Chapter 5

Neutron star Core

5.1 Introduction

In the previous chapter it was seen that the ground state of a neutron star inner crust contains nuclei immersed in electron and neutron gases. With increasing density nuclei come closer to each other and at a density \( \rho \sim 0.5 \rho_0 \) \( (\rho_0 \simeq 2.8 \times 10^{14} \text{ g cm}^{-3} \) is the saturation density of nuclear matter), nuclei no longer exist as they merge together to form a uniform matter of neutrons, protons and electrons. The outer core begins at this point. With increasing density Fermi energy of all the components increase. When the Fermi energy (or, chemical potential) of electrons exceeds the muon rest mass energy \( (m_\mu = 105.7 \text{ MeV}) \), muons appear in the system and take part in maintaining the charge neutrality \( (n_e + n_\mu = n_p) \) and the \( \beta \)-equilibrium \( (\mu_n = \mu_p + \mu_e, \mu_e = \mu_\mu) \) of the system. At density \( \rho \gtrsim 2 \rho_0 \), inner core starts the composition of which is not known and therefore model dependent. There are suggestions that at such high densities transition from \( npe\mu \) matter to various exotic phases such as strange baryons (\( \Lambda, \Sigma \) and \( \Xi \) hyperons), Bose-Einstein condensation of pions and (anti)kaons may take place. At ultrahigh densities, matter can dissolve into deconfined quark matter [105].

There are a host of theoretical models describing matter of neutron star cores. The
main uncertainty in the calculation of dense matter arises from poorly known many-body interactions. However, several methods have been developed over the years to solve this problem and can be divided into two main categories. In the first category one tries to calculate the ground state energy of the matter starting from a bare nucleon-nucleon (NN) interaction. Models based on Brueckner-Bethe-Goldstone (BBG) theory \[106,107\], Green’s function method \[108,109\], variational method \[110\] etc fall under this category. In the other category one starts from an effective NN interaction. The non-relativistic Skyrme-type models \[70\], the relativistic mean field (RMF) model \[105,111,112\] etc belong to this category.

Any many-body theory of neutron star matter must reproduce the empirical results of bulk nuclear matter: the saturation density, \(n_0 = 0.15 - 0.16 \text{ fm}^{-3}\); the binding energy per nucleon, \(E/A = −16 ± 1 \text{ MeV}\); effective nucleon mass \(m^*_N = 0.7 - 0.8 m_N\); the incompressibility defined as \(K = \left[ \rho^2 \frac{d^2}{d\rho^2} \left( \frac{\varepsilon}{\rho} \right) \right]_{\rho=\rho_0}\) in the range 200 – 300 MeV and symmetry energy at the saturation given as \(a_s = \frac{1}{2} \left( \frac{\partial^2 (\varepsilon/\rho)}{\partial t^2} \right)_{t=0} ; t \equiv \frac{\rho_n-\rho_p}{\rho}\) within 30 – 35 MeV. In this thesis, we employ the RMF model to investigate the properties of dense matter in neutron star interior.

5.2 Relativistic Mean Field (RMF) model

Motivated by the experimental observation of large Lorentz scalar and four-vector components in the NN interaction Walecka \[111\] introduced a field-theoretical model, also known as \(\sigma – \omega\) model, to describe the properties of nuclei as well as nuclear matter. In this model the interaction between nucleons is mediated by the exchange of a scalar \((\sigma)\) and a vector meson \((\omega)\). In the static limit of infinitely heavy baryons, these meson exchanges correspond to an effective NN potential of the form \[112\]

\[
V = \frac{g_\omega^2}{4\pi} \frac{e^{-m_\omega r}}{r} - \frac{g_\sigma^2}{4\pi} \frac{e^{-m_\sigma r}}{r},
\]
which with the appropriate choices of the coupling constants \((g_\sigma, g_\omega)\) and and masses \((m_\sigma, m_\omega)\), reproduces the main qualitative behaviors, namely the short range repulsion and the long range attraction of the NN interaction. However, this model was unable to reproduce empirical values of the incompressibility, the effective nucleon mass and the symmetry energy. To gain control over the first two properties Boguta and Bodmer \[113\] introduced non-linear self-interactions of the scalar meson \((\sigma)\) in this model. The model was further extended by including a vector-isovector meson \((\rho)\) which accounts for the symmetry energy of the nuclear matter.

