4.1 Introduction

This chapter presents a description of the methods and procedures adopted to meet the objectives stated in Chapter 3. In order to isolate the different aspects of the research work, without losing continuity, this chapter is divided into various sections and subsections as outlined below.

This chapter is primarily divided into three sections. In the first section, the conceptual methods and computational methods used for modelling infiltration by using a physically based Green and Ampt model, and also by using a model based on the numerical solution of the Richards equation have been discussed. A description of the laboratory and field experiments conducted for estimation of the model parameters as well as for the preparation of the data base is given in the second section. This section also describes the field experiments conducted under various conditions for the verification of the developed models. In the last section, a comparative study on the applicability of the model predictions with regard to published data on layered soils, and a sensitivity analysis of the model parameters are presented.

4.2 Development of Physically Based Model

As discussed in Chapter 2, Green and Ampt (1911) proposed an approximate model utilizing Darcy’s law to estimate infiltration. The original equation was derived for infiltration into a deep homogeneous soil for ponded surface conditions and water entering as piston flow, resulting in sharply defined wetting front separating a zone that has been wetted from a totally un-wetted zone.
Assuming negligible at the surface, and expressing the difference between initial and final volumetric water contents in terms of cumulative infiltration, application of Darcy’s law yields the following form of the Green and Ampt equation:

\[ f = K_s (1 + S_{av} \frac{M}{F}) \]  \hspace{1cm} \text{(4.1)}

where

- \( f \) = infiltration rate, \([\text{m/s}]\)
- \( K_s \) = hydraulic conductivity of the saturated zone, \([\text{m/s}]\)
- \( M \) = initial moisture deficit, \([\text{m}^3/\text{m}^3]\)
- \( S_{av} \) = effective suction at the wetting front, \([\text{m}]\)
- \( F \) = cumulative infiltration, \([\text{m}]\)

Substituting \( f = \frac{dF}{dt} \) in the above equation, and integrating over a time period \( t \), with the condition that \( F=0 \) at \( t=0 \), and \( F=F \) at \( t=t \), we get,

\[ K_s t = F - S_{av} M \ln(1 + \frac{F}{MS_{av}}) \]  \hspace{1cm} \text{(4.2)}

where

- \( t \) = time, \([\text{s}]\)

The above equation relates cumulative infiltration to time from the start of infiltration, and can therefore be more conveniently used than equation 4.1.

Skaggs et al. (1969) found that infiltration capacity and cumulative infiltration were approximately independent of rainfall intensity. This assumption was used by Mein and Larson (1973) to extend the Green and Ampt model to a general case of delaying surface ponding with steady rainfall intensity into equation 4.2. They solved for cumulative infiltration at time to ponding as:
\[ F_p = MS_{av} \left( \frac{i}{K_s} - 1 \right) \]  \hspace{1cm} (4.3)

where

\[ F_p = \text{cumulative infiltration at time to ponding, [m]} \]
\[ i = \text{rainfall intensity, [m/s]} \]

It was shown by Chu (1978) that equation 4.3 could also be used for unsteady rainfall conditions by partitioning the whole unsteady rainfall event into small time intervals with constant rainfall intensity. Hence, time to ponding could be estimated by,

\[ t_p = \frac{(F_p - F_{n-1})}{i} + t_{n-1} \]  \hspace{1cm} (4.4)

where

\[ F_{n-1} = \text{cumulative infiltration at n-1 time level, [m]} \]
\[ t_{n-1} = \text{time at n-1 level, [s]} \]

Mein and Larson (1973) and Chu (1978) modified equation 4.4 for steady rainfall conditions as:

\[ K_s(t - t_p + t_s) = F - MS_{av} \ln(1 + \frac{F}{MS_{av}}) \]  \hspace{1cm} (4.5)

where

\[ t_s = \text{pseudo time, the time shift due to effect of cumulative infiltration at the time of ponding, [s]} \]

Using the above concept, Tan (1987) developed a three-layer rainfall infiltration model, covered by a 5mm thick crust. Based on his concepts, the Green and Ampt equations for a two-layer soil profile can be written as:

for the first layer,

\[ K_{s1}(t - t_p + t_s) = F - A_1 \ln(1 + \frac{F}{A_1}) \]  \hspace{1cm} (4.6)
and for the second layer,

\[ K_{s,2}(t - t_1 + t_s - t_1) = F - F_1 + (B_2 - A_2) \ln(F + A_2)/(F_1 + A_2) \] (4.7)

where

\[ t_1 \] = time when wetting front reaches bottom of the first layer, [s]

\[ F_1 \] = cumulative infiltration at time \( t_1 \), [m]

\[ A_1 = M_1(S_{av,1} + D_p), \]

\[ A_2 = M_2(S_{av,2} + D_p) + L_1(M_2 - M_1), \]

\[ B_2 = L_1(K_{s,2}/K_{s,1}) M_2 - M_1, \]

\[ F = \text{cumulative infiltration, [m]} \]

\[ L_1 = \text{thickness of the first layer, [m]} \]

\[ L_2 = \text{thickness of the second layer, [m]} \]

\[ K_{s,1} = \text{saturated hydraulic conductivity of the first layer, [m/s]} \]

\[ K_{s,2} = \text{saturated hydraulic conductivity of the second layer, [m/s]} \]

\[ S_{av,1} = \text{effective suction at the wetting front of the first layer, [m]} \]

\[ S_{av,2} = \text{effective suction at the wetting front of the second layer, [m]} \]

\[ M_1 = \theta_{s,1} - \theta_{i,1} = \text{initial water deficit of the first layer, [m}^3/m^3] \]

\[ M_2 = \theta_{s,2} - \theta_{i,2} = \text{initial water deficit of the second layer, [m}^3/m^3] \]

\[ \theta_{s,1} = \text{saturated volumetric water content of the first layer, [m}^3/m^3] \]

\[ \theta_{s,2} = \text{saturated volumetric water content of the second layer, [m}^3/m^3] \]

\[ \theta_{i,1} = \text{initial volumetric water content of the first layer, [m}^3/m^3] \]

\[ \theta_{i,2} = \text{initial volumetric water content of the second layer, [m}^3/m^3] \]

With equations 4.6 and 4.7 as the governing equations, a computer programme \textit{GAMPT} is written in FORTRAN language to simulate infiltration characteristics of a
two-layer soil profile. The model is superimposed by a surface crust, whose parameters should be determined from the computer programme *CRUST* which uses observed infiltration data. Development of the programme *CRUST* is discussed in section 4.1.2. Flowchart of the programme *GAMPT* is shown in Figure 4.1 and listing of the programme may be found in Appendix III.
Fig. 4.1: Flowchart for GAMPT programme
Table 4.1: Input parameters to programme GAMPT

<table>
<thead>
<tr>
<th>Line</th>
<th>Variable name</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TITLE</td>
<td>[-]</td>
<td>title of the case</td>
</tr>
<tr>
<td>2</td>
<td>RAINOPT</td>
<td>[-]</td>
<td>option for rainfall</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1 for rainfall infiltration; 0 for without rainfall)</td>
</tr>
<tr>
<td></td>
<td>CRUSTOPT</td>
<td>[-]</td>
<td>option for surface crusting</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1 for crust formation; 0 for without crust)</td>
</tr>
<tr>
<td></td>
<td>R LAYEROP</td>
<td>[-]</td>
<td>option for layered soils</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1 for layered soil profile; 0 for homogeneous soil profile)</td>
</tr>
<tr>
<td>3</td>
<td>RAINRATE</td>
<td>[cm/hr]</td>
<td>rainfall rate (omit this line for no rainfall case)</td>
</tr>
<tr>
<td>4</td>
<td>RKO</td>
<td>[cm/hr]</td>
<td>initial crust conductivity</td>
</tr>
<tr>
<td></td>
<td>RKF</td>
<td>[cm/hr]</td>
<td>final crust conductivity</td>
</tr>
<tr>
<td></td>
<td>T37</td>
<td>[hr]</td>
<td>decay constant</td>
</tr>
<tr>
<td>5</td>
<td>RIWC1</td>
<td>[-]</td>
<td>initial water content of the first layer</td>
</tr>
<tr>
<td></td>
<td>RSWC1</td>
<td>[-]</td>
<td>saturation water content of the first layer</td>
</tr>
<tr>
<td></td>
<td>RKS1</td>
<td>[cm/hr]</td>
<td>hydraulic conductivity of the first layer</td>
</tr>
<tr>
<td></td>
<td>SAV1</td>
<td>[cm]</td>
<td>average suction at the wetting front for the first layer</td>
</tr>
<tr>
<td>6</td>
<td>RIWC2</td>
<td>[-]</td>
<td>initial water content of the second layer</td>
</tr>
<tr>
<td></td>
<td>RSWC2</td>
<td>[-]</td>
<td>saturation water content of the second layer</td>
</tr>
<tr>
<td></td>
<td>RKS2</td>
<td>[cm/hr]</td>
<td>hydraulic conductivity of the second layer</td>
</tr>
<tr>
<td></td>
<td>SAV2</td>
<td>[cm]</td>
<td>average suction at the wetting front for the second layer</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(omit the above line for homogeneous soil)</td>
</tr>
<tr>
<td>7</td>
<td>DP</td>
<td>[cm]</td>
<td>depth of ponding on the surface</td>
</tr>
<tr>
<td></td>
<td>RL1</td>
<td>[cm]</td>
<td>length of the first layer or the entire layer for homogeneous soil</td>
</tr>
<tr>
<td>8</td>
<td>DELF</td>
<td>[cm]</td>
<td>incremental cumulative infiltration depth between two successive time levels</td>
</tr>
<tr>
<td></td>
<td>FLIMIT</td>
<td>[cm]</td>
<td>maximum value of cumulative infiltration depth up to which infiltration rates are to be calculated</td>
</tr>
</tbody>
</table>
4.3 Evaluation of Surface Crusting

