Synopsis

Title of thesis: Some Problems on Modular Forms.

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This thesis contains some of my work on modular forms during my stay at Harish-Chandra Research Institute as a research scholar.

1. Generalized Modular Functions

In [15], Knopp and Mason introduced the notion of a generalized modular function. A generalized modular function f on $\Gamma_0(N)$ is a holomorphic function on the upper half-plane $\mathcal{H}$, meromorphic at cusps, that transforms under $\Gamma_0(N)$ like a usual modular function, namely $f(\frac{az+b}{cz+d}) = f(\gamma \circ z) = \chi(\gamma)f(z)$, for all $\gamma \in \Gamma_0(N)$, where $\chi : \Gamma_0(N) \longrightarrow \mathbb{C}^{\times}$ is a homomorphism with $\chi(\gamma) = 1$ for γ parabolic with trace $= 2$. We abbreviate GMF for a generalized modular function. Let f be a non-zero GMF on $\Gamma_0(N)$. Then by a theorem of Eholzer and Skoruppa [5, 9], each GMF f has a product expansion

$$f(z) = q^h \prod_{n\geq 1}(1-q^n)^{c(n)} \qquad (0 < |q| < \epsilon), \quad q = e^{2\pi i z}, \quad z \in \mathcal{H},$$

where $h \in \mathbb{Z}$ and $c(n)(n \geq 1)$ are uniquely determined complex numbers.
If $f : \mathcal{H} \to \mathbb{C}$ is a function and $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL_2^+(\mathbb{R})$, we define the slash operator by,

$$(f|\gamma)(z) := f(\gamma \circ z) = f\left(\frac{az+b}{cz+d}\right).$$

For $M|N$, let $\mathcal{R} = \{\gamma_1, \ldots, \gamma_r\}$ be a set of representatives for $\Gamma_0(N)\backslash\Gamma_0(M)$. We define a "norm" of f w.r.t. $\mathcal{R}$ by,

$$\mathcal{N}_{\mathcal{R},N}^{M}(f) := \prod_{\nu=1}^{r} f|\gamma_\nu.$$

Let $W_N = \begin{pmatrix} 0 & -1 \\ N & 0 \end{pmatrix}$ be the Fricke involution. In [19], W. Kohnen proved that for a non-zero GMF f on $\Gamma_0(N)$ with empty divisor, the exponents $c(n)(n \in \mathbb{N})$ take infinitely many different values. In Chapter 1, which is a joint work with W. Kohnen [KM], we show that $c(p)(p\ prime)$ take infinitely many different values, by assuming certain extra conditions on the GMF. More precisely, we prove the following theorem. We prove this theorem by using a positive proportion result regarding values not taken by the Fourier coefficients of cusp forms lying in the subspace of newforms, due to Ono and Skinner [39].

Theorem 0.0.1. *Let f be a non-constant GMF on $\Gamma_0(N)$ with $div(f) = \emptyset$ and assume that f has algebraic Fourier coefficients. Suppose further that $\mathcal{N}_{\mathcal{R},N}^{M}(f)$ and $\mathcal{N}_{\mathcal{R},N}^{M}(f|W_N)$ are constant for every divisor M of N, $M \neq N$. Then the exponents $c(p)$ (p prime) take infinitely many different values.*

Next, we prove the following result using Landau's theorem on Dirichlet series with non-negative coefficients, coupled with the fact that the abscissa of absolute convergence of the Hecke L-function of a non-zero cusp form of weight 2 is exactly $3/2$.

Theorem 0.0.2. *Let f be a non-constant GMF on $\Gamma_0(N)$ with $div(f) = \emptyset$ and suppose that the $c(n)\,(n \geq 1)$ are real. Then the $c(n)\,(n \geq 1)$ change signs infinitely often.*

