CHAPTER 8

COHERENCE INDUCED NEGATIVE REFRACTIVE INDEX IN FOUR-LEVEL ATOMIC MEDIUM

8.1 Introduction

The propagation of electromagnetic wave in a medium is governed by its refractive index \( n_r = \sqrt{\varepsilon_r \mu_r} \). Here \( \varepsilon_r \) and \( \mu_r \) are relative dielectric permittivity and permeability which in general are complex functions of frequency. Depending on the value of refractive index, all the available media/materials can be characterized into four quadrants as shown in Fig. 8.1. Conventional optical materials belong to the first quadrant. These are known as right handed materials since electric vector \( \vec{E} \), magnetic vector \( \vec{H} \) and wave vector \( \vec{k} \) form a right handed coordinate frame in them. The second and fourth quadrants constitute non-propagating evanescent waves. While the gaseous
and solid plasma materials belong to the second quadrant, materials which can be structured to behave like magnetic plasma belong to the fourth quadrant.

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<tr>
<th>Electrical Plasma</th>
<th>Ordinary Optical materials</th>
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<td>( \varepsilon_r &lt; 0, \mu_r &gt; 0 )</td>
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**Negative Refractive Index**

Most interesting materials which offer possibilities of controlling light propagation within a medium belong to the third quadrant. These materials are characterized by simultaneous negative \( \varepsilon_r \) and \( \mu_r \) and are referred to as negative refractive index (NRI) materials, double negative materials, backward wave media (having negative group velocity), left-handed materials (LHMs) or metamaterials. \( \vec{E}, \vec{H} \) and \( \vec{k} \) form a left-handed triad of vectors in NRI media. Further the Poynting vector is in opposite direction to wave propagation and hence the group velocity direction, which modifies the conventional route of refraction, diffraction and scattering of waves in these materials. The existence of LHMs was predicted by Veselago in 1968 [115]. He showed that LHMs do not violate any fundamental physical law and some of the most fundamental electromagnetic properties in these materials are opposite to that of ordinary materials, resulting in unusual optics. Some of the counter-intuitive electromagnetic and optical effects exhibited by these materials are reversed Snell’s law, reversed Doppler shift, an obtuse angle for Cherenkov radiation, anomalous refraction, sub-wavelength focusing, negative Goos-Hanchen shift, intense enhancement of the local fields, distinct

**Fig. 8.1:** Quadrant diagram illustrating the classification of materials based on the values of \( \varepsilon_r \) and \( \mu_r \).
phase matching conditions and nonlinear response, photon tunneling etc. [115-121]. Due to nonexistence of such materials naturally this field did not captured the attention of researchers for a long time. However following the demonstration of the key practical application of LHM i.e. perfect lens by Pendry in 2000 [117] the interest in these materials has grown tremendously. He showed that a NRI slab can focus all Fourier components and amplify evanescent modes allowing a complete reconstruction of a point source to a perfect point image, thus making it possible to achieve, in principle, unlimited resolution without any loss of energy [117]. Since then LHMs have become one of the frontline research area. The captivating optical properties of LHM not only bring new conceptual horizons in the basic understanding of physics but make them a potential candidate for diverse applications such as sub-wavelength imaging and beam refocusing, electromagnetic cloaking, slow and stopped light, stimulated Raman scattering, enhanced bio-sensing, quantum computation, in acoustics, photonics etc. [115-121].

8.2 Approaches for Realization of Negative Refraction Index

Several fascinating approaches have been developed for fabrication of LHMs. Most of the LHMs have been artificially realized in the microwave region using transmission line simulation, nanostructures, assembling composite lattice of metallic split ring resonators and metallic wires, or by using anomalous propagation properties of light in two-dimensional photonic crystal structures with periodicity of the order of or much smaller than the wavelength of the electromagnetic field [122,123]. All such materials, also known as artificial metamaterials, require delicate manufacturing of spatially periodic structures. Very recently Yoon et al. [124] have demonstrated NRI by
exploiting inertia of electrons in semiconductor two-dimensional electron gases which promises to open a path to miniaturization in the science and technology of these materials.

Of particular interest is the realization of NRI in optical region. However in this region refraction is always accompanied with absorption. Further it is difficult to realize negative $\mu_r$ with low loss, since magnetic dipole response to an oscillating magnetic field is smaller than the electric dipole response by a factor of $\alpha_{fs}^2$, where $\alpha_{fs} \approx 1/137$ is the fine structure constant. Several elegant suggestions such as magneto cross coupling technique or chirality induction have been made to alleviate this problem. A chiral media is an optically active media capable of producing negative refraction of circularly polarized wave [125]. Coupling a magnetic dipole transition coherently with an electric dipole transition may lead to electromagnetically induced chirality, which can show NRI with suppressed absorption without requiring negative permeability [118]. However such media suffer from losses due to environmental effects. Another proposal suggested a quantum optical approach in which, under certain conditions, electric-dipole and magnetic-dipole transitions in a multilevel EIT atomic/molecular system exhibit NRI [126-131]. As has been established in previous chapters EIT based dispersive media do not suffer from absorption at resonance, and offer low transmission losses even at high frequencies.