As discussed in Chapter 2, hydraulic properties at the soil surface change dramatically during the application of water. Such changes, which depend on the surface conditions, can have a very prominent effect on the rate of infiltration. Indeed in some of the studies of infiltration, the exponential decay of infiltration rate with time has been believed to be due to slaking of aggregates and swelling of colloids which progressively seal the soil surface. The formation of surface crust, and its consequent effect on time to ponding, infiltration rate and cumulative infiltration have already been discussed in Chapter 2.

Because of the complex nature of the crusting process and difficulties encountered in describing the manner in which hydraulic properties of the surface layer change with time, there have been relatively few attempts to characterize the soil crusting phenomena. Chu et al. (1986, 1987) used field data to evaluate a layered Green and Ampt infiltration model with a transient crust model suggested by Brakensiek and Rawls (1983), and they found that the transient crust conductivity had a dominant effect on the infiltration process such that all the other factors became secondary. In the present study, the approach of Chu et al. (1987) has been used for evaluation of soil crusting parameters. These parameters, including initial crust conductivity, final crust conductivity and decay constant have been determined from field experimental data.

The exponential decay of crust was described as:

\[ K = K_f + (K_0 - K_f) \exp(-t/T) \]  \hspace{1cm} (4.8)

where

\[ K_0 = \text{initial crust conductivity or the initial conductivity of the soil, [mm/hr]} \]
\[ K_f = \text{minimum or final crust conductivity, [mm/hr]} \]
\[\begin{align*}
Z_c &= \text{crust thickness, [mm]} \\
&= 0.5 \text{ mm assumed (Moore, 1981; Chu et al., 1987)} \\
S_c &= \text{correction factor for unsaturated flow below the crust, assumed as 0.81} \\
&\quad \text{(Brakensiek and Rawls, 1983)} \\
P_a &= \text{matric potential beneath the crust, [mm]} \\
t &= \text{time, [hr]} \\
T_{37} &= \text{a time constant (the time when } K \text{ reaches a value equal to } K_f, \\
&\quad \text{where } 1/e=0.37; 1/T_{37} \text{ is the rate at which crust conductivity drops)} \\
\end{align*}\]

The infiltration depth may be expressed in terms of time to ponding as:

\[F = T_p \cdot i + K_f \cdot (t - T_p) + (K_0 - K_f) \cdot T_{37} \cdot (\exp(-T_p / T_{37}) - \exp(-t / T_{37}))\]

\[\text{(4.10)}\]

where

\[T_p = \text{time to ponding, [hr]}\]

\[= T_{37} \cdot \ln((K_0 - K_f) / (i - K_f))\]

\[F = \text{infiltration depth, [mm]}\]

As the infiltration depth is more stable than the infiltration rate, it is more advantageous to use equation 4.10 than equation 4.9. To solve for the three parameters, \(T_{37}, K_0\) and \(K_f\), three data points were needed. Choosing the three data points to be \((T_1,F_1), (T_2,F_2)\) and \((T_3,F_3)\), and substituting into equation 4.10 leads to:

\[F_1 = T_p \cdot i + K_f \cdot T_1 - T_p + (K_0 - K_f) \cdot T_{37} \cdot (\exp(-T_p / T_{37}) - \exp(-T_1 / T_{37}))\]

\[\text{(4.11)}\]

\[F_2 = T_p \cdot i + K_f \cdot T_2 - T_p + (K_0 - K_f) \cdot T_{37} \cdot (\exp(-T_p / T_{37}) - \exp(-T_2 / T_{37}))\]
\[ F_3 = T_p^* i + K_f (T_3 - T_p^*) + (K_0 - K_f) * T_{37}^* (\exp(-T_p^*/T_{37}) - \exp(-T_3/T_{37})) \]

(4.12)

Eliminating \( T_p^* i \) from the above equations,

\[ F_2 - F_1 = K_f^* (T_2 - T_1) + (K_0 - K_f) * (\exp(-T_1/T_{37}) - \exp(-T_2/T_{37}) \cdot T_{37}^* \]

(4.13)

\[ F_3 - F_2 = K_f^* (T_3 - T_2) + (K_0 - K_f) * (\exp(-T_2/T_{37}) - \exp(-T_3/T_{37}) \cdot T_{37}^* \]

(4.14)

Solving the above two equations for parameters \( K_f \) and \( K_0 \) gives,

\[ K_f = \frac{(C_1 \cdot B_2 - C_2 \cdot B_1)}{D} \]

\[ K_0 = \frac{(A_1 \cdot C_2 - A_2 \cdot C_1)}{D} \]

where

\[ A_1 = T_2 - T_1 - T_{37}^* (\exp(-T_1/T_{37}) - \exp(-T_2/T_{37})) \]

\[ A_2 = T_3 - T_2 - T_{37}^* (\exp(-T_2/T_{37}) - \exp(-T_3/T_{37})) \]

\[ B_1 = T_{37}^* (\exp(-T_1/T_{37}) - \exp(-T_2/T_{37})) \]

\[ B_2 = T_{37}^* (\exp(-T_2/T_{37}) - \exp(-T_3/T_{37})) \]

\[ C_1 = F_2 - F_1 \]

\[ C_2 = F_3 - F_2 \]

\[ D = A_1 \cdot B_2 - B_1 \cdot A_2 \]

Equations 4.16 and 4.17 were used to eliminate \( K_f \) and \( K_0 \) from equation 4.13, thus obtaining an equation with just one unknown parameter \( T_{37} \). This equation was solved by the Newton-Raphson method to obtain the value of \( T_{37} \). Once the value of \( T_{37} \) was obtained, the other parameters \( K_f \) and \( K_0 \) were computed from equations 4.16
and 4.17. A computer programme *CRUST* was developed to compute the above parameters by using Newton-Raphson iterative method. The inputs to the programme are given in Table 4.2, the listing of the programme can be found in Appendix II. Experimental data were used as an input to the *CRUST* programme to determine soil crusting parameters. For the parameter evaluation process, the last two points were considered to be constant. The first data point from the experimental data was allowed to represent any point in the data sequence from a point corresponding to the time to ponding to any point prior to the second data point. The objectives of moving the first data point was to investigate the variation of parametric value due to variation in experimental data points. Average values of the crusting parameters were considered to describe the crust behaviour.

