CHAPTER I

INTRODUCTION

One of the most important branches of continuum mechanics is the classical theory of elasticity, which deals with the stresses and deformations in elastic solids produced by external forces or change in temperature. By elastic solids we mean that the material may be deformed and will return to its original configuration after release of the deforming loads. An elastic solid that undergoes only an infinitesimal deformation and for which the governing material law is linear is called linear elastic solid. The classical theory of elasticity serves as an excellent model for studying the mechanical behaviour of a wide variety of solid materials and is used extensively in civil, mechanical and aeronautical engineering design. This is the oldest established theory governing the behaviour of deformable solid materials, founded in its present form in the early nineteenth century. In the theory of linear elasticity, we are concerned with an ideal material governed by Hooke’s law, which represents a linear relationship between the stress and strain. Hooke’s law has influenced the scientific thoughts for a considerably long period for the classical linear infinitesimal theory of elasticity and its results agreed with experiments quite well.

Deformation in earth takes place due to the stresses developed in it. The sudden release of stresses in some part of the earth causes an
earthquake. The sudden release of energy following an earthquake or underground explosion propagates through the earth in the form of waves called elastic waves, which are transmitted in the earth with definite velocities depending on the density and the elastic parameters of the materials in the earth. The elastic waves are mainly of two types; body waves and surface waves. The waves, which penetrate deep into the interior of earth, are known as body waves whereas the waves, which are restricted to the neighbourhood of the free surface or at the interface, are called surface waves.

Poisson (1829) was the first who showed theoretically the existence of body waves. He found that two types of body waves propagate with different speeds in a homogeneous, isotropic, infinite elastic solid. The motion is longitudinal in the quicker waves and transverse in the slower waves. Later, Stoke (1849) identified Poisson’s quicker wave as P-wave or dilatational wave propagating with velocity \( \left( \frac{\lambda + 2\mu}{\rho} \right)^{\frac{1}{2}} \) and the slower wave as S-wave or rotational wave or equivoluminal waves propagating with velocity \( (\mu/\rho)^{1/2} \), where \( \lambda \) and \( \mu \) are Lame’s constants and \( \rho \) is the density of the medium. The S-waves are polarized into SV and SH waves. SV waves are shear waves when the particle motion is in the vertical direction and SH
waves are shear waves when the particle motion is in the horizontal direction.

The thermoelasticity was stimulated by the various engineering sciences. A remarkable progress in the field of aircraft and machine structure have given rise to numerous problems in which thermal stresses play a role of primary importance. It comprises the heat conduction and stress and strain that arise due to the flow of heat. Thermoelasticity makes it possible to determine the stresses produced by the temperature field and to calculate the temperature distribution due to action of time dependent forces and heat sources. It is desirable for all elasto-dynamic problems to consider the temperature’s dependence of displacements giving rise to coupled thermoelastic equations.

The assumptions that are usually made are:

(i) the deformation is very small,

(ii) the materials behave elastically at all times and in the same manner in all directions.

(iii) the temperature field is determined by taking into considerations, the effect of coupling of temperature and strain fields. Temperature field is always dependent on the deformation.

The coupling theory between the strain and temperature fields was studied by Duhamel (1838) who derived equations for the distributions of
strains in an elastic medium subjected to temperature gradients. He introduced the dilatation terms in the equation of thermal conductivity but his equation was not a thermodynamically satisfactory basis. Fundamental studies of the thermodynamics of an elastic solid were not made until Biot (1956) gave a satisfactory derivation of the equation of thermal conductivity, which includes the dilatation terms based on the thermodynamics of irreversible process. The coupling between thermal and strain fields gives rise to the coupled theory of thermoelasticity. The theory of coupled thermoelasticity was formulated by Biot to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. Unfortunately, the heat equations for any of the two theories, though different, are of the diffusion type predicting infinite speeds of propagation for heat waves contrary to physical observations.

The governing equations of thermoelasticity in the usual framework of linear coupled thermoelasticity consist of the wave type (hyperbolic) equations of motion and the diffusion type (parabolic) equation of heat conduction. It is seen that a part of the solution of the energy equation tends to infinity. If an isotropic homogeneous, elastic material is subjected to thermal or mechanical disturbances, the effects in the temperature and displacement field are felt at an infinite distance from the source of disturbance. This implies that a part of the disturbance has an infinite
velocity of propagation, which is physically impossible. To remove this paradox of infinite velocity, various theories of generalized thermoelasticity were developed in past three decades. In contrast to the conventional theories based on parabolic-type heat equation, these theories involve hyperbolic type heat equation. Lessen (1956) studied the thermoelasticity and thermal shock. He derived the thermoelastic stress-strain equations and showed that an additional term was required in heat conductivity equation. The thermal shock problem was solved assuming that dynamic terms can be neglected. Lockett (1958) studied the effect of thermal properties of a solid on the velocity of Rayleigh waves. This note was concerned with the propagation of Rayleigh waves in an isotropic thermoelastic solid. It was found that, within the frequency range normally attainable, the velocity of propagation of these waves can be determined from the classical equation merely by replacing the parameter $\beta^2 = (\lambda^2 + 2\mu)/\mu$ occurring in that equation by $(1 + \varepsilon)\beta^2$ where $\varepsilon = \gamma^2 v_p^2 T(1 + \nu)^2/c(1 - \nu)^2$, $\gamma$ is the coefficient of linear expansion, $v_p$ is the velocity of purely elastic longitudinal waves in the solid, $T$ is the absolute temperature of the solid in its interface state of uniformly zero stress and strains, $c$ is specific heat at constant strain, $\nu$ is the Poisson’s ratio. A typical case was examined numerically and it was found that taking into account the thermal properties of the solid produces a difference of less than one percent in the velocity and amplitude of the Rayleigh waves.
Goodier (1958) studied the formula for overall thermoelastic deformation. On the basis of Betti reciprocal theorem generalized to thermoelastic problems, formulas were found for volume change of an arbitrary body and of a cavity, as well as for mean extension, flexural rotation, terminal deflection and torsional rotation of a bar.

Lord and Shulman (1967) incorporated a heat flux-rate term into Fourier's law to formulate a generalized theory with one relaxation time that admits finite speed for thermal signals. The generalized equation of heat conduction is hyperbolic and hence, automatically ensures finite speeds of wave propagation.