2. Newforms of Half-integral Weight on $\Gamma_0(8N)$ and $\Gamma_0(16N)$

In 1973, G. Shimura introduced the theory of modular forms of half-integral weight and obtained a correspondence between modular forms of half-integral weight and integral weight. After the works of Shimura and Niwa, Shintani gave a correspondence in the reverse direction. In [49, 50], J. -L. Waldspurger proved a remarkable result in showing that the square of the Fourier coefficients of a half-integral Hecke eigenform f is proportional to the special value (at the center) of the L- function of the corresponding Hecke eigenform F of integral weight twisted with a quadratic character, where f maps to F by Shimura correspondence. In 1980's, W. Kohnen characterized certain subspace, called the Kohnen plus space w.r.t. certain operators and further extended the Shimura correspondence to the plus space. Kohnen also studied the analogous Atkin-Lehner theory of newforms in the plus space on $\Gamma_0(4N)$, where N is odd and square-free and obtained the Shintani lifting which is adjoint to the Shimura-Kohnen map. As a consequence, he obtained the explicit form of the Waldspurger theorem for the newforms in the plus space. Analogous theory of newforms for the full space on $\Gamma_0(4N)$, N odd and square-free was obtained by Manickam, Ramakrishnan and Vasudevan in [31]. Manickam in his thesis obtained the theory of newform for the full space on $\Gamma_0(8N)$, N odd square-free [27, 28]. In 1990's, M. Ueda initiated the work extending the result of Kohnen for general N. Recently, M. Ueda and S. Yamana [48] established Kohnen's theory on $\Gamma_0(8N)$, N square-free odd integer, but without considering the Shimura-Kohnen lifts and Waldspurger's formula for the newforms. In Chapter 2 of the thesis, which is a joint work with M. Manickam and B. Ramakrishnan [MMR], we establish the theory of newforms for both the plus spaces on $\Gamma_0(8N)$ and on $\Gamma_0(16N)$ for N odd and square-free. We carry out this theory by using the isomorphism proved by Ueda by means of trace formulas [45, 46]. We define the Shimura-Kohnen maps on these plus spaces and prove that the spaces of newforms of half-integral weight (in the respective plus spaces) on $\Gamma_0(8N)$ and $\Gamma_0(16N)$ are isomorphic to the respective spaces of newforms of integral weight on $\Gamma_0(2N)$ and $\Gamma_0(4N)$. Further, by using the results of Ueda [45, 46], we develop the theory of newforms for the full space on $\Gamma_0(16N)$ with trivial character and also for the primitive quadratic character modulo 8. We give below the precise statements of results proved in Chapter 2.

Before stating the theorems, we give the necessary notations. For a positive integer M with $4|M$, let $S_{k+1/2}(M, \chi)$ denote the space of modular forms of weight $k + 1/2$ on $\Gamma_0(M)$ with χ, a Dirichlet character modulo M.

If χ is trivial, it is denoted by $S_{k+1/2}(M)$. For any positive integer n, the operators $U(n)$ and $B(n)$ are defined on formal sums as follows:

$$U(n): \sum_{m\geq 1} a(m)q^m \mapsto \sum_{m\geq 1} a(mn)q^m,$$

$$B(n): \sum_{m\geq 1} a(m)q^m \mapsto \sum_{m\geq 1} a(m)q^{nm}.$$

For a prime $p|M$, $U(p^2)$ is the Hecke operator on the space $S_{k+1/2}(M,\chi)$ and $B(n)$ maps $S_{k+1/2}(M,\chi)$ into $S_{k+1/2}(Mn,\chi\chi_n)$, where χ_n is the quadratic character $\left(\frac{n}{\cdot}\right)$. For $Q|M$ with $(Q,M/Q)=1$, let $W(Q)$ denote the analogous Atkin-Lehner W-operator on the space $S_{k+1/2}(M,\chi)$. The Kohnen plus space on the space $S_{k+1/2}(4M,\chi)$ for $8\nmid M$ is defined by,

$$S^+_{k+1/2}(4M,\chi) = \left\{f \in S_{k+1/2}(4M,\chi) \mid a_f(n) = 0 \text{ if } (-1)^k n \equiv 2,3 \pmod 4\right\}. \tag{0.0.1}$$

From now onwards, N denotes an odd positive square-free integer. Our first result gives the decomposition of $S^+_{k+1/2}(8N)$ into old and newforms.

Theorem 0.0.3. *We have the following decompositions:*

$$S^+_{k+1/2}(8N) = S^{+,new}_{k+1/2}(8N) \bigoplus S^{+,old}_{k+1/2}(8N), \tag{0.0.2}$$

$$S^{+,old}_{k+1/2}(8N) = \bigoplus_{\substack{rd|2N\\ d<2N,2\nmid r}} S^{+,new}_{k+1/2}(4d)|U(r^2) \bigoplus_{\substack{rd|2N\\ d<2N,2|r}} S^{+,new}_{k+1/2}(4d)|U(r^2)L_+, \tag{0.0.3}$$

where the operator L_+ is defined by,

$$L_+: \sum_{m\geq 1} a(m)q^m \mapsto \sum_{\substack{m\geq 1\\ (-1)^k m\equiv 0,1 \pmod 4}} a(m)q^m.$$