**Table 4.2: Input parameters to programme *CRUST***

<table>
<thead>
<tr>
<th>Line</th>
<th>Variable name</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TITLE</td>
<td>[-]</td>
<td>title of input data, experimental setting etc</td>
</tr>
<tr>
<td>2</td>
<td>NNP</td>
<td>[-]</td>
<td>number of first data points used</td>
</tr>
<tr>
<td>2</td>
<td>NIT</td>
<td>[-]</td>
<td>maximum number of iterations to be carried out</td>
</tr>
<tr>
<td>3</td>
<td>T2</td>
<td>[min]</td>
<td>time of the second data point</td>
</tr>
<tr>
<td>3</td>
<td>F2</td>
<td>[cm]</td>
<td>cumulative infiltration corresponding to T2</td>
</tr>
<tr>
<td>4</td>
<td>T3</td>
<td>[min]</td>
<td>time of the third data point</td>
</tr>
<tr>
<td>4</td>
<td>F3</td>
<td>[cm]</td>
<td>cumulative infiltration corresponding to T3</td>
</tr>
<tr>
<td>5</td>
<td>RI</td>
<td>[cm/hr]</td>
<td>rainfall intensity</td>
</tr>
<tr>
<td>5</td>
<td>DELX</td>
<td>[min]</td>
<td>accuracy of determination of T37 in the iterative procedure</td>
</tr>
<tr>
<td>6</td>
<td>T1</td>
<td>[min]</td>
<td>time of the first data point</td>
</tr>
<tr>
<td>6</td>
<td>F1</td>
<td>[cm]</td>
<td>cumulative infiltration corresponding to T1</td>
</tr>
<tr>
<td>6</td>
<td>X</td>
<td>[min]</td>
<td>initial estimate of T37</td>
</tr>
</tbody>
</table>
4.4 Simulating Infiltration by a Numerical Model

The general laws of fluid motion govern the flow of water through porous media like soil. Mathematically, these laws are stated to be in the form of equation of continuity, the equation of state and the dynamic equation of motion. Combining these three equations for liquid phase, the flow in the vertical direction results in a non-linear partial differential equation in which time, \( t \), and position in space, \( z \), are independent variables. The dependent variable is the volumetric water content, \( \theta \) or soil matric potential, \( \psi \). The functional coefficient in the flow model are \( \psi \) versus \( \theta \), and hydraulic conductivity \( K \) versus \( \theta \) relationships.

A solution exists for the flow model at \( t>0 \), provided \( \psi \) versus \( \theta \) and \( K \) versus \( \theta \) relationships are established for the particular solution. Because \( \psi \) versus \( \theta \) and \( K \) versus \( \theta \) relationships appear as functional coefficients in the flow model, the accuracy of the model solution with experimental data is largely determined by the parametric estimates that describe the functional relationships. The empirical equations that describe the \( \psi \) versus \( \theta \), and \( K \) versus \( \theta \) relationships are non-linear. Because of the strong non-linearity in its terms, the flow model is difficult, if not impossible, to solve analytically.

4.5 Theory and Governing Equations of Soil Water Flow

As stated earlier, one-dimensional form of the Richards equation was used for modeling infiltration in the vertical direction as:

\[
C(\psi) \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z} [K(\psi)(\frac{\partial \psi}{\partial z} - 1)]
\]  \hspace{1cm} (4.16)

A source term \( q \) was added to take care of the rainfall or crust conditions. The appropriate boundary conditions in the surface are:

\[
q(t) = \min (r(t), K(t))
\]
where

\[ K(t) = \text{transient crust conductivity calculated by an iterative procedure described in section 4.} \]

\[ r(t) = \text{rainfall rate} \]

\[ q(t) \text{ assumes a value equal to zero for all nodes not lying on the surface. In cases} \]

where infiltration is to be determined without any rainfall conditions and without crust

formations, \( q(t) \) is set at zero for all nodes.

Prior to time of ponding, we may write,

\[ f(0,t) = r(t) \]

where \( t_p = \text{time to ponding} \)

\[ q(t) = f(0,t) = \text{moisture flux into the surface} \]

\[ = -K(\psi)(\frac{\partial \psi}{\partial z} - 1) \quad (4.17) \]

The lower boundary condition may be a stable water table at some depth \( L \) so that

Dirichlet boundary condition may be applied as:

\[ \psi(L,t) = 0 \quad \text{for} \ 0 < t < \alpha \]

Alternately, the lower boundary conditions may be a moisture flux out of the soil

governed by soil moisture conditions on the lower boundary. The moisture flux can be

written as:

\[ q(t) = -K(L,t) * (\frac{\partial \psi}{\partial z}(L,t) - 1) \]

The initial condition is any known moisture content given by:

\[ \theta(z,0) = \theta_i \]

Using the grid system shown in Figure 4.2, an implicit formulation of equation 4.16

may be written as:

52
\[ C_i \frac{\psi_i^{j+1} - \psi_i^j}{\Delta t} = \frac{1}{\Delta z} [K_{i+1/2}^{j} \frac{\psi_{i+1}^{j+1} - \psi_{i+1}^{j+1}}{\Delta z} - 1\] - \[K_{i-1/2}^{j} \frac{\psi_{i-1}^{j+1} - \psi_{i-1}^{j+1}}{\Delta z} - 1\] \tag{4.18}

or,

\[ \frac{C_i^j}{\Delta t} (\psi_i^{j+1}) - \frac{C_i^j}{\Delta t} (\psi_i^j) = \frac{K_{i+1/2}^{j}}{\Delta z^2} (\psi_{i+1}^{j+1}) - \frac{K_{i+1/2}^{j}}{\Delta z^2} (\psi_i^{j+1}) - \frac{K_{i-1/2}^{j}}{\Delta z^2} (\psi_{i-1}^{j+1}) + \frac{K_{i-1/2}^{j}}{\Delta z^2} (\psi_{i-1}^{j+1}) + \frac{K_i^{j}}{\Delta z} \]

or, \( a_i \psi_i^{j+1} + b_i \psi_i^{j+1} + c_i \psi_{i-1}^{j+1} = RHS_i \) \tag{4.19}

where

\[ a_i = \frac{K_{i+1/2}^{j}}{\Delta z^2} \]

\[ b_i = -\left[ \frac{K_{i+1/2}^{j}}{\Delta z^2} + \frac{K_{i-1/2}^{j}}{\Delta z^2} + \frac{C_i^{j}}{\Delta t} \right] \]

\[ c_i = \frac{K_{i-1/2}^{j}}{\Delta z^2} \]

\[ RHS_i = -\frac{K_{i+1/2}^{j}}{\Delta z} - \frac{K_{i-1/2}^{j}}{\Delta z} - \frac{C_i^{j} \psi_i^j}{\Delta t} \]
Defining the grid system shown in Figure 4.2 and using flux boundary condition given by equation 4.19, the implicit finite difference form of the Richards equation for grid number 1 is

\[
\frac{C_i' \psi^{i+1}_i - \psi_i^i}{\Delta t} = \frac{K_{i+1/2}}{\Delta z} \left( \frac{\partial \psi}{\partial z} - 1 \right)_{i+1/2} + \frac{q}{\Delta z}
\]  

(4.20)

Equation 4.21 for grid number 1 then becomes,

\[a_1 \psi_{i+1} + b_1 \psi_i = RHS_1\]

where

\[a_1 = \frac{K_{i+1/2}}{\Delta z^2}\]

\[b_1 = -\left( \frac{K_{i+1/2}}{\Delta z^2} + \frac{C_1}{\Delta t} \right)\]
and \( RHS_1 = -\frac{C^t_{ij}}{\Delta t} + \frac{(K_{i+1/2} - q)}{\Delta z} \)

Similarly, assuming water table at the bottom, and applying Dirichlet boundary conditions with \( \psi_{N+1} = 0 \), we can write,

\[
b_N \psi_N^{j+1} + c_N \psi_{N-1}^{j+1} = RHS_N
\]

(4.21)

where

\[
b_N = -\frac{K_{N-1/2}^{j+1}}{\Delta z^2} - \frac{K_{N+1/2}^{j+1}}{\Delta z^2} - \frac{C_N^{j+1}}{\Delta t}
\]

and

\[
c_N = \frac{K_{N+1/2}^{j+1}}{\Delta z} - \frac{K_{N-1/2}^{j+1}}{\Delta z} - \frac{C_N^j \psi_N^j}{\Delta t}
\]

Equation 4.21 may be written for each interior node, \( i = 2, 3, 4, \ldots, N-1 \) giving \( N-2 \) equations with \( N \) unknowns. With equation 4.22 and 4.23, we have \( N \) equations in \( N \) unknowns. This system of equations for a tri-diagonal matrix and the system of equations may be solved for \( \psi \) at every node point by the Thomas Algorithm as discussed below.

