Chapter 7

Thermal nucleation in protoneutron stars

7.1 Introduction

A first order phase transition from nuclear matter to some exotic form of matter might be possible in protoneutron stars. It could be either a nuclear to quark matter transition or a first order pion/kaon condensation. Consequently, it might have tremendous implications for compact stars [105] and supernova explosions [154]. Here the focus is the first order phase transition proceeding through the thermal nucleation of a new phase in particular, the $K^-$ condensed phase in hot and neutrino-trapped matter. After the pioneering work of Kaplan and Nelson on the kaon condensation in dense baryonic matter formed in heavy ion collisions as well as in neutron stars [115], several groups pursued the problem of the $K^-$ condensation in (proto)neutron stars [114, 118, 135, 148, 149, 155–162]. In most cases, the phase transition was studied using either Maxwell construction or Gibbs rules for phase equilibrium coupled with global baryon number and charge conservation [151]. The first order phase transition driven by the nucleation of $K^-$ condensed phase was considered in a few cases [163, 164]. In particular, the calculation of Ref. [164] dealt
with the role of shear viscosity on the thermal nucleation of antikaon condensed phase in hot and neutrino-free compact stars. It is to be noted here that the first order phase transition through the thermal nucleation of quark matter droplets was also investigated in (proto)neutron stars [153, 163, 165–169] using the homogeneous nucleation theory of Langer [56, 163, 165]. The thermal nucleation is an efficient process than the quantum nucleation at high temperatures [153, 169].

We adopt the homogeneous nucleation theory of Langer [56, 170] for the thermal nucleation of the $K^-$ condensed phase. Nuclear matter would be metastable near the phase transition point due to sudden change in state variables. In this respect thermal and quantum fluctuations are important. Droplets of $K^-$ condensed matter are formed because of thermal fluctuations in the metastable nuclear matter. Droplets of the new and stable phase which are bigger than a critical radius, will survive and grow. The transportation of latent heat from the surface of the droplet into the metastable phase favours a critical size droplet to grow further. This heat transportation could be achieved through the thermal dissipation and viscous damping [152, 170, 173].

A parametrized form of the shear viscosity was used in earlier calculations of the nucleation of quark matter [153]. Recently, the influence of thermal conductivity and shear viscosity on the thermal nucleation time was studied in a first-order phase transition from the nuclear matter to the $K^-$ condensed matter in hot neutron stars [164]. The shear viscosity due to neutrinos was not considered in that calculation. Here we study the effect of shear viscosity on the thermal nucleation rate of droplets of the $K^-$ condensed matter in neutrino-trapped matter relevant to protoneutron stars [171, 172]. Besides shear viscosities due to neutrons, protons and electrons, this involves the contribution of neutrinos to the total shear viscosity.

We organize the paper in the following way. We describe shear viscosities of different species including neutrinos, models for the EOS, and the calculation of thermal nucleation rate in Sec. 7.2. Results of this calculation are discussed in Sec. 7.3. Section 7.4 gives
the summary and conclusions.

7.2 Formalism

It was noted in the last chapter that the main contributions to the total shear viscosity in neutron star matter came from electrons, the lightest charged particles, and neutrons, the most abundant particles. Neutrinos are trapped in protoneutron stars and their contribution might be significant in transport coefficients such as shear viscosity. In principle, we may calculate shear viscosities for different particle species ($n$, $p$, $e$ and $\nu_e$) in neutrino-trapped matter using coupled Boltzmann transport equations of the form of Eq. (6.1) from which we can immediately write a set of relations between effective relaxation times ($\tau$) and collision frequencies ($\nu_{ij}$, $\nu_{ij}'$) as

$$\sum_{i,j=n,p,e,\nu_e} (\nu_{ij}\tau_i + \nu_{ij}'\tau_j) = 1,$$

which can be cast into a matrix equation similar to Eq. (6.11). However, solving the matrix equation becomes a problem because the relaxation time for neutrinos (calculated below) is much larger than those of other species as it is evident from Fig. 7.1. So we calculate the relaxation times of neutrons, protons and electrons in neutrino-trapped matter from the above matrix equation as was done in the previous chapter.