The realization of negative refraction in EIT based $\Lambda$ system was first proposed by Oktel et al., however with a stringent condition that the middle state ($|2\rangle$ in Fig. 1.2 (a)) is involved in both magnetic transition and electric transition at the same frequency [126]. A much realistic four-level EIT system was studied by Thommen and Mandel
[127] for the existence of left-handedness within a restricted domain of parameters and the requirement of degeneracy of the four levels. Further they suggested that atomic hydrogen and neon are good candidates for such experiments [127]. Since then several multilevel schemes based on quantum coherence and interference have been studied to realize NRI [128-131]. This method of coherently prepared atomic media offers various advantages such as realization of NRI in optical frequency range, electric and magnetic responses at atomic level and isotropic macroscopic electromagnetic structure as compared to artificial metamaterials.

In the chapter we demonstrate the use of laser induced coherent preparation of atomic medium to obtain simultaneous negative $\varepsilon_r$ and $\mu_r$ with minimal absorption in four-level systems in two different configurations interacting with trichromatic coherent field. Such systems can be realized within the hyperfine energy level or Zeeman manifold of alkali atoms. The advantage of rf field coupling over the conventional three level $\Lambda$ scheme [126] is the additional control of $\mu_r$ by regulating the rf field parameters. Further the rf field provides flexibility for adjusting frequency, depth and dispersion of the EIT resonance. We obtain $\varepsilon_r$ and $\mu_r$ for a dense atomic medium in the framework of master equation and Classius-Mossotti relation. Local field corrections (arising due to dipole-dipole interaction of the neighboring atoms) to the susceptibilities of the medium enhance the magnetic response and play an important role in reducing the absorptive losses. Our analysis shows that negative $\varepsilon_r$ and $\mu_r$ can be realized simultaneously in certain probe frequency regions with transparent propagation due to EIT. The use of the dispersion property of the negative refractive index to control the group velocity of the probe beam from subluminal to superluminal is also discussed.
8.3 Description of the Models

We consider two $\Lambda$ type four-level schemes coupled by three coherent fields as shown in Fig. 8.2.

![Diagram of four-level systems coherently driven by three laser fields]

**Fig. 8.2**: Schematic representation of four-level systems coherently driven by three laser fields: control, rf and probe of Rabi frequencies $2\alpha_c$, $2\alpha_{rf}$ and $2\alpha_p$ respectively. The corresponding detunings are $\Delta_c$, $\Delta_{rf}$ and $\Delta_p$. Radiative and nonradiative decay rates associated with $|i\rangle \rightarrow |j\rangle$ transition are denoted by $\gamma_{ij}$ and $\Gamma_{ij}$ respectively.

**Model (a)**: The scheme is similar to the DDL system studied in Chapters 4 and 5 with an additional rf field coupling the excited levels as shown in Fig. 8.2(a). Here transitions $|1\rangle \rightarrow |4\rangle$ and $|2\rangle \rightarrow |3\rangle$ are driven by control and probe lasers of Rabi frequencies $2\alpha_c$ and $2\alpha_p$ respectively. The electric dipole forbidden transition $|3\rangle \rightarrow |4\rangle$ is driven by a rf field of Rabi frequency $2\alpha_{rf}$. The relevant electric and magnetic dipole moments are $\bar{d}_{ij} = \langle i|\hat{d}|j\rangle$ and $\bar{m}_{ij} = \langle i|\hat{m}|j\rangle$ where $\hat{d}$ and $\hat{m}$ are the electric and magnetic dipole operators respectively. The detunings of control, rf and probe fields are $\Delta_c = \omega_{41} - \Omega_c$, $\Delta_{rf} = \omega_{43} - \Omega_{rf}$ and $\Delta_p = \omega_{32} - \Omega_p$ respectively. This scheme was earlier studied by Fu et al. [230] to show switching between EIT and EIA depending on the field detunings.
Coherence induced negative refractive index in four-level atomic medium

Model (b): This scheme consists of a triplet ground state \(|1\rangle, |2\rangle\) and \(|3\rangle\) and an excited state \(|4\rangle\). Here transitions \(|1\rangle \rightarrow |4\rangle\), \(|3\rangle \rightarrow |4\rangle\) and \(|1\rangle \rightarrow |2\rangle\) are driven by probe, control and \(rf\) fields of Rabi frequencies \(2\alpha_r\rho\), \(2\alpha_c\rho\) and \(2\alpha_{rf}\) respectively. The detunings of these fields from the corresponding atomic resonances are \(\Delta_p = \omega_{43} - \Omega_p\), \(\Delta_c = \omega_{41} - \Omega_c\) and \(\Delta_{rf} = \omega_{21} - \Omega_{rf}\). This scheme has been earlier studied in context of \(rf\) induced dynamic Stark effect [231], experimental realization of double dark resonances [232], sub-Doppler resonances [232,233] and other quantum interference effects [234].

8.4 Realization of Negative Refractive Index in Model (a)

8.4.1 Theoretical Formulation

In this section we obtain relative permittivity and permeability for a dense atomic medium in the framework of master equation and Classius-Mossotti relation.