Next, we define the Shimura-Kohnen map on $S^+_{k+1/2}(8N)$. Let t be a square-free integer with $(-1)^k t > 0$ and let D ($= t$ or $4t$ according as t is 1 or $2,3$ modulo 4) be the corresponding fundamental discriminant. The t-th Shimura map on $S_{k+1/2}(8N)$ is defined as

$$f|\mathscr{S}_{t,8N}(z) = \sum_{n\geq 1}\left(\sum_{\substack{d|n\\ (d,2N)=1}} \left(\frac{4t}{d}\right) d^{k-1} a_f(|t|n^2/d^2)\right) q^n. \tag{0.0.4}$$

We now define the D-th Shimura-Kohnen map $\mathscr{S}^+_{D,8N}$ on $S^+_{k+1/2}(8N)$ by

$$f|\mathscr{S}^+_{D,8N}(z) = f|\mathscr{S}_{t,8N}|U(2)(z), \quad f \in S^+_{k+1/2}(8N). \tag{0.0.5}$$

Since $U(2) : S_{2k}(4N) \to S_{2k}(2N)$ and $f|\mathscr{S}_{t,8N} \in S_{2k}(4N)$, it follows that $\mathscr{S}^+_{D,8N}$ maps $S^+_{k+1/2}(8N)$ into $S_{2k}(2N)$. Next, we show that the Hecke operator $U(4)$ maps the space $S^{+,new}_{k+1/2}(8N)$ isomorphically onto the space $S^{new}_{k+1/2}(4N)$. As a consequence of this fact, we obtain the Waldspurger theorem for the newforms belonging to $S^{+,new}_{k+1/2}(8N)$. Let F be a normalized newform in $S^{new}_{2k}(2N)$ and let g be the corresponding newform in $S^{new}_{k+1/2}(4N)$ such that F is the Shimura image of g. For a square-free integer $t \equiv 1 \pmod 4$ with $(-1)^k t > 0$, $(t, N) = 1$, let $L(F, \chi_t, k)$ be the special value of the L-function of F twisted with the character $\chi_t = \left(\frac{t}{\cdot}\right)$. Let $f = \sum_{n\geq 1} a_f(n)q^n \in S^{+,new}_{k+1/2}(8N)$ be the newform such that $f|U(4) = g$. Then F is the newform corresponding to f under the Shimura-Kohnen map. With f, F as above, we have the following (Waldspurger theorem):

$$\frac{|a_f(4|t|)|^2}{\langle f|U(4), f|U(4)\rangle} = 2^{\nu(N)-1}\frac{(k-1)!}{\pi^k}|t|^{k-1/2}\frac{L(F,\chi_t,k)}{\langle F, F\rangle}, \tag{0.0.6}$$

where $\nu(N)$ is the number of prime factors of N and $\langle , \rangle$ denotes the Petersson inner product.
Next, we obtain the theory of newforms on the Kohnen plus space $S^+_{k+1/2}(16N)$.

Theorem 0.0.4. *We have the following decompositions:*

$$S^+_{k+1/2}(16N) = S^{+,new}_{k+1/2}(16N) \oplus S^{+,old}_{k+1/2}(16N), \tag{0.0.7}$$

$$\begin{aligned} S^{+,old}_{k+1/2}(16N) = \left(\oplus_{\substack{rd|N\\ d<N}} S^{+,new}_{k+1/2}(16d)|U(r^2)\right) \oplus \left(\oplus_{\substack{d|N\\ rd|2N}} S^{+,new}_{k+1/2}(8d)|U(r^2)^*\right)\\ \oplus \left(\oplus_{\substack{d|N\\ rd|4N}} S^{+,new}_{k+1/2}(4d)|U(r^2)^*\right), \end{aligned} \tag{0.0.8}$$

where $U(r^2)^*$ is $U(r^2)$ if $2 \nmid r$, is equal to $U(r^2)B(4)$ if $2\|r$ and is equal to $U(r^2/16)B(4)$ if $4|r$.

Using the above decompositions of the spaces, we show that the dimensions of $S^{+,new}_{k+1/2}(16N)$ and $S^{new}_{k+1/2}(8N)$ are the same. Moreover, we prove that $S^{+,new}_{k+1/2}(16N)$ and $S^{new}_{2k}(4N)$ are isomorphic under a linear combination of Shimura-Kohnen maps.
For the full space $S_{k+1/2}(16N)$, we prove the following theorem.