**Thomas Algorithm**

Thomas Algorithm is based on Gauss elimination method and is used to solve a tri-diagonal matrix (Murari, 2003; Smith and Woolhiser, 1971) as given below:

\[
\begin{pmatrix}
b_1 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_2 & b_2 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_3 & b_3 & c_3 & 0 & 0 & 0 & 0 & 0 \\
& & & & & & & & \\
& & & & & & & & \\
& & & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & a_{N-2} & b_{N-2} & c_{N-2} \\
0 & 0 & 0 & 0 & 0 & 0 & a_{N-1} & b_{N-1} & c_{N-1} \\
0 & 0 & 0 & 0 & 0 & 0 & a_N & b_N & \psi_N \\
\end{pmatrix}
\begin{pmatrix}
\psi_1 \\
\psi_2 \\
\psi_3 \\
\psi_{N-2} \\
\psi_{N-1} \\
\psi_N \\
\end{pmatrix}
= 
\begin{pmatrix}
RHS_1 \\
RHS_2 \\
RHS_3 \\
RHS_{N-2} \\
RHS_{N-1} \\
RHS_N \\
\end{pmatrix}
\]
\[ a_i \psi_{i-1} + b_i \psi_i + c_i \psi_{i+1} = RHS_i \]

\[ \beta_i = \frac{c_i}{b_i} \]

\[ \beta_i = \frac{c_i}{b_i - a_i \beta_{i-1}}, \quad 2 < i < N - 1 \]

\[ g_i = \frac{RHS_i}{b_i} \]

\[ g_i = \frac{RHS_i - a_i g_{i-1}}{b_i - a_i \beta_{i-1}}, \quad 2 < i < N - 1 \]

\[ \psi_N = g_N \]

\[ \psi_i = g_i - \beta_i \psi_{i+1}, \quad 2 < i < N - 1 \]

As seen in the above equations, this algorithm works backward, first calculating the value of \( \psi_N \) and then \( \psi_{N-1}, \psi_{N-2}, \ldots, \psi_1 \) etc. A computer programme \textit{RICHARD} is written in FORTRAN language to solve for the suction values using equation 4.21. The input parameters to the model have been discussed in the following sections. The flow chart for the programme is given in Figure 4.3. Listing of the programme may be found in Appendices IV.
Fig 4.3: Flowchart for RICHARD programme

Read options for soil layers, B C s, soil water property model

Read soil data and antecedent moisture content for one/two layers

Read output options

Initialize time and cumulative infiltration volume

Initialize soil water properties for the first layer

Initialize soil water properties for the second layer

Homogeneous soil

Estimate crust parameters from CRUST module

Establish functional relationships between matrix potential, water content, unsaturated hydraulic conductivity and soil water capacity

Calculate conductivity at half nodes

Form tri-diagonal matrix using implicit scheme

Solve matrix potential at (j+1)th time level using backward elimination using Thomas algorithm

Calculate infiltration capacity, cumulative infiltration by Darcy's law

Calculate cumulative infiltration, and infiltration capacity using mass balance equation

Set infiltration rate at infiltration rate at infiltration capacity

Compare with rainfall rate and crust conductivity

Set infiltration rate at minimum of infiltration capacity rainfall rate and crust conductivity

Recalculate actual cumulative infiltration for actual infiltration rate

Print results

Set matrix potential, conductivity values at old time level

\[ \psi_i' = \psi_i'^{+1}, K_i' = K_i'^{+1} \]

Go to new time level, \( y = y + 1 \)

TIMCOUNT = TIMCOUNT + ΔT

Check if TIMCOUNT > TIME

Yes

EXIT

No
### Table 4.3: Input parameters to programme RICHARD

<table>
<thead>
<tr>
<th>Line</th>
<th>Variable name</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>IOPTLR</td>
<td>[-]</td>
<td>option for layered soil</td>
</tr>
<tr>
<td></td>
<td>IOPTLBC</td>
<td>[-]</td>
<td>option for lower boundary condition</td>
</tr>
<tr>
<td></td>
<td>IOPTUBC</td>
<td>[-]</td>
<td>option for upper boundary condition</td>
</tr>
<tr>
<td></td>
<td>IOPTSWPR</td>
<td>[-]</td>
<td>option for soil water properties</td>
</tr>
<tr>
<td></td>
<td>IOPTINF</td>
<td>[-]</td>
<td>option for calculating infiltration</td>
</tr>
<tr>
<td></td>
<td>IOPTWRIT</td>
<td>[-]</td>
<td>option for writing output results</td>
</tr>
<tr>
<td>2</td>
<td>Z</td>
<td>[cm]</td>
<td>depth of the soil profile</td>
</tr>
<tr>
<td></td>
<td>DELZ</td>
<td>[cm]</td>
<td>incremental depth between two nodes</td>
</tr>
<tr>
<td></td>
<td>NNODES</td>
<td>[-]</td>
<td>total number of nodes</td>
</tr>
<tr>
<td>3</td>
<td>TIME</td>
<td>[hr]</td>
<td>total duration of simulation</td>
</tr>
<tr>
<td></td>
<td>DELT</td>
<td>[hr]</td>
<td>time step between two consecutive time levels</td>
</tr>
<tr>
<td></td>
<td>NTIMES</td>
<td>[-]</td>
<td>total number of time steps</td>
</tr>
<tr>
<td>4</td>
<td>RL1</td>
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<td>length of the first layer</td>
</tr>
<tr>
<td>5</td>
<td>PSIIN1</td>
<td>[cm]</td>
<td>initial matric potential of the first layer</td>
</tr>
<tr>
<td></td>
<td>RN1</td>
<td>[-]</td>
<td>n-value in the V-G model for the first layer</td>
</tr>
<tr>
<td></td>
<td>RM1</td>
<td>[-]</td>
<td>m-value in the V-G model for the first layer</td>
</tr>
<tr>
<td></td>
<td>ALP1</td>
<td>[-]</td>
<td>$\alpha$-value in the V-G model for the first layer</td>
</tr>
<tr>
<td>6</td>
<td>RKS1</td>
<td>[cm/hr]</td>
<td>saturated hydraulic conductivity of the first layer</td>
</tr>
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<td></td>
<td>THETAS1</td>
<td>[cm$^3$/cm$^3$]</td>
<td>saturation moisture content of the first layer</td>
</tr>
<tr>
<td></td>
<td>THETAR1</td>
<td>[cm$^3$/cm$^3$]</td>
<td>residual moisture content of the first layer</td>
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<td>[cm]</td>
<td>initial moisture content of the second layer</td>
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<td>[-]</td>
<td>n-value in the V-G model for the second layer</td>
</tr>
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<td>RM2</td>
<td>[-]</td>
<td>m-value in the V-G model for the second layer</td>
</tr>
<tr>
<td></td>
<td>ALP1</td>
<td>[-]</td>
<td>$\alpha$-value in the V-G model for the second layer</td>
</tr>
<tr>
<td>8</td>
<td>RKS2</td>
<td>[cm/hr]</td>
<td>saturated hydraulic conductivity of the second layer</td>
</tr>
<tr>
<td></td>
<td>THETAS2</td>
<td>[cm$^3$/cm$^3$]</td>
<td>saturation moisture content of the second layer</td>
</tr>
<tr>
<td></td>
<td>THETAR2</td>
<td>[cm$^3$/cm$^3$]</td>
<td>residual moisture content of the second layer</td>
</tr>
<tr>
<td>9</td>
<td>ND1</td>
<td>[-]</td>
<td>first node number at which moisture content and suction values should be printed</td>
</tr>
<tr>
<td></td>
<td>ND2</td>
<td>[-]</td>
<td>second node number at which moisture content and suction values should be printed</td>
</tr>
<tr>
<td></td>
<td>ND3</td>
<td>[-]</td>
<td>third node number at which moisture content and suction values should be printed</td>
</tr>
<tr>
<td></td>
<td>ND4</td>
<td>[-]</td>
<td>fourth node number at which moisture content and suction values should be printed</td>
</tr>
<tr>
<td></td>
<td>ND5</td>
<td>[-]</td>
<td>fifth node number at which moisture content and suction values should be printed</td>
</tr>
<tr>
<td></td>
<td>ND6</td>
<td>[-]</td>
<td>sixth node number at which moisture content and suction values should be printed</td>
</tr>
<tr>
<td></td>
<td>ND7</td>
<td>[-]</td>
<td>seventh node number at which moisture content and suction values should be printed</td>
</tr>
<tr>
<td></td>
<td>ND8</td>
<td>[-]</td>
<td>eighth node number at which moisture content and suction values should be printed</td>
</tr>
<tr>
<td></td>
<td>ND9</td>
<td>[-]</td>
<td>ninth node number at which moisture content and suction values should be printed</td>
</tr>
<tr>
<td></td>
<td>ND10</td>
<td>[-]</td>
<td>tenth node number at which moisture content and suction values should be printed</td>
</tr>
<tr>
<td>10</td>
<td>RKO</td>
<td>[cm/hr]</td>
<td>initial crust conductivity</td>
</tr>
<tr>
<td></td>
<td>RKC</td>
<td>[cm/hr]</td>
<td>final crust conductivity</td>
</tr>
<tr>
<td></td>
<td>T37</td>
<td>[hr]</td>
<td>decay constant</td>
</tr>
<tr>
<td>11</td>
<td>RAINRATE</td>
<td>[cm/hr]</td>
<td>rainfall rate</td>
</tr>
<tr>
<td></td>
<td>POND</td>
<td>[cm]</td>
<td>depth of ponding</td>
</tr>
</tbody>
</table>
4.5.1 Soil Moisture Absorption Characteristics