Another generalization to the coupled theory of thermoelasticity is what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature rate dependent thermoelasticity. Müller (1971) in a review of thermodynamics of thermoelastic solids, proposed an entropy production inequality with the help of which, he considered restrictions on a class of constitutive equation. A generalization of this inequality was proposed by Green and Laws (1972), Green and Lindsay (1972), by including temperature rate among the constitutive variables. They have developed a temperature-rate dependent thermoelasticity, which does not violate the classical Fourier's law of heat conduction, when the body under
consideration has a centre of symmetry, and this theory also predicts a finite speed for heat propagation.

Green and Naghdi (1993) proposed thermoelasticity without energy dissipation, in which, in comparison to the classical theory, the Fourier law was replaced by a heat flux rate-temperature gradient relation. Hetnarski and Ignaczak (1996) introduced low temperature thermoelasticity, in which, in comparison to the classical theory, both the free energy and heat flux depend not only on the temperature and the strain tensor but also on an elastic heat flow that satisfies a non-linear evolution equation. Chandersekhariah and Tzoû (1995) introduced dual-phase-lag thermoelasticity, in which the Fourier law was replaced by an approximation to a modification of the Fourier law with two different time translations for the heat flux and the temperature gradient.

Hetnarski and Ignaczak (2000) worked out a survey of five different thermoelastic models in which disturbances were transmitted in a wave-like manner: (a) Lord and Shulman (L-S) model; (b) Green and Lindsay (G-L) model; (c) Hetnarski and Ignaczak (H-I) model; (d) Green and Naghd (G-N) model; (e) Chandrasekhariah and Tzou(C-T) model. Except for H-I model, which is strongly non-linear and applicable at low temperatures, the remaining models are linear. For the H-I model a discussion on the fast moving soliton like thermoelastic waves is provided, while for the L-S, G-L,
and G-N models suitable domain of influence results were formulated. The C-T model was treated as an extension of the L-S model in which both a phase-lag of the heat flux and a phase-lag of the temperature gradient are present.

Puri (1972) discussed on exact solution of the frequency equation for plane thermoelastic waves. The solutions for large and small frequencies and for small coupling can be obtained by approximate expansions. The range of validity of approximate solutions for large and small frequencies was also given and numerical verification was obtained.

Puri (1973) discussed the plane waves in generalized thermoelasticity. The properties of two dilatational motions in the context of generalized thermoelasticity were studied. The exact solution to the frequency equation was given and the exact values of real and imaginary parts of the wave number were calculated. Approximate representations of this solution were derived for large and small frequencies, along with the ranges of validity. In this context the wave motions were unchanged for small frequencies, but both were modified at large frequencies. The major modifications concerned only the thermal wave: the phase velocity approaches a constant value as frequency increases without limit; and the specific loss does not have any local maxima. Finally, the behavior of amplitude ratios for small and large frequencies was discussed and the results obtained were compared with
those of earlier investigations in coupled thermoelasticity. Bosch (1974) discussed on the plane strain in a generalized theory of thermoelasticity. The discussion was concerned with the plane strain in the linear theory of generalized thermoelasticity proposed by Green and Lindsay (1972), using the associated matrices method, a representation of Galerkin type was given. This representation was used to derive the solution of the vibration problem corresponding to concentrated body forces and concentrated heat source in an infinite medium.

Sinha and Sinha (1974) studied the reflection of thermoelastic waves at a solid half-space with thermal relaxation. The thermal and elastic plane wave motion of small amplitude in a homogeneous isotropic and thermally conducting solid that occupies a half-space was considered. The presence of the thermal waves effect change in the angle of emergence. Chandrasekharai (1981) studied the wave propagation in a thermoelastic half-space. One-dimensional dynamical disturbances in a thermoelastic half-space with plane boundary due to the application of a step in strain or temperature on the boundary were studied in the context of the linearized Green-Lindsay (1972) thermoelasticity theory. The solution was obtained by the use of integral transforms. Short and long time approximations of solutions were deduced and the exact discontinuities in the mechanical and
thermal fields were analysed. Some of the earlier results were deduced as particular cases of the more general results obtained here.

Jurczyk and Klemans (1985) studied the thermoelastic attenuation of Rayleigh waves. Thermoelastic attenuation of Rayleigh waves was larger than that of bulk waves because heat was conducted not only in the propagation direction, but also normal to the surface. This attenuation was further enhanced if the thermal expansivity is a function of depth below the surface. Expressions were obtained for the attenuation and some experimental data on Rayleigh wave attenuation was discussed.

Chand and Sharma (1988) discussed transient magneto-thermoelastic waves in a half-space with thermal relations. The distribution of temperature, deformation and magnetic field in a homogeneous isotropic, thermally and perfectly electrically conducting elastic half-space, in contact with the vacuum, had been investigated by taking (i) a step in stress and (ii) a thermal shock at the plane boundary, in the context of Green-Lindsay theory of thermoelasticity. The Laplace transform on time had been used to obtain the solutions. Because the "second sound" effects are short-lived, so the small time approximations had been considered. The deformation and temperature were found to be continuous at the wave fronts whereas the magnetic field was found to be discontinuous in the case of normal load; but these quantities were discontinuous in the case of thermal shock. Noda,
Furuk-Awa and Ashida (1989) discussed the generalized thermoelasticity in an infinite solid with a hole.

Sharma and Chand (1993) discussed the propagation of waves in rotating magneto-thermoelastic media. The work was aimed at presenting the distribution of deformation, temperature, perturbed magnetic field and stresses in vacuum as well as in the elastic medium due to thermal shock, acting on the plane boundary; the solutions were derived by formulating a generalized thermoelastic theory that combines both the theories developed by Lord and Shulman as well as by Green and Lindsay. The Laplace transform technique has been used to obtain the short time solutions. The theoretical results obtained had been verified numerically and were represented graphically for the case of carbon steel material. Chandrasekharaiah (1996) studied the thermoelastic plane waves without energy dissipation. Chandrasekharaiah and Srinath (1996) discussed the one-dimensional waves in a thermoelastic half-space without energy dissipation. The theory of thermoelasticity without energy dissipation was employed to study one-dimensional disturbances in a half-space with rigid plane boundary. The disturbances were supposed to be due to a constant step in temperature applied to the boundary. The Laplace transform method was employed to solve the problem. Exact expressions for displacement, temperature and stress fields were obtained. The characteristic features of
the underlying theory were analysed by comparing these expressions with their counterparts in other generalized thermoelasticity theories. Chandrasekharaiah (1997a) discussed the thermoelastic Rayleigh waves without energy dissipation. Chandrasekharaiah and Srinath (1997) discussed the thermoelastic plane waves without energy dissipation in a rotating body. Chandrasekharaiah (1997b) also studied the complete solutions in the theory of thermoelasticity without energy dissipation.