Theorem 0.0.5. *The space $S_{k+1/2}(16N)$ contains no newforms in it and the whole space is obtained upon duplicating forms in $S_{k+1/2}(8N)$. The decomposition* *is given by,*

$$\begin{aligned}S_{k+1/2}(16N) = {} & \oplus_{rd|N} S^{+,new}_{k+1/2}(4d)|U(r^2) \oplus \oplus_{rd|N} S^{+,new}_{k+1/2}(4d)|U(4r^2)\\ & \oplus \oplus_{rd|N} S^{+,new}_{k+1/2}(4d)|U(4)L_+U(r^2) \oplus \oplus_{rd|N} S^{+,new}_{k+1/2}(4d)|U(8)W(8)U(r^2)\\ & \oplus \oplus_{rd|N} S^{+,new}_{k+1/2}(4d)|B(4)U(r^2) \oplus \oplus_{rd|N} S^{+,new}_{k+1/2}(4d)|U(4)B(4)U(r^2)\\ & \oplus \oplus_{rd|N} S^{new}_{k+1/2}(4d)|U(r^2) \oplus \oplus_{rd|N} S^{new}_{k+1/2}(4d)|B(4)U(r^2)\\ & \oplus \oplus_{rd|N} S^{new}_{k+1/2}(4d)|U(2)B(2)U(r^2) \oplus \oplus_{rd|N} S^{+,new}_{k+1/2}(8d)|U(r^2)\\ & \oplus \oplus_{rd|N} S^{new}_{k+1/2}(8d)|U(r^2) \oplus \oplus_{rd|N} S^{new}_{k+1/2}(8d)|U(2)B(2)U(r^2).\end{aligned} \tag{0.0.9}$$

Finally, we study the theory of newforms on $S_{k+1/2}(16N, \chi_2)$, where χ_2 is the even quadratic character modulo 8 given by $\left(\frac{2}{\cdot}\right)$. We prove the following result.

Theorem 0.0.6. *The space $S_{k+1/2}(16N, \chi_2)$ is decomposed as*

$$S_{k+1/2}(16N, \chi_2) = S^{new}_{k+1/2}(16N, \chi_2) \oplus S^{old}_{k+1/2}(16N, \chi_2),$$

where the above directsum is orthogonal with respect to the Petersson inner product and the decomposition of $S^{old}_{k+1/2}(16N, \chi_2)$ is given by

$$\begin{aligned}S^{old}_{k+1/2}(16N, \chi_2) = {} & \oplus_{rd|N} S^{+,new}_{k+1/2}(4d)|B(2)U(r^2) \oplus \oplus_{rd|N} S^{+,new}_{k+1/2}(4d)|U(2r^2)\\ & \oplus \oplus_{rd|N} S^{+,new}_{k+1/2}(4d)|U(8r^2) \oplus \oplus_{rd|N} S^{+,new}_{k+1/2}(4d)|U(8)W(8)U(r^2)\\ & \oplus \oplus_{rd|N} S^{new}_{k+1/2}(4d)|U(2r^2) \oplus \oplus_{rd|N} S^{new}_{k+1/2}(4d)|B(2)U(r^2)\\ & \oplus \oplus_{rd|N} S^{+,new}_{k+1/2}(8d)|B(2)U(r^2) \oplus \oplus_{rd|N} S^{new}_{k+1/2}(8d)|B(2)U(r^2)\\ & \oplus \oplus_{rd|N} S^{new}_{k+1/2}(8d)|W(8)U(r^2) \oplus \oplus_{\substack{rd|N\\ d<N}} S^{new}_{k+1/2}(16d, \chi_2)|U(r^2).\end{aligned} \tag{0.0.10}$$

Moreover, the spaces $S^{new}_{k+1/2}(16N, \chi_2)$ and $S^{new}_{2k}(8N)$ are isomorphic as Hecke modules.

3. Products of Eigenforms

The space of modular forms of a fixed weight on the full modular group has a basis of simultaneous eigenvectors for all the Hecke operators. A modular form is called an eigenform if it is a simultaneous eigenvector for all the Hecke operators. A natural question that arises is that whether the product of two eigenforms (may be of different weights) is an eigenform. This question was taken up by W. Duke [8] and E. Ghate [10]. They proved that there are only finitely many cases where this phenomenon occurs. A more general question (i.e., the Rankin-Cohen bracket of two eigenforms) was studied by D. Lanphier and R. Takloo-Bighash [25]. They showed that in this case also the phenomenon occurs in only finitely many cases. Recently, Beyerl et al. [4] have extended this result to a certain class of nearly holomorphic modular forms. In Chapter 3 of the thesis, we extend these results to some cases of quasimodular and nearly holomorphic eigenforms [M]. We consider the product of two quasimodular eigenforms and the product of nearly holomorphic eigenforms. Finally, we generalize the results of Ghate [11] to the case of Rankin-Cohen brackets.