(a) Density Matrix Equations and Coherences

The time evolution of the system is described by the following master equation,

\[
\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_0,\rho] - \sum_{i,j} \gamma_{ij} (A_{ij}\rho - 2A_{ji}\rho A_{ij} + \rho A_{ij}) - \sum_{i,j} \Gamma_{ij} (A_{ji}\rho - 2A_{ji}\rho A_{ji} + \rho A_{ji}) , \quad (8.1)
\]

where \(\gamma_{ij}\) and \(\Gamma_{ij}\) represent the radiative and non radiative decay rates associated with transitions \(|i\rangle \rightarrow |j\rangle\). The semi classical Hamiltonian of the system under RWA is

\[
H_0 = -\alpha_c(A_{41} + A_{41}^\dagger) - \alpha_p(A_{23} + A_{32}^\dagger) - \alpha_{rf}(A_{34} + A_{43}^\dagger) + (\Delta_c - \Delta_{rf} - \Delta_p) A_{22} + (\Delta_c - \Delta_{rf}) A_{33} + \Delta_c A_{44} . \quad (8.2)
\]

The elements of the density operator satisfy the following equations:
\[\begin{align*} 
\frac{d\rho_{11}}{dt} &= -2\Gamma_{12}\rho_{11} - i\alpha_{c}(\rho_{14} - \rho_{41}) + 2\Gamma_{24}\rho_{22} + 2\gamma_{41}\rho_{44}, \\
\frac{d\rho_{12}}{dt} &= -f_{1}\rho_{12} - i\alpha_{p}\rho_{13} + i\alpha_{c}\rho_{42}, \\
\frac{d\rho_{13}}{dt} &= -i\alpha_{p}\rho_{12} - f_{2}\rho_{13} - i\alpha_{ef}\rho_{14} + i\alpha_{c}\rho_{43}, \\
\frac{d\rho_{14}}{dt} &= -i\alpha_{c}(\rho_{11} - \rho_{44}) - i\alpha_{ef}\rho_{13} - f_{3}\rho_{14}, \\
\frac{d\rho_{22}}{dt} &= 2\Gamma_{12}\rho_{11} - 2\Gamma_{21}\rho_{22} - i\alpha_{p}(\rho_{23} - \rho_{32}) + 2\gamma_{32}\rho_{33}, \\
\frac{d\rho_{23}}{dt} &= -i\alpha_{p}(\rho_{22} - \rho_{33}) - f_{4}\rho_{23} - i\alpha_{ef}\rho_{24}, \\
\frac{d\rho_{24}}{dt} &= -i\alpha_{c}\rho_{21} - i\alpha_{ef}\rho_{23} - f_{5}\rho_{24} + i\alpha_{p}\rho_{34}, \\
\frac{d\rho_{33}}{dt} &= -i\alpha_{p}(\rho_{32} - \rho_{23}) - 2(\gamma_{32} + \Gamma_{34})\rho_{33} - i\alpha_{ef}(\rho_{34} - \rho_{43}) + 2\Gamma_{43}\rho_{44}, \\
\frac{d\rho_{34}}{dt} &= i\alpha_{p}\rho_{24} - i\alpha_{c}\rho_{31} - i\alpha_{ef}(\rho_{33} - \rho_{44}) - f_{6}\rho_{34}, \\
\frac{d\rho_{44}}{dt} &= i\alpha_{c}(\rho_{14} - \rho_{41}) + 2\Gamma_{34}\rho_{33} + i\alpha_{ef}(\rho_{34} - \rho_{43}) - 2(\gamma_{41} + \Gamma_{43})\rho_{44}, 
\end{align*}\]

where the coefficients \( f_{i} \) are defined as

\[\begin{align*} 
f_{1} &= \Gamma_{12} + \Gamma_{21} + i(\Delta_{c} - \Delta_{ef} - \Delta_{p}), \quad & f_{2} &= \Gamma_{12} + \Gamma_{21} + i(\Delta_{c} - \Delta_{ef}), \\
f_{3} &= \Gamma_{12} + \gamma_{41} + \Gamma_{43} + i\Delta_{c}, \quad & f_{4} &= \Gamma_{21} + \gamma_{32} + \Gamma_{34} + i\Delta_{p}, \\
f_{5} &= \Gamma_{21} + \gamma_{41} + \Gamma_{43} + i(\Delta_{p} + \Delta_{ef}), \quad & f_{6} &= \gamma_{32} + \Gamma_{34} + \gamma_{41} + \Gamma_{43} + i\Delta_{ef}. \end{align*}\]

For weak probe laser steady state solutions of Eq. (8.3) can be obtained as follows:

\[\begin{align*} 
\rho_{32}^{(1)} &= \frac{\alpha_{p}[f_{1}\alpha_{ef}\rho_{33}^{(0)} + i(f_{1}f_{3}^{*} + \alpha_{c}^{2})(\rho_{22}^{(0)} - \rho_{33}^{(0)}) + \alpha_{c}\alpha_{ef}\rho_{13}^{(0)}]}{f_{1}f_{3}^{*}f_{5}^{*} + \alpha_{ef}^{2}f_{1} + \alpha_{c}^{2}f_{4}^{*}} = \rho_{23}^{(1)*}, \\
\rho_{12}^{(1)} &= \frac{\alpha_{p}[f_{4}\alpha_{c}\rho_{33}^{(0)} - i(f_{4}f_{5}^{*} + \alpha_{ef}^{2})(\rho_{22}^{(0)} - \rho_{33}^{(0)}) + \alpha_{c}\alpha_{ef}\rho_{13}^{(0)}]}{f_{4}f_{5}^{*}f_{6}^{*} + \alpha_{ef}^{2}f_{1} + \alpha_{c}^{2}f_{4}^{*}} = \rho_{21}^{(1)*}, \\
\rho_{43}^{(0)} &= i\alpha_{ef}[(f_{2}f_{3} + \alpha_{ef}^{2})(\rho_{33}^{(0)} - \rho_{44}^{(0)}) - \alpha_{c}^{2}(\rho_{11}^{(0)} - \rho_{44}^{(0)})]/F_{1}, \\
\rho_{13}^{(0)} &= -\alpha_{c}\alpha_{ef}[f_{6}\rho_{11}^{(0)} + f_{3}\rho_{33}^{(0)} - (f_{3} + f_{6}^{*})\rho_{44}^{(0)}]/F_{1}, 
\end{align*}\]
Coherence induced negative refractive index in four-level atomic medium