7.2.1 Neutrino shear viscosity

To calculate the neutrino shear viscosity we follow the prescription of Goodwin and Pethick [174] and get the following expression

$$\eta_\nu = \frac{1}{5} n_\nu k_{f_\nu} c T_{\nu} \left[ \frac{\pi^2}{12} + \lambda_\eta \sum_{m=\text{odd}} \frac{2(2m+1)}{m^2(m+1)^2[m(m+1)-2\lambda_\eta]} \right].$$

(7.2)
Figure 7.1: Relaxation times corresponding to different species in neutrino-trapped nuclear matter as a function of normalized baryon density at a temperature $T = 10$ MeV and $Y_L = 0.4$.

For neutrino shear viscosity, we only consider scattering processes involving neutrinos and other species. Various quantities in Eq. (7.2) are explained below. The neutrino relaxation time ($\tau_\nu$) is,

$$\tau_\nu^{-1} = \sum_{i=n,p,e} \tau_{\nu i}^{-1},$$

(7.3)

$$\tau_{\nu i}^{-1} = \frac{E_i^2 (k_B T)^2}{64 \pi^2} \langle I^i \rangle,$$

(7.4)

and $\lambda_\eta$ is defined as

$$\lambda_\eta = \tau_\nu \sum_i \lambda^i_\eta \tau_{\nu i}^{-1}$$

(7.5)

$$\lambda^i_\eta = \frac{\int d\Omega_2 d\Omega_3 d\Omega_4 \frac{1}{2} \left[ 3(\hat{k}_1 \cdot \hat{k}_3)^2 - 1 \right] \langle |M|^2 \rangle_i \delta(k_1 + k_2 - k_3 - k_4) \langle M^2 \rangle_i \delta(k_1 + k_2 - k_3 - k_4)}{\int d\Omega_2 d\Omega_3 d\Omega_4 \langle |M|^2 \rangle_i \delta(k_1 + k_2 - k_3 - k_4)}$$

(7.6)

$$= 1 - \frac{3}{2k_{\nu i}^2} < I^i > + \frac{3}{8k_{\nu i}^2} \frac{I^i_1}{I^i_2} < I^i >,$$

(7.7)
\[ < I > = \frac{8G_F^2\pi k_{f\nu}}{3} \left[ 4C_V C_A \left( \frac{k_{f\nu}}{E_{f\nu}} \right) + (C_V^2 + C_A^2) \right] \left\{ 3 + \left( \frac{k_{f\nu}}{E_{f\nu}} \right)^2 + \frac{2}{5} \left( \frac{k_{f\nu}}{E_{f\nu}} \right)^2 \right\} \]
\[ - \left( \frac{m_i}{E_{f\nu}} \right)^2 \left( C_V^2 - C_A^2 \right) \]  
(7.8)

\[ I_1' = \frac{32G_F^2\pi k_{f\nu}^3}{15} \left[ 12C_V C_A \left( \frac{k_{f\nu}}{E_{f\nu}} \right) + (C_V^2 + C_A^2) \right] \left\{ 5 + \left( \frac{k_{f\nu}}{E_{f\nu}} \right)^2 + \frac{12}{7} \left( \frac{k_{f\nu}}{E_{f\nu}} \right)^2 \right\} \]
\[ -3 \left( \frac{m_i}{E_{f\nu}} \right)^2 \left( C_V^2 - C_A^2 \right) \]  
(7.9)

\[ I_2' = \frac{128G_F^2\pi k_{f\nu}^3}{35} \left[ 20C_V C_A \left( \frac{k_{f\nu}}{E_{f\nu}} \right) + (C_V^2 + C_A^2) \right] \left\{ 7 + \left( \frac{k_{f\nu}}{E_{f\nu}} \right)^2 + \frac{10}{3} \left( \frac{k_{f\nu}}{E_{f\nu}} \right)^2 \right\} \]
\[ -5 \left( \frac{m_i}{E_{f\nu}} \right)^2 \left( C_V^2 - C_A^2 \right) \]  
(7.10)

Here \(< |M^2| >\) is the squared matrix element summed over final spins and averaged over initial spins for a scattering process and \(C_V\) and \(C_A\) are vector and axial-vector coupling constants.