(i) Quasimodular forms:

Let M_k and $\widetilde{M}_k$ denote respectively the space of modular forms and the space of quasimodular forms of weight k on $SL_2(\mathbb{Z})$. It is known that the differential operator $D = \frac{1}{2\pi i}\frac{d}{dz}$ takes $\widetilde{M}_k$ to $\widetilde{M}_{k+2}$. Let T_n be the n^{th} Hecke operator. The following result follows by straightforward verification.

Proposition 0.0.7. *If $f \in \widetilde{M}_k$, then $(D^m(T_n f))(z) = \frac{1}{n^m}(T_n(D^m f))(z)$, for $m \geq 0$. Moreover, we have $D^m f$ is an eigenform for T_n iff f is. In this case, if λ_n is the eigenvalue of T_n associated to f, then $n^m\lambda_n$ is the eigenvalue of T_n associated to $D^m f$.*

For $k \geq 2$, let E_k be the Eisenstein series of weight k, which is an eigenform. For $k \geq 4$, E_k is a modular form and E_2 is a quasimodular form. For $k \in \{12, 16, 18, 20, 22, 26\}$, let Δ_k denote the unique normalized cusp form of weight k on $SL_2(\mathbb{Z})$. Following the method of proof of Theorem 3.1 of [4], we prove the following result.

Theorem 0.0.8. *Let $f \in M_k$, $g \in M_l$. For $r, s \geq 0$, assume that $D^r f \in \widetilde{M}_{k+2r}$, $D^s g \in \widetilde{M}_{l+2s}$ are eigenforms. Then $(D^r f)(D^s g)$ is an eigenform only in the following cases.*

1. $E_4^2 = E_8,\quad E_4E_6 = E_{10},\quad E_6E_8 = E_4E_{10} = E_{14},\quad E_4\Delta_{12} = \Delta_{16},$
   $E_6\Delta_{12} = \Delta_{18},\quad E_4\Delta_{16} = E_8\Delta_{12} = \Delta_{20},$
   $E_4\Delta_{18} = E_6\Delta_{16} = E_{10}\Delta_{12} = \Delta_{22},$
   $E_4\Delta_{22} = E_6\Delta_{20} = E_8\Delta_{18} = E_{10}\Delta_{16} = E_{14}\Delta_{12} = \Delta_{26}.$

(*These are the modular cases given in [8] and [10]*)

2. $(DE_4)E_4 = \frac{1}{2}DE_8$

Using the properties of Bernoulli numbers, we prove the next result.

Theorem 0.0.9. *For $k \geq 2$ and $r, s \geq 0$, $(D^r E_2)(D^s E_k)$ is not an eigenform.*

As a consequence of the above theorem, we get the following corollary.

Corollary 0.0.10. *Let $f \in M_k$ be an eigenform. Then $(D^r E_2)f$ is an eigenform iff $r = 0$ and $f \in \mathbb{C}\Delta_{12}$.*

(ii) Nearly holomorphic modular forms:
Let $E_2^*(z) = E_2(z) - \frac{3}{\pi Im(z)}$. Then E_2^* is a nearly holomorphic modular form of weight 2 which is an eigenform. Using the same methods of proof as in the case of quasimodular forms, we prove the following theorem.

Theorem 0.0.11. *Let f be a normalized eigenform in M_k. Then $E_2^* f$ is an eigenform iff $f = \Delta_{12}$.*

(iii) Rankin-Cohen brackets of eigenforms:
Let $M_k(\Gamma_1(N))$ (respectively $M_k(N, \chi)$), $S_k(\Gamma_1(N))$, (respectively $S_k(N, \chi)$) and $\mathcal{E}_k(\Gamma_1(N))$ (respectively $\mathcal{E}_k(N, \chi)$) be the spaces of modular forms, cusps forms and Eisenstein series of weight $k \geq 1$ on $\Gamma_1(N)$ (respectively on $\Gamma_0(N)$ with character χ) respectively. We have an explicit basis $\mathcal{B}$ for $M_k(\Gamma_1(N))$ consisting of common eigenforms for all Hecke operators T_n with $(n, N) = 1$ as described in the work of Ghate [11]. An element of $M_k(\Gamma_1(N))$ is called an almost everywhere eigenform or a.e. eigenform in short, if it is a constant multiple of an element of $\mathcal{B}$.
We have an explicit basis of eigenforms of $\mathcal{E}_k(\Gamma_1(N))$ described in the following theorem (see for example [35], Theorems 4.7.1 and 4.7.2).