Cukic and Trajkovski (1997) studied the plane thermoelastic waves in infinite half-space caused by instantaneous pressure forces on its surface. The propagation of longitudinal thermoelastic waves in a half-space caused by mechanical influences was considered. One-dimensional dynamic problem of coupled thermoelasticity was solved by using integral transforms. Exact and approximate expressions for calculation of thermo-mechanical values at each point of a half-space and at arbitrary moment of time were given. Qualitative and quantitative analysis of the effects of damping and dispersion of thermoelastic wave was performed.

Sharma et al (2000) studied the propagation of thermoelastic waves in homogeneous isotropic plates. They considered isotropic thermoelastic plate of thickness ‘d’ in the context of coupled theory of thermoelasticity. The effect of stress free insulated or isothermal and rigid insulated or isothermal boundaries on the wave propagation had been studied in addition to the
thermo-mechanical coupling phenomenon. The results of uncoupled thermoelasticity were also deduced at appropriate stages. The phase velocities of purely transverse (SH) modes, which get decoupled from rest of motion, were also obtained. The effect of plate thickness on the symmetric and skew symmetric modes of vibration was also studied. It was observed that the plates subjected to rigid or stress free isothermal boundaries admit similar modes of shear, symmetric and skew symmetric wave propagation. The theoretical results obtained for symmetric and skew symmetric modes of wave propagation in various cases had been verified numerically and illustrated graphically for aluminium-epoxy material.

Roychoudhuri and Mukhopadhyay (2000) studied the effects of rotation and relaxation times on plane waves in generalized thermoviscoelasticity. The generalized dynamical theory of thermo-elasticity proposed by Green and Lindsay was applied to study the propagation of harmonically time dependent thermo-visco-elastic plane waves of assigned frequency in an infinite visco-elastic solid of Kelvin-Voigte type, when the entire medium rotates with a uniform angular velocity. A more general dispersion equation was deduced to determine the effects of rotation, visco-elasticity and relaxation time on the phase-velocity of the coupled waves. The solutions for the phase velocity and attenuation coefficient were obtained for small thermoelastic couplings by the perturbation technique. Taking an
approximate material, the numerical values of the phase velocity of the waves were computed and results were shown graphically to illustrate the problem.

Abd-Alla and AL-Dawy (2000) discussed the reflection phenomena of SV-waves in a generalized thermoelastic medium. They discussed the reflection of thermoelastic plane waves at a solid half-space nearby a vacuum. They used the generalized thermoelastic waves to study the effects of one or two thermal relaxation times on the reflection plane harmonic waves. They considered the thermal and elastic waves of small amplitudes in a homogeneous, isotropic and thermally conducting elastic solid. The expressions for the reflection coefficients, which are the ratio of the amplitudes of the reflected waves to the amplitude of the incident waves, were obtained. It was shown analytically that the elastic wave was modified due to the thermal effect. The reflection co-efficients of a shear wave incident from within the solid on its boundary, which depend on the thermoelastic coupling factor and included the thermal relaxation times, had been found in the general case. The numerical values of reflection coefficients against the angle of incidence for different values of thermal relaxation times had been calculated and results were given in the form of graphs. Some special cases of reflection had also been discussed, for
example, in the absence of thermal effects their results reduced to the ordinary pure elastic case.

Chandrasekharaiha and Srinath (2000) studied the thermoelastic waves without energy dissipation in an unbounded body with a spherical cavity. The linear theory of thermoelasticity without energy dissipation was employed to study waves emanating from the boundary of a spherical cavity in a homogeneous and isotropic unbounded thermoelastic body. The waves were supposed to be spherically symmetric and caused by a constant step in temperature applied to the stress-free boundary of the cavity. Small-time solutions for the displacement, temperature and stress fields were obtained by using the Laplace transform technique. It was found that there exist two coupled waves, of which one was predominantly elastic and other was predominantly thermal, both propagating with finite speeds but with no exponential attenuation. Exact expressions for discontinuities in the field functions that occur at the wave fronts were computed and analysed. The results were compared with those obtained earlier in the context of some other models of thermoelasticity.

Singh (2000) studied the wave propagation in heat-flux dependent generalized thermoelasticity. The study was concerned with the reflection and refraction of thermoelastic waves at an interface between two dissimilar heat-flux dependent generalized thermoelastic solid half-spaces. The
amplitude ratios of various reflected and refracted waves with the angle of incidence for Green-Lindsay and Lord-Shulman theories have been obtained. The variations of these amplitude ratios had been shown graphically for Green-Lindsay and Lord-Shulman theories. Some special and particular cases had been discussed. Sharma, Singh and Kumar (2000) discussed the generalized thermoelastic waves in homogeneous isotropic plates. The propagation of thermoelastic waves in homogeneous isotropic plate subjected to stress-free and rigid insulated and isothermal conditions was investigated in the context of conventional coupled thermoelasticity (CT), Lord-Shulman (LS), Green-Lindsay (GL), and Green Naghadi (GN) theories of thermoelasticity. Secular equations for the plate in closed form and isolated mathematical conditions for symmetric and skew-symmetric wave mode propagation in completely separate terms were derived. It was shown that the motion for SH modes gets decoupled from the rest of the motion and remains unaffected due to thermo-mechanical coupling and thermal relaxation effects. The phase velocities for SH modes have been obtained. The results for coupled and uncoupled theories of thermoelasticity have also been obtained as particular cases from the derived secular equations. At short wavelength, limits the secular equations for symmetric and skew symmetric waves in a stress-free insulated and isothermal plate reduce to Rayleigh surface waves frequency equations. Finally, the
numerical solution was carried out for aluminium-epoxy composite material and the dispersion curves for symmetric and skew symmetric wave modes were presented to illustrate and compare the theoretical results.

