter. Experimental measurements [85, 121] are available for different $M$ window.
3.5.4 Results and Discussion on $p_T$ Distributions of Dileptons

For the evaluation of invariant momentum and invariant mass distribution of dilepton, we need to fold the static emission rate with the space-time dynamics. The space-time dynamics is described by relativistic hydrodynamics. The inputs for the initial condition required to solve the hydrodynamic equations are taken from the Table 3.1. With all these ingredients the $p_T$ spectra of dileptons for SPS and RHIC energies are calculated.

![Transverse mass spectra of dimuons in In+In collisions at SPS energy.](image)

Figure 3.13: Transverse mass spectra of dimuons in In+In collisions at SPS energy. Solid lines denote the theoretical results [107].

The transverse mass distribution of dimuons produced in In+In collisions at $\sqrt{s_{NN}} = 17.3$ GeV has been evaluated for different invariant mass ranges ( [119, 122, 63, 123] for details). The quantity $dN/M_TdM_T$ has been obtained by integrating the production rates over invariant mass windows $M_1$ to $M_2$ and $M_T$ is defined as $\sqrt{<M>^2 + p_T^2}$ where $<M> = (M_1 + M_2)/2$. The results are compared with the data obtained by NA60 collaborations [121, 119, 122] at SPS energy (Fig. 3.13). Theoretical results contain contributions from the thermal decays of light vector mesons ($\rho$, $\omega$ and $\phi$) and also from the decays of vector mesons at the freeze-out [61, 63] of the system has also been
considered. The non-monotonic variation of the effective slope parameter extracted from the $M_T$ spectra of the lepton pair with $\langle M \rangle$ evaluated within the ambit of the present model [123] reproduces the NA60 [121] results reasonably well.

![Transverse momentum spectra of dileptons for different invariant mass windows for minimum bias Au-Au collisions at RHIC energy](image)

**Figure 3.14:** Transverse momentum spectra of dileptons for different invariant mass windows for minimum bias Au-Au collisions at RHIC energy [107].

For Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV, we have evaluated the dilepton spectra for different invariant mass bins with the initial condition (min bias) shown in Table 3.1 and lattice QCD equation of state. The results are displayed in Fig. 3.14. The slopes of the experimental data on $p_T$ distribution of lepton pairs for different invariant mass windows measured by the PHENIX collaboration [124] could be reproduced well with the same initial condition that reproduces photon spectra [85]. In fact, the reproduction of data for the mass bins $0.5 < M(\text{GeV}) < 0.75$ and $0.81 < M(\text{GeV}) < 0.99$ do not need any normalization factors (Fig. 3.14). For lower mass windows slopes are reproduced well but fail to reproduce the absolute normalization. Therefore, it should be clarified here that the theoretical results shown in Fig. 3.14 for lower mass windows (to be precise for $0.1 < M(\text{GeV}) < 0.2$, $0.2 < M(\text{GeV}) < 0.3$ and $0.3 < M(\text{GeV}) < 0.5$) contain normalization
constants 660, 220 and 20 respectively.

### 3.5.5 Results and Discussion on Invariant Mass Distributions of Dileptons

With the use of Eq. 3.27, we have evaluated the $M$ distribution of lepton pairs originating from QM and HM without medium effects (the invariant mass distribution with medium effect will be discussed in Chapter-6) on the spectral functions of $\rho$ and $\omega$ for RHIC initial conditions. The invariant mass spectra of lepton pairs may be used to extract (i) the $M$ (GeV)$^{-1}$

<table>
<thead>
<tr>
<th>$M$ (GeV)</th>
<th>$dN/dM$ (GeV$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>1</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>1.5</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>2</td>
<td>$10^{-4}$</td>
</tr>
</tbody>
</table>

Figure 3.15: Invariant mass distribution of lepton pairs from quark matter (red solid line) and hadronic matter (blue dashed line) [126].

This suggests that the dilepton spectra can be used as a clock for heavy ion collision. As mentioned before, the $p_T$ spectra of the lepton pairs are affected by flow. Therefore, the evolution of flow of the evolving QGP may be estimated by studying the transverse medium effects of the vector meson spectra function, (ii) contributions from the (early) QGP phase by selecting $M > M_\phi$ and (iii) from the (late) hadronic phase ($M \sim m_\rho$).
momentum spectra with appropriate selection of invariant mass window. Hence the lepton pairs can also be used as flow-meter [64, 107, 126, 127] for the system formed in relativistic heavy ion collision. The HM dominates in the $M \sim M_\rho$ region whereas QM outshines in $M(> M_\phi)$ domain. Therefore, these two mass windows are selected to extract the flow parameters of the respective phases. In the present work, two procedures have been proposed to estimate the radial flow of the matter, i.e. (i) ratio of the $p_T$ spectra of thermal photons to dileptons and (ii) HBT radii extracted from the dilepton correlation function.

In this chapter, we are interested only on the $p_T$ distribution of photons and lepton pairs in various $M$ bins. The results will be used in the next Chapter to extract flow for various mass domain from the ratio of the $p_T$ spectra of thermal photons to dileptons. However, it should also be mentioned at this point that for the extraction of the flow the experimental data have been used here. Therefore, the non-reproduction of the absolute normalization of the $p_T$ spectra of lepton pairs for the lower mass windows at RHIC may not affect the extraction of the magnitude of the radial flow. We have studied the Bose-Einstein correlation function (BECF) for lepton pairs in Chapter- 5, and studied the effects of radial flow on the HBT radii extracted from the BECF of lepton pairs. Using the dilepton production from the QGP and hadron (with medium effect of vector meson spectral function), the elliptic flow of dilepton is evaluated (in Chapter- 6).