Chapter 5

Role of shape dependent dissipation

5.1 Introduction

Experimental and theoretical studies of heavy-ion-induced fusion-fission reactions at beam energies above Coulomb barriers have made significant contributions to the understanding of nuclear collective dynamics at high excitation energies in recent years. In particular, careful analysis of experimental data have established that the fission dynamics of a hot compound nucleus (CN) is dissipative in nature. Consequently, fission has become a useful probe to study the dissipative properties of the nuclear bulk. The detailed discussion on this topic is given in Chapter 1.

As described earlier in many occasions, Kramers’ fission width can be obtained as

$$\Gamma = \frac{\hbar \omega_g}{2\pi} e^{-V_B/T} \left\{ \sqrt{1 + \left( \frac{\beta}{2\omega_s} \right)^2} - \frac{\beta}{2\omega_s} \right\},$$

(5.1)

considering fission as diffusion of a Brownian particle across the fission barrier \( V_B \) placed in a hot and viscous fluid bath of temperature \( T \) and reduced dissipation coefficient \( \beta \). The frequencies of the harmonic oscillator potentials describing the nuclear potential at the ground-state and the saddle configurations are \( \omega_g \) and \( \omega_s \), respectively. Equation (5.1) was obtained [27] assuming the reduced dissipation coefficient \( \beta \) to be shape independent and constant for all deformations of the nucleus. Subsequently, the aforementioned stationary fission width predicted by Kramers was found to be in reasonable agreement with the asymptotic fission width.
obtained from numerical solutions of the Fokker-Planck [45, 156, 157, 162, 163, 164, 165] and 
Langevin [105, 166, 167, 168, 169] equations where shape-independent and constant values of 
dissipation were used. Kramers’ fission width is extensively used in statistical model calculation 
for decay of CN. The coefficient $\beta$ is often treated as a free parameter to fit experimental 
data. Efforts are also continuing to improve the modeling of the fission process to extract more 
reliable values of the dissipation coefficient [27, 105, 106].

It was first reported by Fröbrich et al. [160] that the experimental data on pre-scission neu-
tron multiplicity and fission cross section cannot be fitted by the same strength of the reduced 
dissipation in Langevin dynamical calculations. While a smaller value of $\beta$ can account for 
fission excitation function, a larger value of $\beta$ is required to describe the pre-scission multiplicity 
data. A shape-dependent nuclear dissipation was found necessary to simultaneously fit the 
pre-scission neutron multiplicity and fission cross-section data [78, 160]. From considerations of 
chaos in single-particle motion within the nuclear volume, shape dependence of a similar nature 
is also predicted for one-body dissipation, considered to be mainly responsible for damping of 
nuclear motion [98, 147]. A smaller dissipation strength in the presaddle region and a larger 
dissipation strength in the postsaddle region is found necessary in subsequent applications of 
Langevin equations for dynamics of fission [175, 176].

Shape-dependent dissipation is also introduced in statistical model calculation for the decay 
of a CN, in the following manner [59, 60, 108, 177]. One considers two dissipation strengths 
here: a smaller one ($\beta_{in}$) operating within the saddlepoint region and a larger one ($\beta_{out}$) effective outside the saddle point. In a statistical model calculation of nuclear fission, it is assumed 
that the fission width is given by $\Gamma_{K}^{in}$ [$\Gamma_{K}$ in Eq. (5.1) with $\beta_{in}$]. For a fission event, $\beta_{out}$ 
is subsequently used to calculate the saddle-to-scission transition time during which further 
neutron evaporation can take place. However, the assumption that the fission width is given 
by $\Gamma_{K}^{in}$ requires close scrutiny, as we are considering a shape-dependent dissipation here, while 
Kramers’ width was originally obtained assuming a shape-independent dissipation.

In the present chapter, we examine [110] the validity of determining the fission width from 
$\beta_{in}$ alone when a shape-dependent dissipation is considered. To this end, we compare $\Gamma_{K}^{in}$
with stationary widths from Langevin dynamical model calculations, considering the latter to represent the true fission width. We also compare the prescission neutron multiplicities ($n_{\text{pre}}$) and evaporation residue (ER) cross sections obtained from the statistical model with a shape-dependent dissipation with those obtained from the corresponding Langevin equations. In the next section, comparison between the Kramers’ fission width and the corresponding Langevin dynamical width is described for the shape-dependent dissipation. For the present purpose, dynamical model calculations are done by including particle and $\gamma$ evaporation channels. A brief account of this calculation procedure is given in Sec. 5.3. The results of the dynamical model calculations are compared with the corresponding statistical model results in Sec. 5.4. Finally, we summarize the results in Sec. 5.5.