Abd-alla (2000) studied the relaxation effects on reflection of generalized magneto-thermoelastic waves, considering the elastic, electromagnetic, and thermal fields. Each of these fields contributes to the total deformation of the body and interacts with each other. Reflection of generalized magneto-thermoelastic waves was employed to study the effects of the two thermal relaxation times and the magnetic field on the reflection plane harmonic waves of a semi-infinite elastic solid nearby a vacuum. The expressions for the reflection coefficients were obtained. The thermal relaxation times, the magnetic and thermal effects on the reflection coefficients were studied by comparing the results with their counterparts in the following cases: (i) in the absence of the thermal relaxation times, (ii) when there is no magnetic field and (iii) in the absence of the thermal effect. Finally, a numerical solution in the case metal aluminium was found and the results were presented graphically.

Zimmerman (2000) discussed the coupling in poroelasticity and thermoelasticity and gave a brief derivation of the equations of linearised poroelasticity and thermoelasticity. Dimensionless parameters were presented that quantify the strength of the coupling between mechanical and
hydraulic (or thermal) effects. The thermoelastic coupling parameter was usually very small, so that, although the temperature field influences the stresses and strains, the stresses and strains did not appreciably influence the temperature field.

Sharma (2001) studied the propagation of thermoelastic waves in homogeneous isotropic plate subjected to stress free and rigid insulated and isothermal conditions, was investigated in the context of conventional coupled thermoelasticity (CT), Lord-Shulman (LS), Green-Lindsay (GL) and Green-Nagdhi (GN) theories of thermoelasticity. Secular equations for the plate in closed form and isolated mathematical conditions for symmetric and anti-symmetric wave mode propagation in completely separate terms were derived. It was shown that the motion for purely transverse (SH) modes get decoupled from rest of the motion and remain unaffected due to thermo-mechanical coupling and thermal relaxation effected. The phase velocities for SH modes were also obtained. The results for coupled and uncoupled thermoelasticities were obtained as particular cases from the derived secular equations. At short wave lengths limits, the secular equations for symmetric and skew symmetric waves in a stress free insulated and isothermal plate reduced to Rayleigh surface waves frequency equations because a finite plate in such situations behave like a semi-infinite medium. Finally, the numerical solution was carried out for aluminium-epoxy
composite material and the dispersion curves for symmetric and antisymmetric wave modes were presented graphically to illustrate and compare the theoretical results. The theory and numerical computations were found to be in close agreement.

El-Karamany (2002) studied uniqueness and reciprocity theorems in generalized linear micropolar thermoviscoelasticity. A model of the equations of generalized linear micropolar thermoviscoelasticity was given. The formulation was applied to the coupled theory as well as to five generalizations, the Lord-Shulman theory with one relaxation time, the Green-Lindsay theory with two relaxation times, the Green-Naghdi theories of type-II (without energy dissipation) and of type-III, and the Chandrasekharaih-Tzoū theory with dual-phase-lag. Using Laplace transforms, a uniqueness theorem for that model was proved, restrictions on relaxation functions were deduced and the dynamic reciprocity theorem was derived. The cases of generalized linear micropolar thermoviscoelasticity of Kelvin-Voigt model, generalized linear micropolar thermoelasticity, generalized thermoviscoelasticity and generalized thermoelasticity could be obtained from the given general mode.

Sharma et al (2003) studied the reflection of generalized thermoelastic waves from the boundary of a half-space. They investigated the problem of thermoelastic wave reflection from the insulated and isothermal stress-free
as well as rigidly fixed boundaries of homogeneous isotropic solid half-space in the context of various linear theories of thermoelasticity, namely, Lord-Shulman, Green-Lindsay, Green-Naghadi, coupled thermoelasticity, and uncoupled thermoelasticity. The ratios of reflection coefficients to that of incident coefficients were obtained for P and SV-wave incidence cases. The results for partition of the energy for various values of the angle of incidence were computed numerically and presented graphically for aluminium-epoxy composite material in case of incident P and SV-waves and the results were discussed and compared in various models of thermoelasticity. Abd-alla et al. (2003) discussed on the reflection of the generalized magneto-thermo-visco-elastic plane waves at the boundary of a semi-infinite solid. The free surface of the solid was considered to be adjacent to vacuum. They assumed that the solid was subjected to a constant temperature and magnetic field. When a rotational wave was incident from within the solid on its boundary, the expressions for the reflections coefficients of the waves generated at the boundary were obtained. The assumptions that the body was a perfect conductor and the choice that the visco-elastic model is of Kelvin-Voigt type were considered. The effects of the viscosity, the applied magnetic field and thermo-elastic coupling factor on the reflection co-efficients were studied. The numerical calculations of the absolute values of the reflection coefficients and their relative changes
for the Lord-Shulman and Green-Lindsay in generalized thermoelastic models were presented graphically.

Sharma and Pal (2004) discussed the Rayleigh-Lamb waves in magneto-thermoelastic homogeneous isotropic plate. The propagation of magneto-thermoelastic plane waves in an initially unstressed, homogeneous isotropic, conducting plate under uniform static magnetic field has been investigated. The generalized theory of thermoelasticity was employed, by assuming the electrical behaviour as quasi-static and the mechanical behaviour as dynamic, to study the problem. The secular equations for both symmetric and skew symmetric waves have been obtained. The magneto-elastic shear horizontal (SH) mode of wave propagation got decoupled from rest of the motion and it was not influenced by thermal variations and thermal relaxation times. At short wavelength limits, the secular equations for symmetric and skew-symmetric modes reduced to Rayleigh surface wave frequency equation, because a finite thickness plate in such a situation behaves like a semi-infinite medium. Thin plate results were also deduced at the end. Dispersion curves were represented graphically for various modes of wave propagation in different theories of thermoelasticity. The amplitudes of displacement, perturbed magnetic field and temperature change were also obtained analytically and computed numerically. The result in case of elastokinetics, magneto-elasticity and coupled magneto-
elasticity has also been deduced as special cases at appropriate stages of this work. Knowles (2004) discussed on shock waves in a special class of thermoelastic solids. He described a thermoelastic model for shock waves in uniaxial strain based on a subclass of the so-called materials of Mie-Grüneisen type. He compared the Hugoniot curve with the isotherms and isentropes for this model, and constructed the shock-wave solution to a simple impact problem.