### 5.2.1 Hadronic phase

To describe the pure hadronic phase we consider an extension of the \(\sigma - \omega\) model where nucleons interact with each other by the exchange of \(\sigma, \omega\) and \(\rho\) mesons. The NN interaction is given by the Lagrangian density \[105, 114\]

\[
\mathcal{L}_N = \bar{\psi}_N \left( i \gamma_\mu \partial^\mu - m_N + g_\sigma N \sigma - g_\omega N \gamma_\mu \omega^\mu - \frac{1}{2} g_\rho N \gamma_\mu \tau_N \cdot \rho^\mu \right) \psi_N \\
+ \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - U(\sigma) \\
- \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega^\mu - \frac{1}{4} \rho_{\mu\nu} \cdot \rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_{\mu} \cdot \rho^{\mu}. \tag{5.2}
\]

where \(\psi_N \equiv (\psi_p, \psi_n)^T\) is the isospin doublet of nucleons with \(\psi_p\) and \(\psi_n\) being the 4-component Dirac spinors for proton and neutron, respectively; \(m_N\) is the nucleon mass; \(m_\sigma, m_\omega, m_\rho\) are masses of mesons; \(g_\sigma N, g_\omega N \) and \(g_\rho N\) are coupling constants; \(\rho\) and \(\tau_N\) are vectors in isospin space (isovectors) and \(\omega_{\mu\nu}, \rho_{\mu\nu}\) are meson field tensors given by

\[
\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \\
\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu. \tag{5.3}
\]
The scalar self-interaction \[105, 113\] is

\[
U(\sigma) = \frac{1}{3} g_1 m_N (g_{\sigma N})^3 + \frac{1}{4} g_2 (g_{\sigma N})^4.
\] (5.4)

The coupling constants are the parameters of the model and can be determined by relating them algebraically to five empirically known quantities of bulk nuclear matter at saturation: \(\rho, E/A, m^*_N, K\) and \(a_s\).

Equations of motion for the fields are obtained from Euler-Lagrange equation

\[
\partial_\mu \frac{\partial L}{\partial (\partial_\mu \phi)} = \frac{\partial L}{\partial \phi},
\] (5.5)

where \(\phi\) represents fields \(\psi_N, \sigma, \omega\) and \(\rho\). Using this Eq. (5.5) along with Eq. (5.2) we get the Dirac equation for nucleons

\[
\left[ i \gamma^\mu \left( i \partial_\mu - g_{\omega N} \omega_\mu - \frac{1}{2} g_{\rho N} \tau_N \cdot \rho_\mu \right) - \left( m_N - g_{\sigma N} \sigma \right) \right] \psi_N = 0.
\] (5.6)

For mesons we get following equations of motions

\[
(\Box + m_\omega^2) \omega_\mu = g_{\omega N} \psi_N \gamma_\mu \psi_N, \quad (\Box + m_\rho^2) \rho_\mu = \frac{1}{2} g_{\rho N} \psi_N \tau_N \cdot \gamma_\mu \psi_N, \quad (5.7, 5.8, 5.9)
\]

with \(\Box \equiv \partial^\mu \partial_\mu\).