Soil moisture characteristic curves during the wetting process for the soils were generated with the help of two replicated runs of capillary rise experiments carried out in the laboratory. The experimental procedure and the set-up adopted for these experiments have been described in detail by Tan (1987). The sectioned columns used in the experiment had an uniform inside diameter of 20mm and thickness of 6mm. A 100 cm soil column consisting of sectioned Lucite columns was packed uniformly with air dried soil. The zero head was established at the bottom of first lucite column, with the help of a constant head burette. The negative pressure head at each column section was taken to be the height at the mid-section of the column section. The top end of the burette was closed with a rubber stopper to make it air-tight. During the experimental process, the cumulative volume of water and the height of the wetting front were recorded as a function of time. The observation of the cumulative volume of water gave an indication of the water flow rate and the water absorbed by the soil, and helped in refilling the burette to provide a continuous supply of water. The experiment was continued until an equilibrium stage was reached. This stage was reached after about 15 days when flow rate was decreased to zero (less than 0.1 ml over a period of two hours). The soil column was then sliced into individual sections taking care that there was no apparent soil loss in the process. The soil from each individual section was transferred to small aluminum containers and covered immediately. The containers were weighed and oven dried to determine volumetric water content.

4.5.2 Unsaturated Hydraulic Conductivity Characteristics

Horizontal absorption experiments were carried out in the laboratory to determine the soil-water diffusivity characteristics of the soil. The experimental set-up and the method have been discussed above, and the theory and the procedure have been outlined
by Elrick et al. (1979) and Tan (1987). The equation used to determine the soil-water diffusivity was

$$D(\theta) = -\frac{1}{2} \frac{d\lambda^*}{d\theta} \int d\theta$$

(4.22)

where

$$D(\theta) = \text{diffusivity of the soil}$$

$$\lambda = \text{scaling factor in the Boltzmann transformation and is a function only of water content, [m}^2/\text{sec}]$$

$$\theta_i = \text{initial water content, [m}^3/\text{m}^3]$$

$$\theta = \text{measured water content, [m}^3/\text{m}^3]$$

$$x = \text{distance of the wet front, [m]}$$

Determination of soil-water diffusivity involves determination of $\lambda$ as a function of $\theta$. Similarly, determination of $\lambda$ involves determination of the wetting front position as a function of $\theta$.

Unsaturated hydraulic conductivity was computed by solving the following equation

$$K(\psi) = D(\theta) \frac{\partial \theta}{\partial \psi}$$

(4.23)

where

$$K(\psi) = \text{unsaturated hydraulic conductivity and matric potential, [m/s]}$$

The matric potential $\psi$ corresponding to water content was determined from soil-water characteristics. Repeated experiment was terminated at different elapsed times. During each experiment, the cumulative volume of water as well as the position of the wetting front was recorded as a function of the square root of the time. At the end of each
experiment, the run time was recorded, the combined soil-water column was sliced into individual sections and the soil in each section of known volume was used to calculate the volumetric water content. A computer programme INTEGRAT was developed to help in the integration process in equation 4.22, and another computer programme DERIVE was used to help in the differentiation involved in equations 4.23 and 4.31 using cubic splines whose theory is discussed below. The listings of the programmes can be found in Appendices V and VI respectively.

**Derivatives from Interpolating Polynomials**

If a function is reasonably well approximated by an interpolating polynomial, it is expected that the slope of the function is also approximated by the slope of the polynomial, though the error of estimating the slope is greater than the error of estimating the function.

We use divided-difference polynomial for approximating a polynomial as:

\[ f(x) = P_n(x) + \text{error} \]

\[ = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + \ldots + \]

\[ f[x_0, x_1, \ldots, x_n](x - x_0)(x - x_1) \ldots (x - x_{n-1}) + \text{error} \]

(4.24)

Ignoring the error term, Newton-Gregory forward polynomial that passes through a group of equispaced points can be written as:

\[ P_n(x) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!} \Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!} \Delta^3 f_0 + \ldots \]

\[ = f_0 + \left( \begin{array}{c} s \\ 1 \end{array} \right) \Delta f_0 + \left( \begin{array}{c} s \\ 2 \end{array} \right) \Delta^2 f_0 + \left( \begin{array}{c} s \\ 3 \end{array} \right) \Delta^3 f_0 + \ldots \]

(4.25)

Differentiating 4.25, remembering that \( f_0 \) and all the \( \Delta \)-terms are constants (they are just the numbers from the difference table), we have
The derivatives of the factorials of rapidly become complicated. We get considerable simplification if we let \( s = 0 \), giving us the derivative corresponding to \( x_0 \) (where \( s = 0 \)). If we let \( s = 0 \), the above equation becomes,

\[
f'(x_0) = \frac{1}{h} (\Delta f_0 + \frac{1}{2} (s - 1) \Delta^2 f_0 + \frac{1}{6} [(s - 1)(s - 2) + s(s - 2) + s(s - 1)] \Delta^3 f_0 + \ldots) \tag{4.27}
\]

Equation 4.27 is the derivative value, approximating to the derivative of the function, is the derivative of the \( n \)th degree polynomial passing through the point \((x_0, f_0)\) and \( n \) additional points to its right, evaluated at \( x = x_0 \). A computer programme \textit{DERIVE} was written to evaluate the derivative value at various points corresponding to \( x_0 \). Dependent variable \( f(x_0), f(x_1), f(x_2), \ldots, f(x_n) \) and independent variable \( x_0, x_1, x_2, x_3, \ldots, x_n \) are given as input parameters to the program. If the derivative is to be calculated at \( x \), the programme locates the position of \( x \), say \( x_{m-1} < x < x_m \) where \( m \) is an integer. The derivative is calculated at either of these limits of \( x \) whichever is nearer to it. The listing of the programme can be found in Appendix V.

\textbf{Newton-Cotes Integration Formula}

The usual strategy in developing formulae for numerical integration is similar to that for numerical integration. We pass a polynomial through points of the function, and then integrate this polynomial approximation to the function. This permits us to integrate a function known only as a table of values. When the values are equispaced, Newton-Gregorey forward polynomial is a convenient starting point, so that,
\[ \int_a^b f(x) \, dx = \int_a^b P_n(x) \, dx \]  
(4.28)

for \( n=1, \)

\[ \int_{x_0}^{x_1} f(x) \, dx = \int_{x_0}^{x_1} \left( f_0 + s\Delta f_0 \right) \, dx = h \int_{s=0}^{s=1} \left( f_0 + s\Delta f_0 \right) \, ds \]

\[ = hf_0s + h\Delta f_0 s^2 / 2 \Big|_0^1 = h(f_0 + 1/2\Delta f_0) \]

\[ = \frac{h}{2} [2f_0 + (f_1 - f_0)] = \frac{h}{2} (f_0 + f_1) \]  
(4.29)

for \( n=2, \)

\[ \int_{x_0}^{x_1} f(x) \, dx = \int_{x_0}^{x_1} \left( f_0 + s\Delta f_0 + \frac{s(s-1)}{2} \Delta^2 f_0 \right) \, dx \]

\[ = h \int_{s=0}^{s=2} \left( f_0 + s\Delta f_0 + \frac{s(s-1)}{2} \Delta^2 f_0 \right) \, ds \]

\[ = \frac{h}{3} (f_0 + 4f_1 + f_2) \]  
(4.30)

for \( n=3, \)

\[ \int_{x_0}^{x_1} f(x) \, dx = \frac{3h}{8} (f_0 + 3f_1 + 3f_2 + f_3) \]  
(4.31)