Singh (2005) studied the reflection of P and SV-waves from free surface of an elastic solid with generalized thermodiffusion. The governing equations for generalized thermodiffusion in an elastic solid were solved. There exist three kinds of dilatational waves and a shear vertical (SV) wave in a two-dimensional model of the solid. The reflection phenomenon of P and SV wave from free surface of an elastic solid with thermodiffusion was considered. The boundary conditions were solved to obtain a system of four non-homogeneous equations for reflection coefficients. These reflection coefficients were found to be depending upon the angle of incidence of P and SV-waves, thermodiffusion parameters and other material constants. The numerical values of modulus of the reflection coefficients were presented graphically for different values of thermodiffusion parameters. The dimensional velocities of various plane waves were also computed for different material constants.
Garbin and Knopoff (1973) discussed the compressional modulus of a material permeated by a random distribution of circular cracks. They considered the solutions to the problem of the propagation of compressional elastic waves through a medium permeated by a random distribution of randomly oriented circular cracks. The cracks are sparsely distributed and have radii small compared with the wavelength. O'Connell and Budinsky (1974) calculated the seismic velocities in dry and saturated cracked solids. The elastic moduli of a solid permeated with an isotropic distribution of flat cracks have been calculated from the energy of a single crack by use of a self-consistent approximation. The results were applicable for a dense network of cracks and give physically reasonable results up to the point that the shear modulus vanishes. Garbin and Knopoff (1975a) calculated the shear modulus of a material permeated by a random distribution of free circular cracks. Garbin and Knopoff (1975b) also calculated the elastic modulii of a medium with liquid filled cracks. Budinansky and O'Connell (1976) obtained the elastic moduli of a cracked solid by self-consistent method. O'connell and Budionsky (1977) discussed the viscoelastic properties of fluid-saturated cracked solids. Relaxation of shear stresses in viscous fluid inclusions also results in dissipation. Viscoelastic moduli are derived, by using a self-consistent approximation that describes the complete range of behaviour. There are two characteristic frequencies near which
dissipation is largest and moduli change rapidly with frequency. The first corresponds to fluids flow between cracks and its value can be estimated from the crack geometry or permeability. The second corresponds to the relaxation of shear stress in an isolated viscous fluid inclusion, its value may also be estimated.

Hudson (1981) studied the wave speeds and attenuation of elastic waves in material containing cracks. Expressions now exists from which may be calculated the propagation constants of elastic waves travelling through material containing a distribution of cracks. The cracks are randomly distributed in position and may be randomly oriented. The wavelengths involved are assumed to be large compared with the size of the cracks and with their separation distances so that the formulae, based on the mean taken over a statistical ensemble, may reasonably be used to predict the properties of a single sample. The results are valid only for small concentrations of cracks.

Explicit expressions, correct to lowest order in the ratio of the crack size to a wavelength, are derived here for the overall elastic parameters and the overall wave speeds and attenuation of elastic waves in cracked materials where the mean crack is circular, and the cracks are either aligned or randomly oriented. The cracks may be empty or filled with solid or fluid
material. These results are achieved on the basis of simply the static solution for an ellipsoidal inclusion under stress.

Crampin (1987) discussed the basis for earthquake prediction. He said that most of the earth's crust is pervaded by distributions of fluid-filled cracks and micro cracks that are aligned by the contemporary stress field so that the cracked rock mass is effectively anisotropic to seismic waves. This causes shear-waves to split and shear-wave splitting is observed whenever shear-waves propagate along suitable ray paths in the crust are recorded by three-component instruments. These distributions of cracks are known as extensive-dilatancy anisotropy. Many characteristics of the crack and stress geometry can be monitored by analyzing shear waves propagating through the cracked rock mass. Observations of temporal variations of the behaviour of shear-wave splitting in seismic gaps confirm these hypotheses and suggest that stress change before earthquakes may be monitored by analyzing shear-waves. In particular, monitoring earthquake preparation zones with three component shear-wave vertical-seismic-profiles could lead to techniques for the routine predictions of earthquakes.

Xu and King (1990) discussed the attenuation of elastic waves in a cracked solid. The spectral ratios technique is used to measure the attenuation and phase dispersion of the compressional wave and two shear waves polarized parallel and perpendicular to the cleavage of a slate, before
an after cracks have been induced in the cleavage plane. The experimental results show that the quality factor $Q$ of the rock sample is affected significantly by the presence of cracks, and that $Q$ is more sensitive to crack parameters than the corresponding wave velocity. Hudson (1990a) discussed the overall elastic properties of isotropic materials with arbitrary distribution of circular cracks. The transmission properties of the mean field in elastic material with a random distribution of circular cracks of small aspect ratio are presented for the general case where the crack normal are distributed in any pre-determined way in space. Special distributions, where the crack normal lie in one direction only, or lie at a fixed angle to a fixed direction, reproduce established results. All results are valid to second order in the crack number density. Peacock and Hudson (1990) discussed the seismic properties of rocks with distributions of small cracks. Angular variation of velocity and attenuation due to parallel and nearly parallel cracks and cracks with normal at a fixed angle to a given axes were discussed. Anisotropy gradually decreases as the distribution of orientations is widened from exactly parallel to random. Increasing the aspect ratio of thin saturated cracks generally causes to elastic properties to approach those due to dry cracks. For fluid-filled cracks the line singularity where the birefringent shear waves have equal velocities moves closer to the symmetry axis as either the distribution of orientations is widened or the aspect ratio is
increased. Calculated attenuations due to Rayleigh scattering and viscous
dissipation in crack-filling fluids are many orders of magnitude less than
reported attenuation in the earth. Viscous dissipation varies linearly with
frequency and aspect ratio of thin fluid-filled cracks. A power-law
distribution of crack radii, intended to represent that found in the earth’s
crust has no effect on the angular variations of seismic velocities or viscous
attenuations but scattering attenuation is sensitive to the power-law
exponent. Lognormal distributions of aspect ratio, also corresponding to
observations in real rocks, affects velocity as well as attenuation. Hudson
(1990b) also discussed the attenuation due to second-order scattering in
material containing cracks. The method of smoothing has lead to the
calculation of overall or effective elastic parameters for wave propagation in
material containing cracks, valid to second order in the number density of
cracks. Wave speeds are obtained for wavelengths long compared with crack
dimensions by working to the lowest order in frequency. To find the
attenuation due to scattering of energy out of the mean wave, calculation to
higher order in frequency are necessary and up to now, attenuation has been
obtained by summing over the scattering cross-sections of the cracks, thus
neglecting any crack-crack interactions. Here we evaluate scattering
attenuation to higher order in the series obtained from the smoothing
approximation in order to allow for multiple scattering. It turns out that, for
crack radii and crack spacing small compared with a wavelength, the term of lowest order taken from the double scattering component exactly cancels out the sum over scattering cross-sections, leaving only higher order terms to account for attenuation due to scattering. In other words, the effective material parameters contain no attenuation component arising from scattering.