The last four equations form a set of coupled nonlinear differential equations and therefore very difficult to solve exactly. Moreover, coupling constants are expected to be large which make perturbation approach inapplicable. However, in the context of studying dense and uniform matter of neutron stars we can use the mean-field approximation where
the meson field operators are replaced by their ground state expectation values as
\[ \sigma \rightarrow \langle \sigma \rangle \]
\[ \omega_\mu \rightarrow \langle \omega_\mu \rangle \]
\[ \rho_\mu \rightarrow \langle \rho_\mu \rangle \]

The validity of this approximation improves with density as at large densities source terms of Eqs. (5.7)-(5.9) increase which in turn increase the justification of the above replacements. The expectation values of space components of \( \omega_\mu, \rho_\mu \) vanish due to the rotational symmetry of the system. The first two components of the isovector field (\( \rho \)) also have vanishing expectation values in the ground state so that only the third component survives. Moreover, in the rest frame of this uniform matter the fields are independent of space and time. Considering all these together we get greatly simplified equations of motion for nucleons as well as mesons

\[
\left[ \gamma^0 \left( i \partial_0 - g_{\omega N} \omega_0 - \frac{1}{2} g_{\rho N} \tau_3 N \rho_{03} \right) \right] \psi_N = 0 , \quad (5.10)
\]
\[
m^2_{\sigma \sigma} = - \frac{\partial U}{\partial \sigma} + g_{\sigma N} < \bar{\psi}_N \psi_N > , \quad (5.11)
\]
\[
m^2_{\omega 0} = g_{\omega N} < \psi_N^\dagger \psi_N > , \quad (5.12)
\]
\[
m^2_{\rho 03} = \frac{1}{2} g_{\rho N} < \psi_N^\dagger \tau_3 N \psi_N > . \quad (5.13)
\]

As there is no space-time dependence in these equations we look for the stationary solution for nucleons of the form
\[
\psi_N = u(\vec{k}) e^{i(\vec{k} \cdot \vec{x} - \varepsilon(k)t)} , \quad (5.14)
\]
where \( u(k) \) is an 8-component spinor. When this \( \psi_N \) is substituted in Eq. (5.10) we get the eigenvalue equation
\[
(\vec{\alpha} \cdot \vec{k} + \beta m_N^* u(\vec{k}) = (\varepsilon(k) - g_{\omega N} \omega_0 - \frac{1}{2} g_{\rho N} \tau_3 N \rho_{03}) u(\vec{k}) , \quad (5.15)
\]
with eigenvalue
\[ \varepsilon(k) - g_{\omega N} \omega_0 - \frac{1}{2} g_{\rho N} \tau_3 N \rho_{03} = \pm (\vec{k}^2 + m_N^*)^{1/2}, \]
which yields,
\[ \varepsilon(k) = g_{\omega N} \omega_0 + \frac{1}{2} g_{\rho N} \tau_3 N \rho_{03} \pm (\vec{k}^2 + m_N^*^2)^{1/2} = \varepsilon_{\pm}(k). \tag{5.16} \]
where \( m_N^* \) is the effective nucleon mass given as
\[ m_N^* = m_N - g_{\sigma N} \sigma. \tag{5.17} \]
The general solution for the field operator \( \psi_N \) is written as
\[ \psi_N = \frac{1}{\sqrt{V}} \sum_s \int d\vec{k} \left[ a_s(\vec{k}) u_s(\vec{k}) e^{-i\varepsilon_{+}(\vec{k}) t + i\vec{k} \cdot \vec{x}} + b_{s}^\dagger(\vec{k}) v_s(\vec{k}) e^{-i\varepsilon_{-}(\vec{k}) t - i\vec{k} \cdot \vec{x}} \right], \tag{5.18} \]
where \( u_s(\vec{k}) \) and \( v_s(\vec{k}) \) are positive and negative energy spinors, respectively; \( a_s(\vec{k}) \) denotes the annihilation operator for particles whereas \( b_{s}^\dagger(\vec{k}) \) stands for the creation operator for antiparticles. However, we do not consider antiparticles in our calculation as there is no antiparticle present in the ground state \( (T = 0) \) of uniform nuclear matter. Nucleons being fermions occupy all the energy levels upto their Fermi energies \( (\varepsilon_f \equiv \varepsilon(k = k_f)) \), which are also their chemical potentials \( (\mu) \), in the ground state.