Equations 4.29, 4.30 and 4.31 are known as Newton-Cotes formulae. The first of the Newton-Cotes formulae, based on approximating \( f(x) \) on \((x_0,x_1)\) by a straight line is known as trapezoidal rule. The Newton-Cotes formulae based on quadratic and cubic polynomials are known as Simpson’s rules. The second degree Newton-Cotes formula integrates a quadratic over \( 2\Delta x \) intervals that are of uniform width and is known as Simpson’s 1/3\textsuperscript{rd} rule. The third Newton-Cotes formula that finds frequent use is obtained by integrating a cubic interpolating polynomial over its range of fit and is known as
Simpson’s 3/8\textsuperscript{th} rule. To apply this, the interval between \( a \) to \( b \) must be divided into a multiple of three panels.

Using Trapezoidal rule, to evaluate \( \int_{a}^{b} f(x) \, dx \), we subdivide the interval from \( a \) to \( b \) into \( n \) sub-intervals. The area under the curve in each sub-interval is approximated by the trapezoid formed by replacing the curve by its secant line drawn between the end-points of the curve. This gives,

\[
\int_{a}^{b} f(x) \, dx = \sum_{i=1}^{n} \frac{h}{2} (f_i + f_{i+1}) = \frac{h}{2} (f_1 + f_2 + f_2 + f_3 + \ldots + f_n + f_{n+1})
\]

\[
= \frac{h}{2} (f_1 + 2f_2 + 2f_3 + \ldots + 2f_n + f_{n+1})
\]

Using the above equation, a computer programme \textit{INTEGRAT} was used to calculate the integral over a function, known only as set of finite number of points, over a given limit lying between the points. The listing of the program can be found in Appendix V.

4.5.3 Soil Water Retention Models

There are mainly two types of soil-water retention models which are widely used, viz. (i) Brooks and Corey model, and (ii) van Genuchten model. Most of the researchers have used these two models, though some have used modified forms of these two models. These two models have been discussed below.

\textbf{Brooks and Corey Model}

One of the most popular functions for describing \( \theta(h) \) was proposed by Brooks and Corey (1964), and is popularly known as \textit{BC}-equation:

\[
S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = (\alpha \psi)^{-\lambda}
\]

where, \( S_e \) is the effective saturation, also known as reduced water content \((0 < S_e < 1)\), \( \theta_r \) and \( \theta_s \) are the residual and saturation water contents respectively, \( \alpha \) is
an empirical constant \( [L^{-1}] \) whose inverse is often referred to as the air entry value or bubbling pressure, and \( \lambda \) is a pore-size distribution parameter affecting the slope of the retention function. For notational convenience, \( \psi \) and \( \alpha \) are taken positive for unsaturated soils (\( \psi \) denotes matric potential). Because of its simple form, equation 4.40 has been used in numerous unsaturated flow studies. The BC-equation has been shown to produce fairly accurate results for many coarse-textured soils characterized by narrow pore or particle size distributions (large \( \lambda \) values). Results have been generally less accurate for fine-textured and undisturbed, structured field soils because of the absence of the well defined air entry value for these soils.

**van Genuchten Model**

Several continuously differentiable (smooth) equations have been proposed to improve the description of soil-water retention near saturation. Most of these equations are mathematically too complicated to be easily incorporated into predictive pore-size distribution models for the hydraulic conductivity, or lack of simple inverse relationships which makes them less attractive for many soils-water studies. A relatively smooth function was proposed by van Genuchten (1980) which is referred to as VG-equation:

\[
S_e = \frac{1}{[1 + (\alpha \psi)^n]^{-m}}
\]

(4.34)

where

\( S_e \) = effective saturation or effective degree of saturation

\( \alpha \), \( n \) and \( m \) are empirical constants affecting the shape of the retention curve

The limiting curve follows from equation 4.34 by removing the factor 1 from the denominator. This shows that VG and BC equations become equivalent at low \( S_e \) when \( \lambda = mn \). The same limiting curve also appears when \( n \) in the equation 4.41 is allowed to go to infinity, while simultaneously decreasing \( m \) such that the product \( mn \), remains at

65
The limiting BC-equation exhibits a sharp break in curve at the air entry value \( h_a = \frac{1}{\alpha} \). For finite values of \( n \) the curve remains smooth and more or less sigmoidally-shaped on a semi-logarithmic plot. However, the curve becomes markedly non-sigmoidal on the regular \( \theta - \psi \) plot, especially when \( n \) is relatively small.

The results of fitting equation 4.34 to retention data of four different soils are shown in Table 4.4. The examples were previously discussed by van Genuchten and Neilsen (1985). The table contains fitted parameter values and the calculated sum of the squares, SSQ, of the fitted versus observed water contents. The SSQ values reflect the relative accuracy of the retention models in describing the retention data. For Weld silty clay loam, the BC-equation matches the data equally well as the variable \( m,n \) case, whereas, the VG curves associated with restrictions \( m=1-1/n \) and \( m=1-2/n \) produced relatively poor results. This situation is different for Touchel silt loam and sand where BC-equation produces an unacceptable fit while the VG-equation with restricted \( m,n \) values produced results which are essentially identical to those for the general case when \( m \) and \( n \) are independent. Finally, there is a progressively better fit to the Sarpy loam data going from the BC limiting curve via the restricted cases \( m=1-2/n \) and \( m=1-1/n \), to the more general case of variable \( m,n \). From these results and many other examples not further discussed here, we conclude that variables \( m, n \) give an excellent fit to the observed retention data for most soils. The only exceptions are structured or aggregated soils characterized by very bimodal pore-size distributions. Of the three cases with restricted \( m \) values, \( m=1-1/n \) seems to perform best for many but not all soils, while the BC-equation generally performs best for selected coarse-textured and/or repacked, sieved soils with relatively narrow pore-size distributions. Although, the variable \( m,n \) case produced always superior results, its use is not necessarily recommended when only a limited range of retention
data (usually in the wet range) is available. Unless augmented with laboratory measurements at relatively low $S_e$ values, such data sets may not lead to accurate description of the retention curve in the dry range.
Table 4.4: Fitted soil hydraulic parameters from the retention curve

<table>
<thead>
<tr>
<th>Type of curve</th>
<th>$\theta_r$ (cm$^3$/cm$^3$)</th>
<th>$\theta_s$ (cm$^3$/cm$^3$)</th>
<th>$\alpha$ (1/cm)</th>
<th>n (-)</th>
<th>$\lambda, m$ (-)</th>
<th>SSQ (10$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weld silty clay loam</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>variable $m,n$</td>
<td>0.116</td>
<td>0.469</td>
<td>0.0173</td>
<td>61.54</td>
<td>$m=0.0308$</td>
<td>18</td>
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<tr>
<td>$m=1-1/n$</td>
<td>0.159</td>
<td>0.496</td>
<td>0.0136</td>
<td>5.45</td>
<td>$m=0.816$</td>
<td>487</td>
</tr>
<tr>
<td>$m=1-2/n$</td>
<td>0.155</td>
<td>0.495</td>
<td>0.0143</td>
<td>5.87</td>
<td>$m=0.659$</td>
<td>425</td>
</tr>
<tr>
<td>$n \to \infty$</td>
<td>0.116</td>
<td>0.465</td>
<td>0.0172</td>
<td>-</td>
<td>$\lambda=1.896$</td>
<td>21</td>
</tr>
<tr>
<td>Touchet silt loam</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>variable $m,n$</td>
<td>0.081</td>
<td>0.524</td>
<td>0.0313</td>
<td>3.98</td>
<td>$m=0.493$</td>
<td>14</td>
</tr>
<tr>
<td>$m=1-1/n$</td>
<td>0.102</td>
<td>0.526</td>
<td>0.0278</td>
<td>3.59</td>
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</tr>
<tr>
<td>$m=1-2/n$</td>
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<td>0.524</td>
<td>0.0312</td>
<td>3.98</td>
<td>$m=0.497$</td>
<td>14</td>
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<td>0.018</td>
<td>0.499</td>
<td>0.0377</td>
<td>-</td>
<td>$\lambda=1.146$</td>
<td>367</td>
</tr>
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<td>Sand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>variable $m,n$</td>
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<td>0.369</td>
<td>0.0227</td>
<td>4.11</td>
<td>$m=4.80$</td>
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<tr>
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<td>0.367</td>
<td>0.0364</td>
<td>5.05</td>
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<td>0.0382</td>
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<td>0.0462</td>
<td>-</td>
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<td>354</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
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<td>variable $m,n$</td>
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4.5.4 Hydraulic Conductivity Models