Liu et al. (1995) discussed the seismic properties of a general fracture. In modelling the wave behaviour through fractured and jointed rocks, different models have been proposed to describe the fractures. A fracture can be modelled (1) as a parallel-walled thin layer, or (2) as a planar array of distributions of small cracks or voids (rough surface), or (3) empirically as a linear slip interface. For the first two cases, approximate boundary conditions can be derived and it is found that they are similar to the third empirical linear slip interface model. The results according to exact elastodynamic theory had been compared with solutions based on the approximate boundary conditions. Good agreement was obtained. Consequently these three models could be cast into a universal form. The relatively simple boundary conditions, that have been derived, could be used to model the much more complicated microstructures of natural fractures, joints, and faults in the earth.
Hudson et al. (1996) studied the mechanical properties of materials with interconnected cracks and pores. They studied the effect on the overall properties of a cracked solid of the existence of connections between otherwise isolated cracks and of small-scale porosity within the 'solid' material. The intention was to provide effective medium models for the calculation of elastic wave propagation with wavelengths greater than the dimensions of the cracks. The method follows that of earlier papers in which the overall elastic properties were directly related to parameters governing the microstructure, such as crack number, density and the mean radius and spacing distance of the cracks. Expressions derived by the method of smoothing were evaluated to second order in the number density of cracks, thereby incorporating crack-crack interactions through both the strain field in the solid and the flow field of fluids in the pores. Flow of interstitial liquids tends to weaken the material; the limit of zero flow is equivalent to isolating the cracks and the limit of free flow is equivalent to dry (gas-filled) cracks. It also introduces additional attenuation. The inclusion of small-scale porosity gives a model of 'equant porosity' which is more closely constrained by the details of crack dynamics than earlier models.

Craster (1998) discussed the scattering by cracks beneath fluid-solid interfaces. The scattering of incident plane elastic, or fluid, body waves and
interfacial waves by an arbitrary orientated subsurface crack was considered. The crack lies in an infinite elastic half-space that is coupled to an overlying fluid half-space. Material parameters relevant for water-metal and water-rock combinations were taken and far field scattering patterns were given; these demonstrate beam formation along critical angles. For light or moderate, fluid coupling, it was shown that the beams form along different critical angles depending upon the magnitude of the coupling. In addition, reciprocity relations relating the far field scattering coefficients viewed along an angle $\theta$, and generated by one type of plane wave incident along $\phi$, to the scattering coefficient viewed along $\phi$, generated by another plane wave incident along $\theta$, were found. Reciprocity relations involving interfacial waves were also given. Power flow theorems were derived; these relate the time averaged scattered power to a combination of far field scattering coefficients. This was used to determine the proportion of scattered power converted into the different types of scattered wave. The reciprocity and power flow theorems provide a powerful consistency check upon the numerical accuracy of the results. The boundary value problem was recast as a system of coupled integro-differential equations for the unknown jump in displacement across the crack faces. These integral equations were solved in an efficient and fast numerical manner by
performing the integrations over the crack faces analytically, thus reducing the computational effort substantially.

Hudson et al (2001) studied the effective-medium theories for fluid-saturated materials with aligned cracks. There is general agreement between different theories giving expressions for the overall properties of materials with dry, aligned cracks if the number density of cracks is small. There is also very fair agreement for fluid filled isolated cracks. However, there are considerable differences between two separate theories for fluid-filled cracks with equant porosity. Comparison with recently published experimental data on synthetic sandstones gave a good fit with theory for dry samples. However, although the crack number density in the laboratory sample was such that first-order theory was unlikely to apply, expressions correct to second order (in the number density) provide a worse fit. It also appeared that the ratio of wavelength to crack size was not sufficiently great for any detailed comparison with effective-medium theories, which were valid only when this ratio was large. The data show dispersion effects for dry cracks and scattering, neither of which would occur at sufficiently long wavelengths. Data from the water-saturated samples indicated that the effect of equant porosity was significant, although the two theories differ strongly as to just how significant. Once again, and in spite of the reservations mentioned above, a reasonable fit between theory and observation could be