Theorem 0.0.12. *Let ψ_1 and ψ_2 be Dirichlet characters mod M_1 and mod M_2 respectively, such that $\psi_1\psi_2(-1) = (-1)^k$, where $k \geq 1$. Also assume that:*

- *if $k = 2$ and ψ_1 and ψ_2 both are trivial, then $M_1 = 1$ and M_2 is a prime number.*
- *otherwise, ψ_1 and ψ_2 are primitive characters.*

Put $M = M_1M_2$ and $\psi = \psi_1\psi_2$. Then there is an element $f = f_k(z, \psi_1, \psi_2) = \sum_{n=0}^{\infty} a_n q^n$ such that $L(s; f) = L(s, \psi_1)L(s-k+1, \psi_2)$, with explicit description of a_n, $n \geq 0$. The modular form $f_k(z, \psi_1, \psi_2) \in \mathcal{E}_k(M, \psi)$ is an eigenvector for the Hecke operators of level M. Modulo the relation $f_1(z, \psi_1, \psi_2) = f_1(z, \psi_2, \psi_1)$ when $k = 1$, the set of elements $f_k(Qz, \psi_1, \psi_2)$, where $QM_1M_2|N$ form a basis of $\mathcal{E}_k(N, \psi)$ consisting of common eigenforms of all the Hecke operators T_n of level N, with $(n, N) = 1$.

For $f \in M_{k_1}(N, \chi)$ and $g \in M_{k_2}(N, \psi)$, the m^{th} Rankin-Cohen bracket of f and g is defined by,

$$[f, g]_m(z) = \sum_{r+s=m} (-1)^r \binom{m+k_1-1}{s} \binom{m+k_2-1}{r} f^{(r)}(z) g^{(s)}(z),$$

where $f^{(r)}(z) = D^r f(z)$ and $g^{(s)}(z) = D^s g(z)$.

Following the methods of [11], we prove the following theorems.

Theorem 0.0.13. *Let $k_1, k_2 \geq 1$ be integers and $N \geq 1$ be a square-free integer and $m \geq 1$. If $g \in S_{k_1}(\Gamma_1(N))$ and $h \in S_{k_2}(\Gamma_1(N))$ are a.e. eigenforms, then $[g, h]_m$ is not an a.e. eigenform.*

Theorem 0.0.14. *Let k_1, k_2, k, m be positive integers such that $k = k_1 + k_2 + 2m$ and N be square-free.*

1. *Let $k_1 \geq 3$ and $k_2 \geq k_1 + 2 > 2$. Suppose that $g \in S_{k_1}(N, \chi)$ is an a.e. eigenform which is a newform and $h \in \mathcal{E}_{k_2}(N, \psi)$ is an a.e. eigenform. If dim $(S_k^{new}(N, \chi\psi)) \geq 2$, then $[g, h]_m$ is not an a.e. eigenform.*
2. *Let $k_1, k_2 \geq 3$, $|k_1 - k_2| \geq 2$. Let $g = f_{k_1}(z, \chi_1, \chi_2) \in \mathcal{E}_{k_1}(N, \chi)$ and $h = f_{k_2}(z, \psi_1, \psi_2) \in \mathcal{E}_{k_2}(N, \psi)$ be a.e. eigenforms as mentioned in Theorem 0.0.12 with χ and ψ primitive characters. If dim $(S_k^{new}(N, \chi\psi)) \geq 2$, then $[g, h]_m$ is not an a.e. eigenform.*

List of publications and preprints:

1. [KM] W. Kohnen and J. Meher, *Some remarks on the q-exponent of generalized modular functions*, Ramanujan J. **25**, (2011), no.1, 115-119.
2. [MMR] M. Manickam, J. Meher and B. Ramakrishnan, *Newforms of Half-integral weight on $\Gamma_0(8N)$ and $\Gamma_0(16N)$*, Preprint 2011.
3. [M] J. Meher, *Some Remarks on Rankin-Cohen Brackets of Eigenforms*, (Revised version submitted to Int. J. Number Theory).