\[
\rho_{11}^{(0)} = -\Gamma_2 [j_{32} \gamma_{41} + \Gamma_3 \gamma_{41} + \gamma_{32} \Gamma_4] - \alpha_c^4 \gamma_{41}^2 j_2^2 + \alpha_c^2 \gamma_{41}^2 \{j_1 j_3 - j_2 (\Gamma_3 - \Gamma_4 + 2 \gamma_{32})\} + \alpha_c^2 \gamma_{41}^2 \{j_1 (\gamma_{32} + \Gamma_4) + \alpha_c^2 \gamma_{41}^2 j_3 (\gamma_{32} + \gamma_{41})\}/F_2^2, \tag{8.6a}
\]

\[
\rho_{33}^{(0)} = -\alpha_c^2 \Gamma_2 [j_1 \gamma_{43} + \gamma_{43}^2 (j_1 j_3 + j_2 \gamma_{41}) - \alpha_c^2 \gamma_{41}^2 j_2^2]/F_2, \tag{8.6b}
\]

\[
\rho_{44}^{(0)} = -\alpha_c^2 \Gamma_2 [j_1 (\gamma_{32} + \Gamma_4) + \gamma_{32}^2 (j_1 j_3 - j_2 \gamma_{32}) - \alpha_c^2 \gamma_{41}^2 j_2^2]/F_2, \tag{8.6c}
\]

\[
\rho_{22}^{(0)} = 1 - \rho_{11}^{(0)} - \rho_{33}^{(0)} - \rho_{44}^{(0)}, \tag{8.6d}
\]

where

\[
j_1 = \text{Re}[(f_2 f_6^* + \alpha_c^2)/F_1], \quad j_2 = \text{Re}(F_1)/|F_1|^2, \quad j_3 = \text{Re}[(f_2 f_3 + \alpha_c^2)/F_1], \tag{8.7a}
\]

\[
F_1 = f_2 f_3 f_6^* + \alpha_c^2 f_3 + \alpha_c^2 f_6^*, \tag{8.7b}
\]

\[
F_2 = -\alpha_c^4 \gamma_{41}^2 j_2^2 (\Gamma_12 + 3 \Gamma_{21} + \gamma_{32}) + \alpha_c^2 \gamma_{41}^2 \{j_1 j_3 (\Gamma_12 + 3 \Gamma_{21} + \gamma_{32}) - j_2 (2 \gamma_{32} \gamma_{41} + \Gamma_12 - 2 \gamma_{32} - \Gamma_{34} + \gamma_{43}) + \Gamma_{21} (\gamma_{41} + \Gamma_{43} - 3 \gamma_{32} - \Gamma_{34})\}
\]

\[
+ \alpha_c^2 \gamma_{41}^2 j_1 [\Gamma_{12} + 2 \Gamma_{21}] (\gamma_{32} + \Gamma_{34}) + \Gamma_{43} (\gamma_{41} + \Gamma_{43} + \gamma_{41} + \gamma_{32} \Gamma_{43}) + \alpha_c^2 j_3 \quad \Gamma_{12} + \Gamma_{21} (\gamma_{32} + \Gamma_{34}) + (\Gamma_{12} + \Gamma_{21}) (\gamma_{41} + \Gamma_{43} + \gamma_{32} \gamma_{41} + \gamma_{32} \Gamma_{43})]. \tag{8.7c}
\]

(b) Electric and Magnetic Response

The atomic coherences can be used to analyze the possibility of observing NRI in the medium. Transitions \(|2\rangle \rightarrow |3\rangle\) and \(|1\rangle \rightarrow |2\rangle\) are driven by electric and magnetic field of the weak probe respectively. Therefore the coherences related to the electric and magnetic response are \(\rho_{32}^{(l)}\) and \(\rho_{21}^{(l)}\) respectively. The induced electric dipole moment of an atom due to the interaction with probe field is given by

\[
\tilde{P}_e(\omega_p) = \text{Tr}(\hat{\rho} \tilde{d}) = \tilde{d}_{23} \rho_{32}^{(l)} + \tilde{d}_{23} \rho_{23}^{(l)}. \tag{8.8}
\]