For non-relativistic nucleons \((m_i/E_{f\nu}) \simeq 1, (k_{f\nu}/E_{f\nu}) \ll 1\) and if also \((k_{f\nu}/E_{f\nu}) \ll 1, \lambda^i_n\) reduces to [174]
\[ \lambda^i_n = \frac{11}{36} C_V^2 + \frac{2}{7} C_A^2 \]  
(7.11)

However, we do not assume non-relativistic approximation in this calculation. The total shear viscosity is given by
\[ \eta_{total} = \eta_n + \eta_p + \eta_e + \eta_{\nu} \]  
(7.12)

with
\[ \eta_i(=n,p,e) = \frac{n_i k_{f\nu}^2 \tau_i}{5m_i^*} \]  
(7.13)

where \(m_i\) and \(k_{f\nu}\) denote the effective mass and Fermi momentum respectively, of \(i\)-th species. The relaxation times \((\tau_i)\) are calculated using the method described in the previous chapter and are given by
\[ \tau_n = \frac{\left[ \nu_p \nu_e - \nu_{pe} \nu_{ep} \right] - \nu_{np} \left( \nu_{pe} - \nu_e \right)}{D} , \]
\[ \tau_p = \frac{\nu_n \left( \nu_e - \nu_{pe} \right) - \nu_{pn} \nu_e}{D} , \]
\[ \tau_e = \frac{\nu_n \left( \nu_p - \nu_{ep} \right) - \nu_{pn} \left( \nu_{np} - \nu_{ep} \right)}{D} , \] \hspace{1cm} (7.14)

where \( D = \nu_n \left( \nu_p \nu_e - \nu_{pe} \nu_{ep} \right) - \nu_{pn} \nu_e \). Collision frequencies appearing in Eq. (7.14) are obtained from Eqs. (6.12), (6.13), (6.17)-(6.18) and (6.20)-(6.22).

### 7.2.2 The EOS

The knowledge of the EOS for the nuclear as well as the \( K^- \) condensed phases is essential for the computation of shear viscosity and thermal nucleation rate. As discussed in earlier chapters we consider here a first-order phase transition from the charge neutral and \( \beta \)-equilibrated \((\mu_e + \mu_p = \mu_n + \mu_{\nu_e})\) nuclear matter to the \( K^- \) condensed matter in a protoneutron star. Those two phases are composed of neutrons, protons, electrons, electron type neutrinos and of \( K^- \) mesons only in the \( K^- \) condensed phase. Both phases are governed by baryon number conservation and charge neutrality conditions. The critical droplet of the \( K^- \) condensed matter is in total phase equilibrium with the metastable nuclear matter. The mixed phase is governed by Gibbs phase rules along with global baryon number conservation and charge neutrality conditions (Sec. 5.2.3). We adopt the bulk approximation [163] which does not consider the variation of the meson fields with position inside the droplet. Relativistic field theoretical models described in Chapter 5 are employed to calculate the EOS in nuclear and antikaon condensed phases. Expressions for the energy density and pressures in the nuclear matter and the \( K^- \) condensed matter are given in Secs. (5.2.1), (5.2.2). As we are interested in neutrino-trapped matter of protoneutron stars, we have to add the contributions of neutrinos in Eqs. (5.26) and (5.27) given by

\[ \epsilon_{\nu_e} = \frac{k_{\text{F}e}}{8\pi^2} , \hspace{1cm} P_{\nu_e} = \frac{k_{\text{F}e}}{24\pi^2} . \] \hspace{1cm} (7.15)
Here we use the zero temperature EOSs because it was noted earlier that the temperature of a few tens of MeV did not modify the EOS considerably \cite{164}.