There are two types of hydraulic conductivity models as discussed below:

Mualem’s Model

The model of Mualem (1976) for predicting the hydraulic conductivity, $K$, can be written as:

$$K(S_e) = K_s S_e^l f(S_e) / f(l)^2$$

with

$$f(S_e) = \left[ \frac{1}{\psi(x)} \right] dx$$

where $K_s$ is the hydraulic conductivity at saturation and $l$ is the pore-connectivity parameter estimated by Mualem (1976) to be about 0.5 as an average for many soils.
Substituting the inverse of the equation 4.43 into equation 4.42 leads to the following expression for $K$:

$$K(S_e) = K_s S_e^{I_e(p,q)^2} \quad (4.37)$$

where $I_e(p,q)$ is the incomplete Beta function and $p = m + 1/n$, $q = 1 - 1/n$ and $\xi = S_e^{1/m}$.

Equation 4.37 holds for the general case of independent $m$ and $n$ in equation 4.34. Simpler expressions for $K$ may be obtained when the permissible values for $m$ and $n$ are restricted such that $m = l + 1/n$ becomes an integer (van Genuchten, 1980). The simplest case arises when $K = 0$, which leads to restriction $m = l - 1/n$. Equation 4.37 then reduces to the form:

$$K(S_e) = K_s S_e^{I_e[1 - (1 - S_e^1/n)^n]^2} \quad (4.38)$$

When the BC retention equation is substituted into 4.45, the hydraulic conductivity function according to Mualem becomes,

$$K(S_e) = K_s S_e^{I_e+2/\lambda} \quad (4.39)$$

The predictive equation for $K$ used so far thus assume that $K_s$ is well defined and easily measured with hydraulic parameters. This assumption is probably correct for many repacked, coarse-textured and other soils characterized by narrow pore-size distributions. However, direct field measurements of $K_s$ are generally very difficult for undisturbed field soils. Also, the hydraulic conductivity near saturation is primarily determined by soil structural properties which are known to be subject to considerable spatial variability in the field. This is in contrast to soil textural properties which generally are less variable and have a more dominant effect on $K$ in dry range. The rapid decrease of the predicted hydraulic conductivity near saturation when $n$ relatively small is intuitively realistic. It suggests that $K$ near saturation is determined only by a very few large macro-pores or
cracks which have little relation to the overall pore-size distribution that determines the
general shape of predictive conductivity curve at intermediate water contents. Thus, both
theoretical and experimental considerations suggest that $K_s$ should not be as a matching
point for the hydraulic conductivity models. Instead, it seems more accurate to match the
predicted and observed unsaturated hydraulic conductivity functions at water contents
less than saturation. The same holds for the saturated water content, $\theta_s$, which is best
regarded as an empirical parameter to be used in the context of a specific water retention
model, and hence must be fitted to observed unsaturated soil-water retention data points.

Burdine's Model

The model of Burdine can be written in general form as follows:

$$K(S_e) = K_s S_e^r \frac{g(S_e)}{g(0)}$$  \hspace{1cm} (4.40)

where

$$g(S_e) = \int_{0}^{S_e} \left[ \frac{1}{(\psi(x))^2} \right] dx$$  \hspace{1cm} (4.41)

The pore-connectivity parameter $l$ accounts for the presence of tortuous flow path.
A variety of values have been suggested for $l$, Burdine (1953) assumed a value of 2.

Results analogous to those of Mualem's model can be derived also for Burdine's
model. For independent $m$ and $n$ the hydraulic conductivity function can be written as:

$$K(S_e) = K_s S_e^r I_l^s (r, s)$$

where

$$r = m + 2/n \quad \text{and} \quad q = s - 2/n$$  \hspace{1cm} (4.42)

Equation 4.38 for variable $m, n$ may again be simplified by imposing restrictions on
the permissible values of $m$ and $n$. The restriction $m=1-2/n$ leads to:

$$K(S_e) = K_s S_e^l \left[ 1 - (1 - S_e^{1/m})^n \right]$$  \hspace{1cm} (4.43)
For completeness, we also give the conductivity expressions when the $BC$ limiting retention curve (i.e. $n \to \alpha$ with the product $\lambda = mn$ remaining finite) is used in conjunction with Burdine's model.

$$K(S_e) = K_e^{l+1/2/n}$$ (4.44)

Since many soils have $n$ values which are less than 2, the Burdine based models have less applicability than Mualem based model.

**Parameter Estimation by RETC**

RETC (Retention Curve) is a computer program developed by van Genuchten uses a non-linear least-square optimization approach to estimate the unknown parameters from the observed retention and/or conductivity or diffusivity data. The soil water retention curve ($\psi$ vs. $\theta$) contains five potentially unknown parameters (equation 4.40), i.e. $\theta_r$, $\theta_s$, $\alpha$, $n$ and $m$. The predictive equation for $K$ introduces $l$ and $K_s$ as two additional unknowns. Hence, the soil hydraulic functions contain a maximum of seven independent parameters. The model parameters are represented here schematically by the parameter vector $b = (\theta_r, \theta_s, \alpha, n, m, l, K_s)$. RETC code may be used to fit any one, several or all of these parameters simultaneously to observed data.

The most general formulation arises when the parameter $m$ and $n$ assumed to be independent. The hydraulic conductivity function is then given by equation 4.42 or 4.46 when the predictive models of Mualem or Burdine are used, respectively. The restrictions $n - \alpha$ (i.e. the $BC$ function), $m = l - 1/n$ and $m = l - 2/n$ will reduce the maximum number of independent parameters from seven to six. In addition to imposing restrictions on $m$ and $n$, the RETC user can keep one or more of the other coefficients (e.g. $\theta_s$, $l$, $K_s$) constant during the parameter optimization process, provided that an estimate of those coefficients is available. For example, the model parameter vector reduces to $b = (\theta_r, \theta_s, \alpha, n)$ when
Mualem restriction $m=l_1-l/n$ is implemented and only retention data are used in the optimization. The aim of the curve fitting process is to find an equation that maximizes the sum of squares associated with the model, while minimizing the residual sum of squares, SSQ.

**Limitations of using RETC**

While using RETC program, one must restrict to the following limitations:

(i) Soil textural class: USDA classification: (sand, silty loam, clay loam etc.)

(ii) Textural percentages: percentage sand, silt and clay should sum up to 100% (permissible range: 90-101%)

(iii) Bulk density: range between 0.5 and 2.0 gm/cm$^3$

(iv) Volumetric water content: water content should range between 0.0 and 1.0 cm$^3$/cm$^3$

**4.6 Measurement of Infiltration**

For establishing the predictability of the physically based and numerically based infiltration models, infiltration experiments were conducted in the field as well as in the laboratory to determine the hydraulic properties of the soil. The sites for the field experiments were the plots of land in front of the Mechanical engineering department and behind the hydraulic laboratory of the Civil engineering department of the Assam Engineering College, Guwahati. Infiltration rates are required in many hydrologic studies such as runoff estimation, soil moisture information, water budgeting and irrigation. Commonly used methods for determining infiltration capacity are hydrograph analysis and infiltration tests. There are many direct and indirect methods available for determining infiltration rates as mentioned below:

- Infiltrometer (single and double ring)
- Observation pits and ponds

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- Placing a catch basin below a laboratory sample
- Artificial rain simulator
- Hydrograph analysis
- Small rainfall simulator
- Rogers infiltrometer
- $\phi$-index method
- W-index method
- $f_{ave}$ method
- Lysimeters

For determination of infiltration characteristics, double ring infiltrometer are widely used due to its simplicity and cost-effectiveness besides being quick and giving results reasonably well. Infiltration rates observed by an infiltrometer are influenced by the thickness of the cylinder diameter, levelling of the cylinder bottom, the method of driving the cylinder into the soil and the installation depth. The variability of the data caused by the ring placement could be overcome by leaving the cylinders in place over a long period of time during a series of measurement. In the earlier studies, only a single cylinder was used and many of the data indicated a high degree of variability. The variability was mainly due to uncontrolled lateral movement of water from the cylinder after the wetting front reached the bottom of the cylinder. After the initiation of infiltration while the wetting front is in the cylinder, the water subsidence rate corresponds to the infiltration rate. When the wetting front passes below the cylinder, a more or less divergence of flow will occur the lateral movement of flow form cylinder is minimized by ponding water in an outer guard cylinder to provide a buffer area around the outer cylinder. A double ring infiltrometer as shown in Figure 4.5, essentially comprises of the following components:
- Inner cylinder of G.I. sheet 2 mm thick and of 300 mm internal diameter and 450 mm height.