Electric and magnetic response of the medium can also be given in terms of electric polarizability \(\alpha_e\) and magnetizability \(\alpha_m\). We choose the probe field \(\tilde{E}_p\) parallel
to the electric dipole moment $\tilde{d}_{23} (= \tilde{d}_{32})$ so that $\alpha_e$ is a scalar quantity. $\alpha_e$ is related to the induced dipole moment as $\vec{P}_e(\omega_p) = \varepsilon_o \alpha_e \vec{E}_p(\omega_p)$ where $\varepsilon_o$ is permittivity of free space. We therefore obtain,

$$\alpha_e = \frac{|d_{23}|^2 \rho_{32}^{(1)}}{2\varepsilon_o \hbar \alpha_p}.$$  (8.9)

Similarly magnetization $\vec{P}_m$ is given as

$$\vec{P}_m(\omega_p) = \text{Tr}(\hat{\rho} \hat{\mu}) = \tilde{m}_{12} \rho_{12}^{(1)} + \tilde{m}_{12} \rho_{21}^{(1)}, \quad \mu_o \vec{P}_m(\omega_p) = \alpha_m \vec{B}_p(\omega_p),$$  (8.10)

where $\mu_o$ is permeability of free space, and $\vec{B}_p$ is the magnetic field given by

$$\vec{B}_p = \vec{k}_p \times \vec{E}_p / \omega_p.$$  (8.11)

We further assume that magnetic dipole moment is perpendicular to the induced electric dipole moment, i.e. $\tilde{m}_{12}$ is parallel to $\vec{k}_p \times \vec{E}_p$ so that $\vec{B}_p = \vec{E}_p / c$ and therefore we have,

$$\alpha_m = \frac{m_{12}d_{23} \mu_o c \rho_{21}^{(1)}}{2\hbar \alpha_p}.$$  (8.12)

The electric and magnetic Classius-Mossotti relations connect the macroscopic and microscopic variables of a media. For macroscopic polarization electric susceptibility of the medium can be obtained as

$$\chi_e = N \alpha_e / (1 - N \alpha_e / 3).$$  (8.13)

The relative electric permittivity of the medium is therefore given as

$$\varepsilon_r = \frac{1 + 2N\alpha_e / 3}{1 - N\alpha_e / 3}.$$  (8.14)

Similarly relative permeability of the medium can be obtained as follows:

$$\mu_r = \frac{1 + 2N\alpha_m / 3}{1 - N\alpha_m / 3},$$  (8.15)
where \( N \) is the atomic density. In order to modify \( \varepsilon_r \) and \( \mu_r \) simultaneously the foremost condition is that the electric and magnetic dipoles should oscillate at the same frequency, which implies that \( \omega_{32} \) and \( \omega_{21} \) should be equal to the probe frequency. The relevant electric and magnetic dipole moments chosen for the subsequent studies are \( 2.534 \times 10^{-29} \text{ Cm} \) and \( 1.312 \times 10^{-23} \text{ J/T} \) respectively which correspond to \(^{87}\text{Rb} \) atom.

For a dilute atomic vapor the microscopic local fields are very weak and hence neglected. However under this condition it is impossible to obtain negative permittivity and permeability as clear from Eqs. (8.9) – (8.15). Thus we consider a dense atomic media of closely packed atoms \( (N = 10^{23} \text{ m}^{-3}) \) where dipole - dipole interactions i.e the Lorentz-Lorenz local fields play a crucial role in the response of the medium. Finally for a left-handed media the absorption coefficient and refractive index are defined as

\[
A = 2\pi \text{Im}[\sqrt{\varepsilon_r \mu_r}] \quad \text{and} \quad n_r = \text{Re}[\sqrt{\varepsilon_r \mu_r}]
\]

respectively. The group velocity of the medium \( (v_g = c/n_g) \) becomes negative in certain frequency range of NRI. Thus by the tailoring the group index \( n_g = n_r(\omega_p) + \omega_p \left[ \frac{dn_r(\omega_p)}{d\omega_p} \right] \) one can tune the velocity of probe propagation from subluminal to superluminal.

### 8.4.2 Results and Discussion

We first consider that the control and rf fields are at resonance. Fig. 8.3 shows the effect of control strength on the permittivity, permeability and refractive index of the media. For \( \alpha_c = 0 \), the system effectively becomes a cascade type system comprising of levels \( |2\rangle, |3\rangle \) and \( |4\rangle \), therefore no coherence is established between levels \( |1\rangle \) and \( |2\rangle \).
This implies \( \alpha_m = 0 \) (cf. Eq. (8.12)) and hence \( \mu_r = 1 \). Thus left handedness of the medium is not possible in the absence of control field.

