Mathematical Physics is the most useful branch of Mathematics for the development of mathematical methods for application to the problems in Seismology, Earthquake and Civil Engineering, Acoustics, Applied Physics, Applied Mathematics.

Deformations produced are generally related to the applied loads and therefore to the stress tensor. These relations are called stress-strain relations or the constitutive equations of the material. By removing the load, material regains its original shape i.e. equilibrium state or natural state. This property is known as elasticity. It is the ability of the material to return to its original shape after removing the applied load. Elasticity can be explained by the Hooke’s law. In other words, amount of compression or stretching is directly proportional to the applied force.

Waves originate due to forced action of applied force in a portion of a deformable continuum. Wave disturbance transfers energy from one point to another due to their interaction pulling and pushing. Propagation of an elastic wave is associated with energy transfer. The idea about the energy flow carried by waves was introduced by the Russian physicist N.A. Umor (1846-1915) in the year 1874. Throwing a stone into water creates waves which propagate from the place of incidence. The amplitude of these waves decrease rapidly with depth, so that their energy propagates practically only in the superficial layer. Consequently, these waves are referred to as surface waves.

In the present thesis, wave propagation in non-homogeneous elastic rod is studied which can be useful for studying the dynamic behavior of rods. Also the propagation of surface waves (Love waves and Rayleigh waves) is the most fascinating field which is very useful in earthquake engineering and seismology. Some observations are also drawn by considering the earth as a composite material.

The present thesis entitled “A Mathematical Analysis of Wave Propagation Models” is divided into four chapters. A brief chapter wise description is given below.

Chapter-1 is an introductory chapter which comprises some basic concepts and laws related to elastic media. This chapter deals with fundamentals of linear theory, classification of materials, universal laws, types of waves, orthogonal curvilinear coordinate system. The classification of wave propagation problems with various assumptions is discussed. Some general methods used for calculating
boundary value problems are also discussed. It also consists of brief survey of the literature related to wave propagation.

**Chapter-2** introduces the effect of wave propagation in non-homogeneous isotropic thin elastic rod. For this study, a rod with elastic parameters depending on one space coordinate taken along the axis of the rod is considered by assuming Young’s Modulus $E = E_0 \cos \alpha x$ and density $\rho = \rho_0 \cos \alpha x$. Also, the body forces are omitted in this propagation of waves and the results for free vibrations are derived.

The general solution is obtained in terms of Legendre’s polynomials. Displacement patterns for the particular cases viz. modified sawtooth, square, triangular and half sine wave rectifier wave patterns as input are studied for time=2 sec, $\alpha = 0.01/m$ and $c_0 = \sqrt{E_0/\rho_0} = 3.5 \text{ m/sec}$. It has been observed that there are distortions from the original wave forms which are due to non-homogeneity. It has been found that the distortion in half sine wave rectifier input is minimum as compared to other three wave inputs.

**Chapter-3** deals with Love wave propagation due to a point source in non-homogeneous sandwiched layer between homogeneous upper half-space and non-homogeneous lower half-space. The rigidity and density of sandwiched layer and lower half space are assumed to vary exponentially as function of depth. Application of harmonic analysis converts the role of time factor into the frequency which is an important aspect for non-destructive dynamic testing mechanism. Here, body forces are taken in the form of Dirac-delta function.

The dispersion equation is obtained for the generated Love waves by using Fourier transform and Green’s function technique. The various curves are plotted for dispersion equation to show the effects non-homogeneity of layers on Love waves. It is found that dimensionless phase velocity of Love wave increases with the increase of non-homogeneity parameters whereas decreases with the increase of dimensionless wave number. Further, in the absence of non-homogeneity factors, the dispersion equation of Love waves reduces to the classical wave equation given by Ewing et. al. (1957).

**Chapter-4** is based on the surface wave propagation in composite materials with fiber-reinforcement in a preferred direction. The problem is being analyzed through wave propagation technique by harmonic sinusoidal motion and using Helmholtz theory of potentials for displacement vector field to study surface waves i.e. Love, Stoneley and Rayleigh waves in the neighborhood of the interface of two different media. Here, body forces are considered in the form of gravity and initial compression.
This chapter is further divided into two parts. **In part-1**, reinforcement is considered in the direction of wave propagation i.e. x-axis. Only gravity is taken as body force. **In part-2**, reinforcement is taken along the normal direction of wave propagation i.e. z-axis. Both gravity and initial compression are considered as body forces.

The results are drawn for Rayleigh waves and Love waves. For isotropic elastic medium, these results are in complete agreement with the corresponding classical results. Graphs are plotted for showing the effects of fiber-reinforcement and initial compression on Love wave propagation. It has been observed from the graphs that phase velocity (dimensionless) of love wave in fiber-reinforced materials decreases with the increase of dimensionless wave number. Inferences and analytical observations drawn show wide range of applications in industrial materials just as textile and paper industries as many of them deals composite materials with fiber-reinforcement.