Capuani and Willis (1999) discussed the wave propagation in elastic media with cracks. The transient dynamic response of an elastic medium containing a random array of aligned penny-shaped cracks was studied. Non-linearity in the response due to contact between crack faces during the motion was taken into account. The mean value of wave fields was considered and an effective constitutive relation was adopted which contains an internal variable defining the mean opening of the cracks. An integral representation of the effective displacement field was given using the Green function of the uncracked body. One-dimensional non-linear waves were studied using the solution of the single scattering problem proposed by the authors in a previous paper. Numerical examples, showing the role of the characteristic crack dimension on the overall non-linear response of the cracked medium, were presented. Sharma (1999) discussed the reflection and refraction at an interface between cracked elastic solid and ordinary elastic solid. Reflection and transmission of plane waves was considered at an interface between two dissimilar elastic solid half-space. Upper medium in which incidence takes place was assumed to be containing microcracks. A critical saturation level was examined for each value of crack density. Numerical study is restricted to a particular model and incidence of only P-
wave is considered. Effect of presence of cracks and their saturation on energy partition at the interface was studied numerically, for different (i) values of crack density, (ii) degrees of saturation of cracks, and (iii) regimes of connection between cracks. Energy ratios are plotted against angle of incidence varying from 0 to normal incidence. Critical angles of incidence for transmitted P and SV waves were discussed. Complete reflection was observed at incidence other than normal incidence. Pointer et al. (2000) studied the seismic wave propagation in cracked porous media. The movement of interstitial fluids within a cracked solid could have a significant effect on the properties of seismic waves of long wavelength propagating through the solid. They considered three distinct mechanisms of wave-induced fluid flow: flow through connections between cracks in an otherwise non-porous material, fluid movement within partially saturated cracks, and diffusion from the cracks into a porous matrix material. In each case the cracks may be aligned or randomly oriented, leading respectively to anisotropic or isotropic wave speeds and attenuation factors. In general, seismic velocities exhibit behaviour that was intermediate between that of empty cracks and that of isolated liquid-filled cracks if fluid flow was significant. In the range of frequencies for which considerable fluid flow occurs, there was high attenuation and dispersion of seismic waves. Fluid flow may be on either a wavelength scale or a local scale depending on the
model and whether the cracks were aligned or randomly oriented, resulting in completely different effects on seismic wave propagation. A numerical analysis showed that all models could have an effect over the exploration seismic frequency range.

Kharun and Loboda (2003) discussed a problem of thermoelasticity for a set of interface cracks with contact zones between dissimilar anisotropic materials. A thermoelastic problem for a set of interface cracks, which were assumed to be fully open, partially closed with frictionless perfect-conducted contact regions and fully closed, situated on the interface of two dissimilar arbitrary oriented orthotropic half-spaces or half-planes which were in a combined uniform tension-shear field and a heat flow, was considered. The problem was reduced to the boundary value problem for an analytical function, which was solved in a closed form. The closed-form expressions for the stresses, displacement jump derivatives on the interface and for the stress intensity factors (SIFs) were derived. For the determination of the contact zone lengths, a set of transcendental equations was obtained. A crack with two contact zones at the crack tips was considered for the numerical illustration and the influence of the orientation of principle axes of the materials, their thermoelastic constants and the applied thermo-mechanical load on the contact zone lengths and SIFs, was investigated. Kharun and Loboda (2004) studied a thermoelastic problem
for interface cracks with contact zones. A problem of thermoelasticity for a set of cracks situated on the interface of two dissimilar isotropic solids under a combined tension shear loading and uniform heat flow was considered. The cracks considered were assumed to be completely open, partially closed with frictionless thermally conducted contact zones and completely closed. By means of the complex-function method, the problem was reduced to a non-homogeneous Dirichlet-Riemann boundary value problem, which had been solved in closed form. For the determination of the contact zone lengths the condition of smooth closure of the crack faces had been used and a set of transcendental equations had been obtained. The closed form expressions for the stresses on the interface and the derivatives of the displacement jumps across the interface as well as the stress intensity factors had been obtained. The numerical examples for a crack with one contact zone and for a crack with two contact zones had been presented. For these cases the dependencies of the stress intensity factors and the relative contact zone lengths with respect to the coefficients of the intensity of thermal and mechanical loading for various thermoelastic constants were presented, and a comparison of the results concerning the crack with one and two contact zones had been performed.

Initial stresses are developed in the medium due to many reasons, resulting from difference of temperature, process of quenching, shot pinning
and cold working, slow process of creep, differential external forces, gravity variations, etc. The Earth is supposed to be under high initial stresses. It is therefore of great interest to study the effect of these stresses on the propagation of stress waves.

Chattopadhyay and Chakraborty (1982) studied the reflection of elastic waves under initial stress at a free surface. The phenomenon of reflection of elastic waves at free surface of an initially stress sandy medium has been studied here. The reflection coefficients have been computed numerically for both P and SV motion. The variations of reflection coefficients for different values of \( \eta \), initial stress parameter, and angle of incidence have been represented graphically. The effects of initial stress and sandiness of the layer on surface wave has also been studied. To the best of our knowledge the idea of a sandy medium under initial stress is a new one, which is physically possible. Taking into consideration of the theories given by Weiskopf (1945) and Biot (1965) the equations of motion have been deduced here. Sidhu and Sarva Jit Singh (1983) gave the comments on reflection of elastic waves under initial stress at a free surface. In the paper under discussion, Chattopadhyay et al. [J. Acoust. Soc. Am. 72, 255-263(1982)] studied the problem of the reflection of elastic waves at the plane free boundary of an initially stressed sandy medium. They showed the effects of the decoupling of the P and SV motion for waves propagating in
the horizontal and vertical directions which are the principal directions of the initial stress tensor considered. However, while discussing the reflection of P and SV-waves, the authors use the above decoupling for waves propagating in an arbitrary direction. In fact, the expressions for the displacement potentials assumed by the authors do not satisfy the equations of motion. Consequently, most of the equations and results of the subject paper are either irrelevant or incorrect. Similarly, the results obtained by the authors for Rayleigh waves in an initially stressed sandy medium are also wrong. Norris (1983) studied the propagation of plane waves in a pre-stressed elastic medium. A homogeneous, isotropic elastic medium having initial axial stresses in two orthogonal directions was considered. The dynamic equations for superimposed small deformations were stated. The propagation of plane waves in such a medium was discussed. It was shown that pure longitudinal and shear waves could propagate only in certain specific directions, which are defined.

Dey et al. (1984) studied P and S-waves in a medium under initial stresses and under gravity. In their studies, the velocities of P and S-waves in an unbounded medium under normal initial stresses and under gravity field were derived and it was shown that these velocities not only depend on the initial stresses and gravity but also on the direction of propagation. The results were shown totally with the classical case is the absence of initial
stress and gravity. The velocities of P and S-waves along different
directions, in presence of different combinations of normal initial stresses
and in presence of gravity field, were calculated numerically and the results
presented graphically. Dey and Roy (1984) discussed on the effect of initial
stresses in two dimensional finite element stress analysis. An attempt had
been made to obtain the solutions of initially stressed materials using finite
element method. In contrast to other traditional techniques the finite element
method could handle such complicated aspects very easily. The formulations
used here eliminate most of the difficulties, which were present in earlier
classical formulations. It had been found that the total stiffness matrix
become non-symmetric when there were initial stresses and it become
symmetric when there were no initial stresses, and this changed nature of the
stiffness matrix was due to the fact that the medium does not remain
isotropic in the presence of initial stresses.