**Fig. 8.3**: Probe field dependence of (a): \( \text{Re}(\varepsilon_r) \), (b): \( \text{Im}(\varepsilon_r) \), (c): \( \text{Re}(\mu_r) \), (d): \( \text{Im}(\mu_r) \), (e): \( n_r \) and (f): \( A \). Here \( \Delta_c = \Delta_{cf} = 0 \), \( \alpha_{cf} = 5\gamma \), \( \gamma_{ij} = \gamma \) and \( \Gamma_{ij} = 0.001\gamma \). \( \alpha_c = \gamma \) (solid curve), \( 5\gamma \) (dashed curve) and \( 10\gamma \) (dotted curve). \( \omega_{n1} \) and \( \omega_{n2} \) are indicated for solid curve in frame (e).
We observe that control field strength is proportional to the maximum amplitude of NRI and frequency range over which both $\varepsilon_r$ and $\mu_r$ are negative. This is due to broadening of EIT resonance at $\Delta_p = 0$ with increase in $\alpha_c$. We denote the frequency of maximum NRI as $\omega_{n1}$ and $\omega_{n2}$ (cf. solid curve in Fig. 8.3(e)). Further from Fig. 8.3(e) it is clear that in the region $\Delta_p > \omega_{n1}$ and $\Delta_p < \omega_{n2}$ refractive index increases with decreasing probe frequency ($dn_r/d\omega_p = -dn_r/d\Delta_p$) showing anomalous behavior. Thus $v_g$ becomes negative in this region which indicates superluminal propagation. On the other hand for $\omega_{n2} < \Delta_p < \omega_{n1}$, group index $n_g >> 1$ indicating subluminal velocity. Thus in the NRI region one can control the propagation of probe beam with proper choice of laser atom interaction parameters. In the probe region $\Delta_p < \omega_{n2}$, the refractive index rises steeply compared to the region $\Delta_p > \omega_{n2}$ which means that the velocity of light can be made much faster than $c$. Further in the vicinity of $\omega_{n2}$ absorption coefficient is negative. This region is of interest for obtaining probe amplification in the NRI media.

Another interesting observation is that one can tune the transparency region, and hence $\omega_{n1}$, $\omega_{n2}$ and the range of NRI by controlling field detunings as shown in Fig. 8.4. This is an important advantage of coherent preparation method over artificial fabrication to realize NRI. Fig. 8.5 shows the dependence of rf field strength ($\alpha_{rf}$) on refractive index and absorption coefficient. For $\alpha_{rf} = 0$, the system can be considered as two independent two-level systems comprising of levels $|1\rangle-|4\rangle$ and $|2\rangle-|3\rangle$. In this condition also, $\rho_{h2} = 0$ which implies $\alpha_m = 0$ and hence $\mu_c = 1$. Thus it can be concluded that left handedness is possible in this scheme only if both electric and magnetic coupling
are present simultaneously. Further from Fig. 8.5(a) it is evident that as the \( rf \) field strength increases the maximum value of negative \( n_r \) attained increases. Fig. (8.3) – (8.5) thus which highlight the importance of choosing optimum values of field strengths and detunings to obtain desired NRI values and range.

**Fig. 8.4:** Refractive index versus probe detuning for \( \alpha_c = \alpha_{rf} = 10\gamma \). Solid and dashed lines correspond to \((\Delta_c, \Delta_{rf}) = (0, 20\gamma)\) and \((20\gamma, 0)\) respectively. Other data are same as in Fig. 8.3. \( \omega_{n1} = -6.2\gamma \) and \( 20.58\gamma \) for solid and dashed curves.

**Fig. 8.5:** Probe field dependence of (a): refractive index and (b): absorption coefficient. Here \( \Delta_c = \Delta_{rf} = 0, \quad \alpha_c = 10\gamma, \quad \alpha_{rf} = \gamma \) (solid curve) and \( 10\gamma \) (dashed curve). Other data are same as in Fig. 8.3.

The effect of control and \( rf \) field parameters on the range of NRI and maximum value of \( n_r \) achieved is summarized in Table-7. It is observed that a very large NRI value can be obtained when the \( rf \) field is detuned.
Table-7: Dependence of strengths and detunings of control and rf fields on the range and maximum amplitude of negative refractive index

<table>
<thead>
<tr>
<th>$\alpha_c$</th>
<th>$\alpha_{rf}$</th>
<th>$\Delta_c$</th>
<th>$\Delta_{rf}$</th>
<th>Range of negative $\epsilon_r$ and $\mu_r$</th>
<th>$n_r$ (max)</th>
</tr>
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<tbody>
<tr>
<td>$\gamma$</td>
<td>$5\gamma$</td>
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<td>$0$</td>
<td>$-6.25\gamma$ to $7.05\gamma$</td>
<td>$-3.28$</td>
</tr>
<tr>
<td>$5\gamma$</td>
<td>$5\gamma$</td>
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<td>$0$</td>
<td>$-9.94\gamma$ to $11.54\gamma$</td>
<td>$-3.26$</td>
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<tr>
<td>$10\gamma$</td>
<td>$5\gamma$</td>
<td>$0$</td>
<td>$0$</td>
<td>$-13.95\gamma$ to $15.73\gamma$</td>
<td>$-3.54$</td>
</tr>
<tr>
<td>$10\gamma$</td>
<td>$\gamma$</td>
<td>$0$</td>
<td>$0$</td>
<td>$-10.72\gamma$ to $11.29\gamma$</td>
<td>$-2.86$</td>
</tr>
<tr>
<td>$10\gamma$</td>
<td>$10\gamma$</td>
<td>$0$</td>
<td>$0$</td>
<td>$-17.62\gamma$ to $19.89\gamma$</td>
<td>$-3.74$</td>
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<tr>
<td>$10\gamma$</td>
<td>$10\gamma$</td>
<td>$20\gamma$</td>
<td>$0$</td>
<td>$-13.99\gamma$ to $13.76\gamma$, $20.34\gamma$ to $29.02\gamma$</td>
<td>$-9.57$</td>
</tr>
<tr>
<td>$10\gamma$</td>
<td>$10\gamma$</td>
<td>$0$</td>
<td>$20\gamma$</td>
<td>$-34.06\gamma$ to $-6.18\gamma$, $-0.35\gamma$ to $9.03\gamma$</td>
<td>$-39.12$</td>
</tr>
</tbody>
</table>