### 7.2.3 Nucleation rate

We are interested in a first order phase transition driven by the nucleation of droplets of antikaon condensed phase in the neutrino-trapped nuclear matter. Droplets of antikaon condensed phase are born in the metastable nuclear matter due to thermal fluctuations. Droplets of antikaon condensed matter above a critical size \( (R_c) \) will grow and drive the phase transition. According to the homogeneous nucleation formalism of Langer and others, the thermal nucleation per unit time per unit volume is given by \cite{56, 170}

\[
\Gamma = \Gamma_0 \exp \left( -\frac{\Delta F(R_c)}{T} \right), \tag{7.16}
\]

where \( \Delta F \) is the free energy cost to produce a droplet with a critical size in the metastable nuclear matter. The free energy shift of the system as a result of the formation of a droplet is given by \cite{167, 168}

\[
\Delta F(R) = -\frac{4\pi}{3} (P^K - P^N) R^3 + 4\pi \sigma R^2, \tag{7.17}
\]

where \( R \) is the radius of the droplet, \( \sigma \) is surface tension of the interface separating two phases and \( P^N \) and \( P^K \) are the pressures in the neutrino-trapped nuclear and the \( K^- \) condensed phases, respectively as discussed above. We obtain the critical radius of the droplet from the maximum of \( \Delta F(R) \) i.e. \( \delta_R \Delta F = 0 \),

\[
R_c = \frac{2\sigma}{(P^K - P^N)}. \tag{7.18}
\]

This relation also demonstrates the mechanical equilibrium between two phases.

We write the prefactor in Eq. (7.16) as the product of two parts - statistical and
dynamical prefactors \[152\,170\,173\]

\[\Gamma_0 = \frac{\kappa}{2\pi} \Omega_0. \quad (7.19)\]

The available phase space around the saddle point at \(R_C\) during the passage of the droplet through it is given by the statistical prefactor (\(\Omega_0\)),

\[\Omega_0 = \frac{2}{3\sqrt{3}} \left(\frac{\sigma}{T}\right)^{3/2} \left(\frac{R_C}{\xi}\right)^4, \quad (7.20)\]

Here \(\xi\) is the kaon correlation length which is considered to be the width of the interface between nuclear and antikaon condensed matter. The dynamical prefactor \(\kappa\) is responsible for the initial exponential growth rate of a critical droplet and given by \[152\,173\]

\[\kappa = \frac{2\sigma}{R_C^2(\Delta w)^2} \left[\lambda T + 2 \left(\frac{4}{3}\bar{\eta} + \bar{\zeta}\right)\right]. \quad (7.21)\]

Here \(\Delta w = w_K - w_N\) is the enthalpy difference between two phases, \(\lambda\) is the thermal conductivity and \(\eta\) and \(\zeta\) are the shear and bulk viscosities of neutrino-trapped nuclear matter. We neglect the contribution of thermal conductivity because it is smaller compared with that of shear viscosity \[164\]. We also do not consider the contribution of bulk viscosity in the prefactor in this calculation.

We can now calculate the thermal nucleation time (\(\tau_{nuc}\)) in the interior of neutron stars as

\[\tau_{nuc} = (VT)^{-1}, \quad (7.22)\]

where the volume \(V = 4\pi/3R_{nuc}^3\). We assume that pressure and temperature are constant within this volume in the core.

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7.3 Results and Discussion

Like the previous chapter here also we use the GM1 parameter set (table 5.1) for nucleon-meson coupling constants. Procedure to get the kaon-meson coupling constants is discussed in Sec. 5.3.2. Here we consider an optical potential depth of $U_K(n_0) = -120$ MeV at normal nuclear matter density and the corresponding kaon-scalar meson coupling constant is $g_{\sigma K} = 1.6337$ (table 5.2). The value of $K^-$ optical potential adopted in this calculation resulted in a maximum neutron star mass of $2.08 M_\odot$ in earlier calculations using the Maxwell construction [135]. This is consistent with the recently observed $2M_\odot$ neutron star [6].

Fig. 7.2 shows EOSs of neutrino-trapped matter ($Y_L = 0.4$) for entropy density $S = 0$ and $S = 2$. It is evident from the figure that thermal effects can’t change the EOS.
Figure 7.3: Shear viscosities corresponding to different particle species in neutrino-trapped nuclear matter as a function of the normalized baryon density at a temperature $T = 10$ MeV and $Y_L = 0.4$.