The expectations value appearing in the RHS of the field equation \[ \text{(5.12)} \] gives the total baryon number density of the system
\[ n \equiv \langle \psi_N^\dagger \psi_N \rangle = \frac{2}{(2\pi)^3} \int_0^{k_{fp}} d\vec{k} + \frac{2}{(2\pi)^3} \int_0^{k_{fn}} d\vec{k} = \frac{k_{fp}^3}{3\pi^2} + \frac{k_{fn}^3}{3\pi^2} = n_p + n_n. \tag{5.19} \]
Similarly, for other expectation values we get scalar and vector baryon densities as

\[
n_s \equiv \langle \bar{\psi}_N \psi_N \rangle = \frac{2}{\pi^2} \int_0^{k_{fp}} k^2 dk \frac{m_N^*}{\sqrt{k^2 + m_N^2}} + \frac{2}{\pi^2} \int_0^{k_{fn}} k^2 dk \frac{m_N^*}{\sqrt{k^2 + m_N^2}}, \tag{5.20}
\]

and

\[
n_v \equiv \langle \bar{\psi}_N \tau_3 \psi_N \rangle = \frac{2}{(2\pi)^3} \int_0^{k_{fp}} dk \bar{k} - \frac{2}{(2\pi)^3} \int_0^{k_{fn}} dk \bar{k} = n_p - n_n. \tag{5.21}
\]

Expression for energy density and pressure for nucleons can be obtained from Energy-momentum tensor as

\[
\epsilon_N = \langle T^{00} \rangle = - \langle \mathcal{L}_N \rangle + \langle \bar{\psi}_N \gamma_0 \partial_0 \psi_N \rangle, \tag{5.22}
\]

\[
P_N = \frac{1}{3} \langle T^{ii} \rangle = \langle \mathcal{L}_N \rangle + \frac{1}{3} \langle \bar{\psi}_N \gamma^i \partial_i \psi_N \rangle. \tag{5.23}
\]

After evaluating the expectation values we get

\[
\epsilon_N = \frac{1}{3} g_1 m_N (g_{N\sigma} \sigma)^3 + \frac{1}{4} g_2 (g_{N\sigma} \sigma)^4 + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_0^2
\]

\[
+ \frac{1}{\pi^2} \left[ \int_0^{k_{fp}} \sqrt{k^2 + m_N^2} k^2 dk + \int_0^{k_{fn}} \sqrt{k^2 + m_N^2} k^2 dk \right], \tag{5.24}
\]

\[
P_N = - \frac{1}{3} g_1 m_N (g_{N\sigma} \sigma)^3 - \frac{1}{4} g_2 (g_{N\sigma} \sigma)^4 - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_0^2
\]

\[
+ \frac{1}{3\pi^2} \left[ \int_0^{k_{fp}} \frac{k^4 dk}{\sqrt{k^2 + m_N^2}} + \int_0^{k_{fn}} \frac{k^4 dk}{\sqrt{k^2 + m_N^2}} \right]. \tag{5.25}
\]

Leptons form uniform Fermi gases and their energy density and pressure can be easily calculated as

\[
\epsilon_L = \sum_{l=e,\mu} \frac{1}{\pi^2} \int_0^{k_{fl}} \left( k^2 + m_l^2 \right)^{1/2} k^2 dk, \tag{5.26}
\]

\[
P_L = \frac{1}{3} \sum_{l=e,\mu} \frac{1}{\pi^2} \int_0^{k_{fl}} \frac{k^4 dk}{\left( k^2 + m_l^2 \right)^{1/2}}. \tag{5.27}
\]
Now, we have the total energy density and pressure of the system:

$$\epsilon = \epsilon_N + \epsilon_L, \quad (5.28)$$

$$P = P_N + P_L. \quad (5.29)$$

So, knowing the masses and coupling constants the EOS of the nuclear matter can be calculated using Eqs. (5.11)-(5.13), (5.19)-(5.21) and (5.24)-(5.29).