8.5 Realization of Negative Refractive Index in Model (b)

8.5.1 Theoretical Formulation

The interaction Hamiltonian of this system under RWA is given as

$$H_0 = -\alpha_{rf} (A_{12} + A_{21}) - \alpha_c (A_{41} + A_{43}) - \alpha_p (A_{34} + A_{43}) - \Delta_{rf} A_{22} - (\Delta_c - \Delta_p) A_{33} - \Delta_c A_{44}. \quad (8.16)$$

The elements of the density operator satisfy the following equations:

$$d\rho_{11}/dt = -2(\Gamma_{12} + \Gamma_{13})\rho_{11} - i\alpha_{rf}(\rho_{12} - \rho_{21}) - i\alpha_c(\rho_{14} - \rho_{41}) + 2\Gamma_{21}\rho_{22} + 2\Gamma_{31}\rho_{33} + 2\gamma_{41}\rho_{44}, \quad (8.17a)$$

$$d\rho_{12}/dt = -i\alpha_{rf}(\rho_{11} - \rho_{22}) - g_1\rho_{12} + i\alpha_c\rho_{42}, \quad (8.17b)$$

$$d\rho_{13}/dt = -g_2\rho_{13} - i\alpha_p\rho_{14} + i\alpha_{rf}\rho_{23} + i\alpha_c\rho_{43}, \quad (8.17c)$$

$$d\rho_{14}/dt = -i\alpha_c(\rho_{11} - \rho_{44}) - i\alpha_p\rho_{13} - g_3\rho_{14} + i\alpha_{rf}\rho_{24}, \quad (8.17d)$$
where the coefficients $g_i$ are defined as

$$g_1 = \Gamma_{12} + \Gamma_{13} + \Gamma_{21} + \Gamma_{23} + i\Delta_{s}, \quad g_2 = \Gamma_{12} + \Gamma_{13} + \Gamma_{31} + \Gamma_{32} + i(\Delta_p - \Delta_c),$$

$$g_3 = \Gamma_{12} + \Gamma_{13} + \gamma_{41} + \gamma_{43} + i\Delta_p, \quad g_4 = \Gamma_{21} + \Gamma_{23} + \Gamma_{31} + \Gamma_{32} + i(\Delta_p - \Delta_c - \Delta_{s}),$$

$$g_5 = \Gamma_{21} + \Gamma_{23} + \gamma_{41} + \gamma_{43} + i\Delta_{42}, \quad g_6 = \Gamma_{31} + \Gamma_{32} + \gamma_{41} + \gamma_{43} + i\Delta_{43}. \quad (8.18)$$

Under the weak probe approximation the solutions of Eq. (8.17) are obtained as

$$\rho_{43}^{(1)} = \frac{\alpha_s [g_4 \alpha_c \rho_{43}^{(0)} + i((g_2 g_4 + \alpha_{s}^2) (\rho_{33}^{(0)} - \rho_{44}^{(0)}) + \alpha_s \alpha_{s} \rho_{34}^{(0)})]}{g_2 g_4 + \alpha_{s}^2 g_4 + \alpha_{s}^3 g_6} = \rho_{34}^{(1)*}, \quad (8.19a)$$

$$\rho_{13}^{(1)} = \frac{\alpha_p [\alpha_c g_4 (\rho_{44}^{(0)} - \rho_{33}^{(0)}) + \alpha_{s} g_6 (\rho_{24}^{(0)} - ig_4 g_6 \rho_{14}^{(0)})]}{g_2 g_4 + \alpha_{s}^2 g_4 + \alpha_{s}^3 g_6} = \rho_{31}^{(1)*}, \quad (8.19b)$$

$$\rho_{14}^{(0)} = -i\alpha_s [(g_5 g_5 + \alpha_{s}^2) (\rho_{11}^{(0)} - \rho_{44}^{(0)}) - \alpha_{s}^2 (\rho_{11}^{(0)} - \rho_{22}^{(0)})] / G_1, \quad (8.19c)$$

$$\rho_{24}^{(0)} = \alpha_c \alpha_{s} [g_3 (\rho_{11}^{(0)} - \rho_{22}^{(0)}) + g_{1}^* (\rho_{11}^{(0)} - \rho_{44}^{(0)})] / G_2, \quad (8.19d)$$

$$\rho_{11}^{(0)} = \{\alpha_{s}^4 \alpha_{s}^4 j^2 (\Gamma_{31} + \Gamma_{32}) + \alpha_{s}^2 \alpha_{s}^2 j^2 (\Gamma_{31} + \Gamma_{32})(\gamma_{41} + \gamma_{43})\} / G_2, \quad (8.20a)$$

$$\rho_{22}^{(0)} = \{\alpha_{s}^2 \alpha_{s}^2 j^2 (\Gamma_{31} + \Gamma_{32}) + \alpha_{s}^2 \alpha_{s}^2 j^2 (\Gamma_{31} + \Gamma_{32})(\gamma_{41} + \gamma_{43})\} / G_2, \quad (8.20b)$$
The electric and magnetic polarizability of the medium are obtained as
\[ \alpha_e = \frac{|d_{43}|^2 \rho_{43}^{(1)}}{2e_0\hbar \alpha_p}, \quad \alpha_m = \frac{m_1 d_{43} \mu_e \rho_{43}^{(1)}}{2\hbar \alpha_p}. \] (8.22)

In this model the dipole synchronization restriction requires \( \omega_{43} = \omega_{31} \). The relevant dipole moments and the atomic density are chosen similar to that in model (a).