First we calculate shear viscosities of neutrons, protons and electrons in neutrino-trapped nuclear matter using Eq. (7.13) in the same fashion as it was done in the last chapter. We take lepton fraction $Y_L = 0.4$ in this calculation. Shear viscosities of different species are shown as a function of normalized baryon density at a temperature $T=10$ MeV in Fig. 7.3. Here the electron viscosity is higher than the neutron and proton shear viscosities. The total shear viscosity excluding the viscosity due to neutrinos in neutrino-trapped nuclear matter is shown as a function of normalized baryon density for appreciably. Therefore, we use the EOS of neutrino-trapped nuclear and $K^-$ condensed phases at zero temperature. In this calculation, the EOS enters in Eq. (7.17) as the difference between pressures in two phases and in Eq. (7.21) as the enthalpy difference between two phases. Here we exploit the zero temperature EOS for the the calculation of shear viscosity and thermal nucleation time. The thermal nucleation of exotic phases was earlier investigated using zero temperature EOS in Ref. [164, 168].
temperatures \(T = 1, 10, 30\) and \(100\) MeV in Fig. 7.4. The shear viscosity is found to increase with baryon density. Furthermore, the shear viscosity decreases with increasing temperature. It is observed that the shear viscosities of neutrons, protons and electrons in neutrino-trapped nuclear matter are of the same orders of magnitude as those of the neutrino-free case presented in the last chapter.

Next we calculate the shear viscosity due to neutrinos. As a prelude to it, we compare effective relaxation times corresponding to different species in neutrino-trapped nuclear matter in Fig. 7.1. Relaxation time is plotted with normalized baryon density at a temperature \(T=10\) MeV in Fig. 7.1. Relaxation times of different particle species owing to scattering under strong and electromagnetic interactions are much much shorter than that of neutrinos undergoing scattering with other species through weak interactions. Consequently, particles excluding neutrinos come into thermal equilibrium quickly on the time scale of weak interactions. We calculate the shear viscosity due to neutrinos only treating others as background and it is shown as a function of normalized baryon density for temperatures \(T = 1, 10, 30\) and \(100\) in Fig. 7.5. Like Fig. 7.4 the neutrino shear
Figure 7.6: Prefactor including the contribution of shear viscosity as a function of temperature at a fixed baryon density and surface tension and compared with that of the $T^4$ approximation.

viscosity decreases with increasing temperature. However, the neutrino shear viscosity is several orders of magnitude larger than shear viscosities of neutrons, protons and electrons shown in Fig. 7.4. It is the neutrino shear viscosity which dominates the total viscosity of Eq. (7.12) in neutrino-trapped matter. We perform the rest of our calculation using the neutrino viscosity in the following paragraphs.

We calculate the prefactor ($\Gamma_0$) according to Eqs. (7.19)-(7.21). The dynamical prefactor not only depends on the shear viscosity but also on the thermal conductivity and bulk viscosity. However, it was already noted that the thermal conductivity and bulk viscosity in neutrino-trapped nuclear matter were negligible compared with the shear viscosity [173]. We only consider the effect of shear viscosity on the prefactor. Besides transport coefficients, the prefactor in particular, the statistical prefactor is sensitive to the correlation
length of kaons and surface tension. The correlation length is the thickness of the interface between nuclear and kaon phases [152, 153] having a value $\sim 5$ fm [165]. The radius of a critical droplet is to be greater than the correlation length ($\xi$) for kaons [152, 165]. We perform our calculation with $K^-$ droplets with radii greater than 5 fm. The other important parameter in the prefactor is the surface tension. The surface tension between nuclear and kaon phases was already estimated by Christiansen and collaborators [175] and found to be sensitive to the EOS. We perform this calculation for a set of values of surface tension $\sigma = 15, 20, 25$ and 30 MeV fm$^{-2}$. The prefactor ($\Gamma_0$) is shown as a function of temperature in $\text{Fig. 7.6}$ It is shown for a baryon density $n_b = 4.235n_0$ which is just above the critical density $3.9n_0$ for the $K^-$ condensation at zero temperature [159], and surface tension $\sigma = 15$ MeV fm$^{-2}$. The prefactor was also approximated by $T^4$ according to the dimensional analysis in many calculations [152, 168]. The upper curve in Fig. 7.6 shows the prefactor of Eq. (7.19) including only the contribution of neutrino shear viscosity whereas the prefactor approximated by $T^4$ corresponds to the lower curve. It is evident from Fig. 7.6 that the approximated prefactor is very small compared with our result.