### 5.2.2 Kaon condensed phase

By using a chiral $SU(3)_L \times SU(3)_R$ Lagrangian, Kaplan and Nelson [115] first demonstrated the possibility of the existence of an (anti)kaon ($K^-$) condensed phase in dense nuclear matter. (Anti)kaon begins to appear in the system when its effective in-medium energy or the chemical potential becomes equal to the chemical potential of electrons i.e.

$$\omega_{K^-} = \mu_{K^-} = \mu_e. \quad (5.30)$$

Generally $\mu_e$ increases as the baryon density increases. But the effective energy of $K^-$ in the nuclear medium decreases with increasing density because of their attractive s-wave interaction with the nuclear medium. Therefore, at some density the above threshold condition may be fulfilled and $K^-$ may appear through the following strangeness changing processes

$$n \rightarrow p + K^-, \quad e^- \rightarrow K^- + \nu_e. \quad (5.31)$$

The appearance of $K^-$ lowers the energy of the system in two ways:

i) Because of its bosonic character $K^-$ mesons may form a condensate in the lowest momentum ($k = 0$) state and thereby save the kinetic energy of electrons they replace.

ii) With the appearance of $K^-$ mesons the proton fraction of the matter also increases
which in turn reduces the symmetry energy of nuclear matter.

A pure (anti)kaon condensed phase contains nucleons (we don’t consider hyperons), leptons (electron, muon) as well as the (anti)kaon. For the interaction between nucleons we consider the same Lagrangian as before in Eq. 5.2. Interaction between kaons and nucleons can be treated in two ways - within the chiral perturbation theory [115–117] or within the kaon-meson coupling scheme. To treat the interactions of kaons on the same footing as nucleons we adopt the latter approach and use the Lagrangian density for the kaon in the minimal coupling scheme introduced by Glendenning and Schaffner-Bielich (1999) and is given by [118]

\[ L_K = D_\mu \bar{K} D^\mu K - m_K^* \bar{K} K, \tag{5.32} \]

where \( K \equiv (K^+, K^0) \) and \( \bar{K} \equiv (K^-, \bar{K}^0) \) denote kaon and antikaon isospin doublets, respectively. \( D_\mu \) is the covariant derivative

\[ D_\mu = \partial_\mu + ig_\omega K \omega_\mu + ig_\rho K \tau_\mu, \tag{5.33} \]

\( m_K^* \) is the effective mass of the kaon

\[ m_K^* = m_K - g_{\sigma K} \sigma, \tag{5.34} \]

and \( g_{\sigma K}, g_{\omega K} \) and \( g_{\rho K} \) are the kaon-meson coupling constants. The equation of motion for the kaon is

\[ (D_\mu D^\mu + m_K^2)K = 0. \tag{5.35} \]

At this point we employ the mean-field approximation discussed in Sec. (5.2) and for static and uniform neutron star matter we get the expression for the in-medium energy
of $K^-$ mesons as
\[
\omega_{K^-} = \sqrt{\left(k^2 + m_{K^*}^2\right) - g_{\omega K} \omega_0 - \frac{1}{2} g_{\rho K} \rho_{03}}. \quad (5.36)
\]

For the s-wave condensation ($\vec{k} = 0$) we obtain
\[
\omega_{K^-} = m_{K^*}^* - g_{\omega K} \omega_0 - \frac{1}{2} g_{\rho K} \rho_{03}. \quad (5.37)
\]

So, we see that the interaction of $K^-$ mesons with the nuclear medium causes a reduction in its energy which has also become density dependent through the meson fields $\sigma, \omega_0$ and $\rho_{03}$. With increasing density meson fields generally increase which in turn decreases $\omega_{K^-}$ and when it becomes equal to the $\mu_e$, $K^-$ mesons appear in the system.