![Figure 5.1: Collective potential and shape-dependent reduced dissipation ($\beta$) for $^{224}\text{Th}$. Different forms of $\beta$ corresponding to different values of $c_\beta$ are shown [110].](image)

### 5.2 Comparison between Kramers’ and Langevin dynamical fission widths

We choose the CN $^{224}\text{Th}$ for the present calculation and solve the one-dimensional Langevin equations [110]. Figure 5.1 shows the collective potential for $^{224}\text{Th}$ along with the shape-dependent dissipation coefficients used in the Langevin calculations. Denoting the elongation
at which the dissipation changes its strength from $\beta_{in}$ to $\beta_{out}$ by $c_\beta$, the Langevin equations are solved for different values of $c_\beta$. Figure 5.2 shows the time-dependent fission widths from the Langevin equations for different values of $c_\beta$. The values of Kramers’ fission widths $\Gamma_{in}^K$ and $\Gamma_{out}^K$ obtained with $\beta_{in}$ and $\beta_{out}$, respectively, in Eq. (3.1) are also shown in this figure. The values of stationary fission widths $\Gamma_L$ from Langevin dynamics are subsequently plotted as a function of $c_\beta$ in Fig. 5.3. It is immediately noted from Fig. 5.3 that for $c_\beta = 1.6$, which corresponds to the elongation at saddle, the stationary fission width ($\Gamma_L$) from Langevin equations is substantially smaller than the $\Gamma_{in}^K$ obtained with a constant value of $\beta_{in}$. This observation is contrary to the interpretation made in statistical model calculations employing shape-dependent dissipations, that $\Gamma_{in}^K$ accounts for the fission rate. We further note in Fig. 5.3 that as $c_\beta$ is shifted outward beyond the saddle point, $\Gamma_L$ approaches $\Gamma_{in}^K$. When $c_\beta$ is moved inward, $\Gamma_L$ approaches $\Gamma_{out}^K$.

The preceding observations are made when we choose $\beta_{out} \gg \beta_{in}$ in accordance with the applications of shape-dependent dissipation in statistical model calculations [59, 60, 108, 177]. However, when the value of $\beta_{out}$ is reduced toward $\beta_{in}$, Fig. 5.4 shows that the stationary fission width from Langevin dynamics gets closer to Kramers’ width for $\beta_{in}$, as expected.
To understand the foregoing observations qualitatively, we proceed as follows. Kramers' width ($\Gamma_K$) [Eq. (3.1)] represents the steady-state diffusion rate of phase-space density ($\rho$) of Brownian particles across the fission barrier satisfying the appropriate Liouville equation, and the net flux or current across the saddle is [Eq. (4.15)]

$$j = \int_{-\infty}^{+\infty} \rho(c = c_s, p) \frac{p}{m_s} dp,$$

where both the outward (positive-$p$) and the inward (negative-$p$) fluxes are considered to obtain the net flux [63]. In terms of Langevin fission trajectories, while the outward flux is controlled by the dissipation within the saddle, the inward flux (from outside to inside the saddle) or the back-streaming trajectories experience the dissipation outside the saddle. Hence the net flux ($j$) depends on both the “pre-saddle” and the “post-saddle” dissipation strengths, and the fission width is no longer determined by the pre-saddle dynamics alone. The stochastic nature of nuclear fission makes it dependent on the fission dynamics around the saddle, the extent of which is illustrated in Fig. 5.3.
The one-dimensional Langevin dynamical calculation for the fission width is demonstrated in Chapter 2. For the present purpose, we need to include the evaporations of neutrons, protons, \( \alpha \) particles, and statistical \( \gamma \)-rays along with the fission channel. Earlier, the statistical model calculation including the evaporation channels is described in Sec. 3.3, where the fission width can be calculated either from the Bohr-Wheeler formula [Eq. (1.4)] or the Kramers’ formula [Eq. 5.1]. A Monte-Carlo sampling is then performed at each time step to decide over the all possible decay modes. On the other hand, in a dynamical trajectory calculation, the shape evolution of an CN nuclear is followed with time and the evaporation channels are sampled at the end of each time step of this dynamical evolution. More specifically, two Monte-Carlo samplings have been performed, first, to decide whether any evaporation is happening or not and if it occurs then a second one to select a particular decay mode out of neutrons, protons, \( \alpha \) particles, and \( \gamma \) evaporation channels. During the dynamical evolution, if fission does not
Figure 5.5: Pre-scission neutron multiplicities from statistical (dashed line) and dynamical (solid line) model calculations with a shape-independent dissipation $\beta = 3.5 \text{ MeV}/\hbar$. Experimental points are from Ref. [54].