Now we discuss the nucleation time of a critical droplet of the $K^-$ condensed phase in neutrino-trapped nuclear matter and the effect of neutrino shear viscosity on it. The thermal nucleation rate of the critical droplet is calculated within a volume with $R_{\text{nuc}} = 100$ meters in the core of a neutron star where the density, pressure and temperature are constant. The thermal nucleation time is plotted with temperature for a baryon density $n_b = 4.235n_0$ in Fig. 7.7. Furthermore, this calculation is done with the kaon correlation length $\xi = 5$ fm and surface tension $\sigma = 15, 20, 25$ and 30 MeV fm$^{-2}$. The size of the critical droplet increases with increasing surface tension. Radii of the critical droplets are 7.1, 9.4, 11.7 and 14.1 fm corresponding to $\sigma = 15, 20, 25$ and 30 MeV fm$^{-2}$, respectively, at a baryon density $4.235n_0$. The nucleation time of the critical droplet diminishes as temperature increases for all cases studied here. However, the temperature corresponding to a particular nucleation time for example $10^{-3} \text{ s}$, increases as the surface
tension increases. There is a possibility that the condensate might melt down if the temperature is higher than the critical temperature. The critical temperature of the $K^-$ condensation was investigated in neutrino-free matter in Ref. [162] and for neutrino matter in Ref. [176]. We compare thermal nucleation times corresponding to different values of the surface tension with the early post bounce time scale $t_d \sim 100$ ms in the core collapse supernova [154] when the central density might reach the threshold density of the $K^-$ condensation. The time scale $t_d$ is much less than the neutrino diffusion time $\sim 1$ s as obtained by Ref. [174]. Thermal nucleation of the $K^-$ condensed phase may be possible when the thermal nucleation time is less than $t_d$. For $\sigma = 15 \text{ MeV fm}^{-2}$, the thermal nucleation time of $10^{-3}$ s occurs at a temperature 16 MeV. It is evident from Fig. 7.7 that the thermal nucleation time is strongly dependent on the surface tension. Further thermal nucleation of a $K^-$ droplet is possible so long as the condensate might survive the melt down at high temperatures [162,176]. Our results of thermal nucleation time are compared with the calculation taking into account the prefactor approximated by $T^4$ in Fig. 7.8 for surface tension $\sigma = 15 \text{ MeV fm}^{-2}$ and at a density $n_b = 4.235n_0$. The upper

Figure 7.7: Thermal nucleation time is displayed with temperature for different values of surface tension

Figure 7.8: Comparison of our results for a fixed surface tension with the calculation of the $T^4$ approximation.
curve denotes the calculation with $T^4$ approximation whereas the lower curve corresponds to the influence of neutrino shear viscosity on the thermal nucleation time. The results of the $T^4$ approximation overestimate our results hugely. For a nucleation time of $10^{-3}$ s at a temperature $T=16$ MeV, the corresponding time in the $T^4$ approximation is larger by several orders of magnitude.

### 7.4 Summary and Conclusions

We have studied shear viscosities of different particle species in neutrino-trapped $\beta$-equilibrated and charge neutral nuclear matter. We have used equations of state of the nuclear and the $K^-$ condensed phases in the relativistic mean field model for the calculation of shear viscosity. It is noted that neutrons, protons and electrons come into thermal equilibrium in the weak interaction time scale. The shear viscosity due to neutrinos is calculated treating other particles as background and found to dominate the total shear viscosity.

Next we have investigated the first-order phase transition from the neutrino-trapped nuclear matter to the $K^-$ condensed matter through the thermal nucleation of a critical droplet of the $K^-$ condensed matter using the same relativistic EOS as discussed above. Our emphasis in this calculation is the role of the shear viscosity due to neutrinos in the prefactor and its consequences on the thermal nucleation rate. We have observed that the thermal nucleation of a critical $K^-$ droplet might be possible well before the neutrino diffusion takes place. Furthermore, a comparison of our results with that of the calculation of thermal nucleation time in the $T^4$ approximation shows that the latter overestimates our results of thermal nucleation time computed with the prefactor including the neutrino shear viscosity. Though we have performed this calculation with the $K^-$ optical potential depth of $U_{K}(n_0) = -120$ MeV, we expect qualitatively same results for other values of the $K^-$ optical potential depth (see table 5.2).