- Outer buffer cylinder of G.I. sheet 2 mm thick and of 450 mm internal diameter and 450 mm height.

- One M.S. plate and a hammer to drive the cylinders into the ground, a linear scale to measure water level depletion, water bucket for filling and refilling, and a stop watch to record time.

Top outer surface of cylinders is collared by a 2 mm thick G.I. sheet by continuous welding process up to 4 cm height so that the top surface of cylinders does not get distorted by hammering during driving into the ground. Both inner and outer sides of the cylinders are painted with waterproof paint to prevent corrosion.

Figure 4.4: A typical infiltrometer

**Experimental procedure**

The following steps were followed while using the double ring infiltrometer to record infiltration characteristics:
1. The possible test sites are examined and necessary soil conditions with regard to texture, structure from open soil profile or by visual interpretation with particular reference to the top 30 cm depth that may influence rate of water intake, specific land use etc.

2. Selected sites should be free from soil cracks, worm holes, vegetations or humus. The site should not be deep, sloping and undulating.

3. The inner cylinder is placed on a leveled ground and is pressed firmly into the soil. Then the cylinder is driven uniformly into the ground for a depth of 15 cm with the help of a driving guide i.e. a wooden plank or a M.S. plate and a metallic hammer. In the similar fashion, the outer cylinder is driven up too the same depth and leveled around the inner one. Care should be taken so that hammering is done in the middle of the wooden plank and not towards the edge of the cylinder so that the hammering impact is equally distributed and driving is uniform with minimum distortion of soil mass.

4. A scale touching the side of the cylinder is inserted to measure water level with respect to time.

5. Enclosed soil surface should be covered to prevent erosion, distortion or puddling during filling. Both the cylinders should be filled simultaneously up to the top zero reference level of the vertical scale.

6. Depletion of water level with time is recorded, initially at very short intervals viz. 1, 2, 3 minutes and thereafter at 5 to 6 minutes and finally at longer intervals as infiltration rate approaches a constant rate.

**Assumptions and Limitations**

The following assumptions were inherently made while making observations from an infiltrometer:
1. There is no flow of water from inner cylinder to soil when water is filled up to the reference point at the starting point.

2. There is no structural change of the soil particles due to hammering vibration while driving the cylinders into soil.

3. There is no lateral movement of water from the inner cylinder.

4. There is no evaporation loss during the test.

The disadvantages of this type of experiment are that the soil structure may be greatly disturbed while driving the cylinder into position. When the soil is not completely protected by vegetation or by layer of humus, the soil structure may be altered by aggregation of eroded soil particles due to impact of water.

**Grain Size Analysis**

Sieve analysis was carried out using IS sieves to determine the textural classification and particle size distribution of the soil. Representative soil samples were collected from each experimental site from the top 300 mm of the soil. The details of soil collection, conditions imposed and test procedure are described by Malakar(2006).

The grain size distribution curve is plotted between the representative diameter of soil grain and percent finer than that size. The aggregate and cumulative weight, of all grains smaller than any given diameter, is plotted in the ordinate using an arithmetic scale, and the size of the soil particle in mm, is plotted on the abscissa using a logarithmic scale. Logarithmic scale becomes useful particularly when there is a very large range of particle sizes. Further, a semi-logarithmic scale has the merit that soils of equal uniformity exhibit the same shape of grain size curves, irrespective of particle size fractions present in the soil.
The position and the shape of a curve indicate the type and gradation of the soil. A well-graded soil has a good representation of grain sizes over a wide range and its gradation curve is smooth. On the other hand, a poorly-graded soil has either an excess or a deficiency of certain particle sizes or has most of the particles about the same size. In the later case, the soil is also called uniformly graded. The Coefficient of uniformity \((C_u)\) is a shape parameter and is defined as:

\[
C_u = \frac{D_{60}}{D_{10}}
\]

(4.45)

where, \(D_{60}\) is the grain diameter (mm) corresponding to 60% finer than that size. And \(D_{10}\) corresponds to 10% of the sample finer in weight on the grain size distribution curve. The diameter \(D_{10}\) is also called effective size. Another shape parameter that is often used is the Coefficient of Curvature \((C_c)\) which is defined as:

\[
C_c = \frac{D_{30}^2}{D_{10} \cdot D_{60}}
\]

(4.46)

where, \(D_{30}\) is the grain diameter (mm) corresponding to 30% finer than that size.

For a soil to be well-graded, \(C_c\) must lie between 1 and 3, and in addition to this, \(C_u\) must be greater than 4 for gravels and greater than 6 for sands.

**Density Test**

The in-situ density or the bulk density of the soil may be found out by any of the following methods:

- sand replacement method
- core cutter method
- water replacement method
- rubber balloon method

Out of these methods, core cutter method was adopted for convenience because it is less bulky, easy to separate and it yields results quickly. The
experimental conditions imposed, materials and equipments used, test procedure followed and calculations performed are described by Malakar (2006).

4.7 Comparison of Model Predictions with Published Results

The infiltration results obtained in this study, from the finite difference solution of the Richards equation, as well as from the physically based Green and Ampt approach, were compared with the results published by Hanks and Bowers (1962) who also used finite difference techniques. Hanks and Bouwers simulated infiltration into a layered combination of two soils: Geary silt loam and Sarpy loam. On the basis of the data provided by them, infiltration into a layered combination of the two soils was simulated. Unsaturated hydraulic conductivity characteristics for the two soils were generated from available diffusivity and absorption characteristics of the two soils (given in Table 4.5) using computer programme RETC. The generated relative hydraulic conductivity versus matric potential curves for Sarpy loam and Geary silt loam are shown in Figures 4.5 and 4.6. Both the numerical model and Green and Ampt model were applied to a layered combination comprising of 11 cm thick layer of Sarpy loam placed over a 29 cm thick layer Geary silt loam. The positions of the two soil layers were interchanged to study the effects of sequencing the soil layers. No surface crusting was considered in either case. Uniform moisture contents of 0.05 and 0.184 m3/m3, as used by Hanks and Bouwers, were considered for Sarpy loam and Geary silt loam, respectively. The results of the model simulations are presented and discussed in sections 5.2.3.
Table 4.5: Relation of moisture content, $\theta$, to moisture diffusivity, $D$, and Pressure head, $\psi$

<table>
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<th>$\theta$</th>
<th>Vol. fraction</th>
<th>Sarpy loam $D$ cm$^2$/sec</th>
<th>Geary silt loam $D$ cm$^2$/sec</th>
<th>$\psi$ cm</th>
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Figure 4.5: Relative hydraulic conductivity versus matric potential for Sarpy loam

Figure 4.6: Relative hydraulic conductivity versus matric potential for Geary silt loam
4.8 Summary

In the above discussions, the detailed methods and procedures followed to meet the objectives of the research work have been presented. Based on the review of literature work carried out, appropriate equations were selected to model infiltration, using implicit scheme of finite difference technique as well as using a lumped physical infiltration model. The governing equations were transformed to user-friendly forms to make them applicable under various initial and boundary conditions. Necessary computer models have been developed for the convenience of computational works. The listings as well as sample output files of the programmes may be found in the appendices, and the input parameters used in the programmes have been discussed in this chapter. A description of the laboratory and field experiments conducted for estimation of model parameters and for preparation of the data base is also presented. Surface crusting has been taken into consideration by developing a crust model which uses observed infiltration data.