occur within a time which is sufficiently long so that the fission width reaches a stationary value, then the decay process is shifted to a statistical model calculation where the fission width is now either calculated from the Kramers’ formula or interpolated from pre-calculated Langevin dynamical widths. This type of calculation is known as combined dynamical and statistical model (CDSM). The dynamical trajectory calculations including particle and $\gamma$ evaporations are very much time consuming. Therefore, in CDSM the decay algorithm is followed with the statistical model code as soon as the fission width reaches the stationary value. Usually, the Kramers’ fission width is used in the statistical model part [78, 99]. However, the applicability of Kramers’ width is the main issue in the present calculation and hence we use the interpolated values of Langevin fission width computed initially for different combinations of compound nuclear spin and temperature [110]. An elaborate discussion on the CDSM code is given in Appendix B.
5.4 Comparison between statistical and dynamical model results

We now compare the pre-scission neutron multiplicities ($n_{pre}$) and ER cross sections obtained from statistical model calculation of compound nuclear decay with those from Langevin dynamical model calculation. Evaporation of neutrons, protons, $\alpha$ particles, and statistical $\gamma$-rays are considered along with the fission channel in both the calculations. While the particle and the $\gamma$ emission widths used in both approaches are obtained from the Weisskopf formula [78], the fission width for the statistical model calculation is taken as $\Gamma_K^{in}$. We first consider the results obtained with a shape-independent strength of dissipation. Figure 5.5 shows the statistical and dynamical model predictions of $n_{pre}$ excitation function calculated for the system $^{16}$O+$^{208}$Pb along with the experimental data [54]. The dissipation strength is obtained here by fitting the data. A close agreement between the results from the two calculations is observed here, which reflects the validity of Kramers’ width for shape-independent dissipation as demonstrated in

![Graph showing comparison between statistical and dynamical model results](image-url)

Figure 5.6: Evaporation residue cross sections from statistical (dashed line) and dynamical (solid line) model calculations for a shape-dependent dissipation with $\beta_{in} = 1.5$ MeV/$\hbar$ and $\beta_{out} = 15$ MeV/$\hbar$. Experimental points are from Ref. [24].
Figure 5.7: Pre-scission neutron multiplicities from statistical and dynamical model calculations for a shape-dependent dissipation with $\beta_{in} = 1.5 \text{ MeV/\hbar}$ and $\beta_{out} = 15 \text{ MeV/\hbar}$. Dash-dotted and dashed lines represent statistical model calculation results with and without the saddle-to-scission neutrons, respectively. Langevin dynamical results are shown by the solid line. Experimental points are from Ref. [54].

Fig. 5.4.

We next perform statistical and Langevin dynamical model calculations where a shape-dependent dissipation is used. In the statistical model calculation, $\Gamma_K^{in}$ is used as the fission width, while $\beta_{out}$ is used to calculate the saddle-to-scission transition time given by Eq. (3.7). Additional neutrons are allowed to evaporate during this period [60]. The pre-saddle dissipation strength $\beta_{in}$ and hence $\Gamma_K^{in}$ are first obtained by fitting the experimental ER excitation function. The strength of $\beta_{out}$ is subsequently adjusted to reproduce the experimental $n_{pre}$ excitation function. Excitation functions for $n_{pre}$ and ER are also obtained from the Langevin dynamical calculation using a shape-dependent dissipation given by the preceding values of $\beta_{in}$ and $\beta_{out}$. Figure 5.6 shows the calculated ER cross sections along with the experimental data. The dynamical model results are considerably larger than the statistical model predictions. This shows that the post-saddle dynamics controlled by $\beta_{out}$ plays an important role in determining
the fission probability of a CN, which in turn demonstrates the inadequacy of using only the $\beta_{in}$ value in Eq. (3.1) to obtain the fission width. Figure 5.7 shows the calculated $n_{pre}$ multiplicities and the experimental values. The statistical model results without including the additional saddle-to-scission neutrons are also given in this figure. The dynamical model predictions, however, turn out to be much higher than the statistical model results. Because the Langevin equations give the true description of dynamics of fission, the preceding differences between statistical and dynamical model results show that for shape-dependent dissipation, the assumptions of $\beta_{in}$ accounting for the fission width and $\beta_{out}$ controlling the saddle-to-scission neutrons are not consistent with the dynamical model results. Consequently, the fitted values of $\beta_{in}$ and $\beta_{out}$ from statistical model calculations when used in dynamical model calculations give rise to substantially different values of $n_{pre}$ and ER cross sections.

5.5 Summary

We therefore conclude that due caution should be exercised when using Kramers’ expression for fission width for systems with shape-dependent dissipation. In such cases, the Kramers’ width obtained with a pre-saddle dissipation strength does not represent the true fission width, and consequently, the “pre-saddle dissipation strength” fitted to reproduce the experimental data in statistical model calculations does not represent the true strength of pre-saddle dissipation.