### 8.5.2 Results and Discussion

For \( \alpha_c = 0 \), this configuration can be considered as two independent two-level systems comprising of levels \( |1\rangle - |2\rangle \) and \( |3\rangle - |4\rangle \). Thus no coherence is established between levels \( |1\rangle \) and \( |3\rangle \) which means \( \alpha_m = 0 \) (cf. Eq. (8.22)). Hence negative refraction is not possible. However when \( \alpha_{rf} = 0 \) the response of the system is similar to
that of a three-level Λ system formed by levels $|1\rangle$, $|3\rangle$ and $|4\rangle$ where one can find probe regions exhibiting NRI [126]. Fig. 8.6 show the effect of control strength on $\varepsilon_r$, $\mu_r$, $n_r$ and $A$.

**Fig. 8.6:** Probe field dependence of (a): $\text{Re}(\varepsilon_r)$, (b): $\text{Im}(\varepsilon_r)$, (c): $\text{Re}(\mu_r)$, (d): $\text{Im}(\mu_r)$, (e): $n_r$ and (f): $A$. Here $\Delta_c = \Delta_{\sigma} = 0$, $\alpha_c = 5\gamma$, $\alpha_c = \gamma$ (solid curve), $5\gamma$ (dashed curve) and $10\gamma$ (dotted curve). $\omega_+$ and $\omega_-$ are indicated for the solid curve in frame (e).
There exist two maxima in the NRI region located at $\Delta_p = \omega_+$ and $\Delta_p = \omega_-$ (cf. Fig. 8.6(e)). For $\Delta_p > \omega_+$ and $\Delta_p < \omega_-$, refractive index $n_r$ increases sharply with increase in $\Delta_p$, thereby indicating anomalous behavior. Comparing Fig. 8.6(e) – (f) we can identify the frequency regions where $n_r < 0$ and $A \sim 0$, which is of significant interest experimentally. Interestingly in the region $\Delta_p \sim \omega_-$ we observe $A < 0$ and that corresponds to the amplification of the probe beam. Fig. 8.7 shows the effect of control and rf detunings on the refractive index of the media. Similar to the previous model, here also one can tune the range and amplitude of NRI by controlling the field detunings.

![Fig. 8.7: Refractive index versus probe detuning for $\alpha_c = \alpha_{rf} = 10\gamma$. The solid and dashed curves correspond to $(\Delta_c, \Delta_{rf}) = (0, 20\gamma)$ and $(20\gamma, 0)$ respectively.](image)

Fig. 8.8 show the effect of rf strengths and detunings on $n_r$ and $A$. From Figs. 8.6(e) and 8.8(e) it is clear that the range and maximum amplitude of NRI increases with increase in strengths of both the fields. The effect of laser atom interaction parameters on NRI is summarized in Table-8.

Finally a comment on the group velocity ($v_g$) in this coherently driven system is appropriate at this stage. Here $v_g > 0$ in the region $\omega_- < \Delta_p < \omega_+$, while $v_g < 0$ in the anomalous dispersion region. This provides a prospect for observing both subluminal and...
superluminal light propagation in the media similar to the previous model.

**Fig. 8.8:** Dependence of probe detuning on (a): refractive index and (b): absorption coefficient. Here $\Delta_c = \Delta_{rf} = 0$, $\alpha_c = 5\gamma$, $\alpha_{rf} = \gamma$ (solid curve), $6\gamma$ (dashed curve) and $10\gamma$ (dotted curve).

**Table-8:** Dependence of strengths and detunings of control and $\text{rf}$ field on the maximum value and range of negative refractive index

<table>
<thead>
<tr>
<th>$\alpha_c$</th>
<th>$\alpha_{rf}$</th>
<th>$\Delta_c$</th>
<th>$\Delta_{rf}$</th>
<th>Range of negative $\varepsilon_r$ and $\mu_r$</th>
<th>$n_r$ (max)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>$5\gamma$</td>
<td>0</td>
<td>0</td>
<td>$-6.53\gamma$ to $6.53\gamma$</td>
<td>$-3.34$</td>
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<tr>
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<td>0</td>
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<td>$5\gamma$</td>
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<td>0</td>
<td>$-17.88\gamma$ to $17.88\gamma$</td>
<td>$-4.73$</td>
</tr>
<tr>
<td>$5\gamma$</td>
<td>$\gamma$</td>
<td>0</td>
<td>0</td>
<td>$-11.03\gamma$ to $11.03\gamma$</td>
<td>$-4.07$</td>
</tr>
<tr>
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<td>$6\gamma$</td>
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<td>0</td>
<td>$-12.55\gamma$ to $12.55\gamma$</td>
<td>$-4.36$</td>
</tr>
<tr>
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<td>$10\gamma$</td>
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<td>0</td>
<td>$-14.85\gamma$ to $14.85\gamma$</td>
<td>$-4.73$</td>
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<tr>
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