The meson field equations (5.11)-(5.13) get modified in the presence of the $K^-$ condensate as
\[
m_{\sigma}^2 \sigma = -\frac{\partial U}{\partial \sigma} + g_{\sigma N} n_s + g_{\sigma K} n_K, \quad (5.38)
\]
\[
m_{\omega_0}^2 \omega_0 = g_{\omega N} n - g_{\omega K} n_K, \quad (5.39)
\]
\[
m_{\rho_{03}}^2 \rho_{03} = \frac{1}{2} g_{\rho N} (\rho_p - \rho_n) - g_{\rho K} n_K. \quad (5.40)
\]

where $n_K$ is the vector density of the kaon and for the s-wave condensation considered here this is also the scalar density and is given by
\[
n_K = 2(\omega_K + g_{\omega K} \omega_0 + g_{\omega K} \rho_{03}) \bar{KK} = 2m_{K^*}^* \bar{KK} \quad (5.41)
\]

The total charge density of the $K^-$ condensate phase is calculated as
\[
Q_K = n_p - n_e - n_\mu - n_k \quad (5.42)
\]

As all the $K^-$ mesons have zero momentum in the s-wave condensation, they don’t have any direct contribution to the total pressure and can be calculated from Eq. (5.29) using
modified meson field equations (5.38)-(5.40) in the presence of a $K^-$ condensate. But they do contribute to the total energy density of the system through the term

$$\epsilon_K = m_K^* n_K,$$  \hspace{1cm} (5.43)

so that the total energy density becomes

$$\epsilon = \epsilon_N + \epsilon_L + \epsilon_K.$$  \hspace{1cm} (5.44)

### 5.2.3 Mixed phase

The transition from the hadronic phase to the $K^-$ condensed phase in neutron stars is mainly of first order, but can also be of second order. The first order transition goes through a mixed phase where both the phases coexist in equilibrium. For a single component system having only one conserved quantity this equilibrium is governed by the Gibbs conditions:

$$\mu^I = \mu^{II} = \mu$$

$$T^I = T^{II} = T$$

$$P^I(\mu, T) = P^{II}(\mu, T) = P$$  \hspace{1cm} (5.45)

For a fixed temperature the last equation of (5.45) gives an unique solution for $\mu$ which is generally determined by using the Maxwell construction. However, the ground state of neutron star matter is a two component system with two conserved quantities namely the total baryon density and the total electric charge. This gives two independent chemical potentials which we can choose as $\mu_n$ and $\mu_e$ so that the Gibbs conditions are now given
Here, $H$ and $K$ stand for the hadronic and the $K^-$ condensed phases, respectively. Unlike the Maxwell case here the equilibrium quantities $(P, \mu)$ don’t remain constant throughout the mixed phase, instead they depend on the proportion of the two equilibrium phases. In the mixed phase the charge neutrality condition reads as

$$ (1 - \chi)Q_H + \chi Q_K = 0, \quad (5.47) $$

where $\chi$ is the volume fraction occupied by the condensed phase. Similarly for the total baryon number density and the energy density of the mixed phase are given by

$$ n_{MP} = (1 - \chi)n_H + \chi n_K, \quad (5.48) $$

$$ \epsilon_{MP} = (1 - \chi)\epsilon_H + \chi \epsilon_K. \quad (5.49) $$

where $n_H$ and $\epsilon_H$ are the baryon number density and the energy density in the hadronic phase respectively and $n_K$ and $\epsilon_K$ are the corresponding quantities in the $K^-$ condensed phase.