Sidhu and Singh (1984) discussed the reflection of P and SV-waves at
the free surface of prestressed elastic half-space. The propagation of plane
waves in a prestressed elastic solid with incremental elastic coefficients
possessing orthotropic symmetry was discussed. Two types of plane waves,
called quasi-P and quasi-S waves, were shown to exist. An expression for
the group velocity of these waves was obtained. The velocities of quasi-P
and quasi-S waves were found to depend on the angle of propagation. The
reflection coefficients of these waves incident at the free surface of a prestressed elastic half-space were derived. Critical reflection of quasi-P and quasi-S waves had been discussed. It was found that while quasi-S waves always had critical reflection, quasi-P waves could have critical reflection only if incremental elastic coefficients satisfied a certain condition. Dey et al. (1985) discussed the reflection and refraction of P-waves under initial stresses at an interface. The discussion deals with the phenomena of reflection and refraction of P-waves at a plane interface between two initially stressed elastic half-spaces. It had been shown analytically that both reflected and refracted P and SV-waves depend on the prestresses present in the media. The numerical values of reflection and refraction coefficients for different initial stresses and the angle of incidence were computed and the results were given in the form of graphs. The results of this work may be used for non-destructive evaluation of prestresses.

Elnaggar (1992) studied the on the dynamical problem of a generalized thermoelastic granular infinite cylinder under initial stress. The object of the present paper is to investigate the influence of initial stress on wave propagation in a generalized thermoelastic granular medium subjected to the boundary condition that the outer surface is traction free. In addition, it is subjected to temperature boundary conditions. The wave velocity equation for the generalized thermoelastic granular medium Rayleigh wave
under the influence of initial stress is obtained. The classical result was derived as a limiting case similar to those obtained by Ewing, Jardetzky and Press (1957).

Montanaro (1999) discussed on singular surfaces in isotropic linear thermoelasticity with initial stress. It would be shown that the theory of a fundamental paper of Chadwick and Powdrill (1965) on singular surfaces, propagating in a linear thermoelastic body, which was stress free, homogeneous and isotropic, also holds when the medium was subjected to hydrostatic initial stress provided the two characteristic speeds were suitably changed. The result was obtained by using Biot’s linearization of the constitutive law for the stress.

Kolpakov (2004) studied the effect of inflation of initial stresses on the homogeneized characteristics of composite. The thermoelastic problem for a composite solid with initial thermal stresses was considered on the basis of asymptotic homogenization method. The major results of the study was that the effective (homogenized) elastic and thermoelastic characteristics of the composite material depend not only on local distributions of all types of material characteristics: local elastic properties, local thermoelastic properties, but also on local initial stresses. Therefore, it was shown that for the inhomogeneous (composite) material, local initial stresses contribute towards values of the effective characteristics of the
material. This kind of interaction was not possible for the homogeneous materials. From the mathematical viewpoint, the above noted phenomenon was based on the fact that in the considered case, the G-limit of a sum was not equal to the sum of G-limits. The developed general homogenized model was investigated in the particular case of the small (as compared with the elastic constants) initial stress, which was common for the practical mechanical problems. The explicit formulas for the effective thermoelastic characteristics and numerical results were obtained for laminated composite solid with the initial stress. Kalpokov and Kalamkarov (2004) discussed the thermoelastic characteristics of a composite solid with initial stresses. The thermoelastic problem for a composite solid with initial stresses was considered on the basis of the asymptotic homogenization method. The homogenized model was constructed by means of the two-scale asymptotic homogenization techniques.

Addy and Chakraborty (2005) studied the thermal effect on gravity waves in a compressible liquid layer over a solid half-space under initial hydrostatic stress. This paper deals with the effect of temperature on gravity waves in a compressible liquid layer over a solid half-space. It has been assumed that the liquid layer is under the action of gravity, while the solid half-space is under the influence of initial compressible hydrostatic stress. When the temperature of the half-space is altered, gravity waves propagate
through the liquid layer along with sub-oceanic Rayleigh waves in the system. A new frequency equation has been derived here for gravity waves and sub-oceanic Rayleigh waves. It has been shown graphically that the phase velocity of gravity waves influenced significantly by the initial compressive hydrostatic stress present in the solid half-space, for a particular value of the phase velocity of sub-oceanic Rayleigh waves and different coupling co-efficients of the temperature.

CONTENTS OF THE THESIS

The present thesis contains eight chapters. Chapter I is the review of an extensive literature on thermoelasticity, cracked elasticity and initial stresses. Chapter II consists of list of symbols (Nomenclature) and deals with a review of cracked thermoelastic solid. Chapter III deals with the reflection of thermoelastic waves from free surface of cracked elastic solid half-space. The amplitude ratios for various reflected waves are calculated and are shown graphically with the angle of incidence, for Lord-Shulman and Green-Lindsay cases. These results are compared with those in absence of cracks. The effect of second thermal relaxation time on reflection coefficients is also observed. Chapter IV contains the problem of reflection and refraction of plane thermal wave at an interface between a thermally conducting liquid half space and a cracked generalized thermoelastic solid half-space. Numerical study is restricted to a particular model. The results
are derived in terms of reflection and refraction co-efficients. These co-efficients are plotted against angle of incidence, incident thermal wave for the saturated cracks. Effects of cracks are observed on various reflected and refracted waves. Chapter V is “On thermoelastic wave propagation at liquid-solid interface in the presence of circular cracks”. Chapter VI deals with a problem on reflection and refraction of plane thermal waves at an interface between a generalized thermoelastic solid half-space and a cracked generalized thermoelastic solid half-space. Effects of saturated cracks are observed numerically on reflected and refracted waves. Chapter VII deals with reflection of generalized thermoelastic waves from a solid half-space under hydrostatic initial stress. Effect of hydrostatic initial stress is observed numerically on reflected waves. Chapter VIII contains conclusions of this thesis.