### 5.3 Parameter sets

The EOS of the neutron star matter can be obtained within the framework of RMF theory if all the coupling constants appearing in the theory are known. To determine the coupling constants we rely on the empirical values of the bulk nuclear matter properties defined

by [105] (for $T = 0$)

$$ \mu_e^H = \mu_e^K, \quad (5.46) $$

$$ \mu_n^H = \mu_n^K, $$

$$ P^H(\mu_e, \mu_n) = P^K(\mu_e, \mu_n). $$

$$ (5.46) $$

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### 5.3 Parameter sets

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Table 5.1: GM1 parameter set that reproduces saturation density \( n_0 = 0.153 \text{ fm}^{-3} \), binding energy \( E/A = -16.3 \text{ MeV} \), \( m_N^*/m_N = 0.70 \), incompressibility \( K = 300 \text{ MeV} \) and symmetry energy coefficient \( a_s = 32.5 \text{ MeV} \). Masses are taken as \( m_N = 938 \text{ MeV} \), \( m_\sigma = 550 \text{ MeV} \), \( m_\omega = 783 \text{ MeV} \) and \( m_\rho = 770 \text{ MeV} \).

<table>
<thead>
<tr>
<th>( g_{\sigma N} )</th>
<th>( g_{\omega N} )</th>
<th>( g_{\rho N} )</th>
<th>( g_1 )</th>
<th>( g_2 ) (fm(^{-1}))</th>
</tr>
</thead>
</table>

Table 5.2: Kaon-scalar meson coupling constants for the GM1 parameter set at different values of \( K^- \) optical potential depth \( U_K \).

<table>
<thead>
<tr>
<th>( U_K ) (MeV)</th>
<th>-100</th>
<th>-120</th>
<th>-140</th>
<th>-160</th>
<th>-180</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM1</td>
<td>0.9542</td>
<td>1.6337</td>
<td>2.3142</td>
<td>2.9937</td>
<td>3.6742</td>
</tr>
</tbody>
</table>

earlier in the chapter, at the saturation density.

### 5.3.1 Nucleon-Meson coupling constants

There are five unknown coupling constants \( g_{\sigma N}, g_{\omega N}, g_{\rho N}, g_1 \) and \( g_2 \) in the nucleon-meson interaction Lagrangian given by Eq. (5.2). For our calculation we use the GM1 parameter set introduced by Glendenning and Moszkowski (1991) as it predicts a maximum neutron star mass compatible with the most recent observed neutron star mass of \( 1.97 M_\odot \).

The values of the parameters of this set are listed in table 5.1.

### 5.3.2 Kaon-Meson coupling constants

Coupling constants for the kaon-meson interaction (Eq. 5.32) are determined by using the quark model and isospin counting rule. For the vector coupling constants we have

\[
g_{\omega K} = \frac{1}{3} g_{\omega N} \quad \text{and} \quad g_{\rho K} = g_{\rho N} .
\] (5.50)
The real part of the $K^-$ optical potential depth at the saturation density provides the scalar coupling constant as

$$U_K(n_0) = -g_{\sigma K} \sigma - g_{\omega' K} \omega_0 .$$ (5.51)

We have seen in the Section [5.2.2] that the interaction of $K^-$ mesons with the nuclear matter is attractive in nature. On the one hand, the analysis of $K^-$ atomic data indicated that the real part of the optical potential could be as large as $U_K = -180 \pm 20$ MeV at normal nuclear matter density [120, 121]. On the other hand, chirally motivated coupled channel models with a self-consistency requirement predicted shallow potential depths of $-40$-60 MeV [122, 123]. Further, the highly attractive potential depth of several hundred MeV was obtained in the calculation of deeply bound $K^-$-nuclear states [124, 125]. An alternative explanation to the deeply bound $K^-$-nuclear states was given by others [126]. This shows that the value of $K^-$ optical potential depth is still a debatable issue. The values of kaon-scalar meson coupling constants corresponding to the GM1 parameter set and for various values of $K^-$ optical potential depths are listed in table 5.2.