4.1 Introduction

In this chapter, we deal with a more realistic and generally applicable representation of an epidemic. We not only consider the essential ingredient of any epidemic process provided by the transfer of infection but also the possibility of removal of infectives from circulation by death or isolation. The model considered here possess characteristics which are sufficiently near to reality because of positive latent period and the effect of direct contact on susceptibles, however, certain limitations are involved. For example, we assume an independent isolated group of given size, subject to homogeneous mixing. In the deterministic model infection and removal occur at certain specified density-dependent rates and in case of stochastic model we suppose that infection and removal occur as a random poisson process. Another important aspect of general model is that generally we do not know, and can not observe, when a susceptible becomes infected. It is only when symptoms appear that the existence of disease is recognized and hence the patient is then removed by isolation, which is the observable event. The patient may die or he
may recover while he is assumed to be immune to further infection in the present study. We have studied deterministic and stochastic models. Also probabilities, $P(T)$ that up to time $T$, epidemic does not break out, and partial differential equation for the probability generating function have been obtained. The probabilities $P(T)$ are calculated for different values of the parameters and population size. These values are graphed also and shown in the figure 4.1 to 4. Some numerical values are also given in the tables 4.1 to 4.

4. Deterministic Model

Now we study the more general deterministic model, which allows the removal of infectives from circulation by death or isolation in addition to the simple deterministic model already studied. Let us consider a community of total size $n$. Let at time $t$, there are $x$ susceptibles, $y$ inactive infectives, $z$ active infectives and $r$ individuals who are isolated, dead, or recovered and immune. Hence $x(t) + y(t) + z(t) + r(t) = n$. As before, taking $\beta$ as the rate of physical contact between the susceptibles and active infectives, $\lambda$ as the rate of direct contact, there will be exactly $(\beta x z + \lambda x) \frac{\delta t}{t}$ inactive infectives in time $\delta t$. If $\gamma$ is the inner-development then in a short interval of time $\delta t$, $\gamma y \frac{\delta t}{t}$ will be new active infectives in the population.
Considering \( \alpha \) as the removal rate, there will be exactly \( \alpha z \delta t \) removals in time \( \delta t \). Hence the whole process is described by the following equations:

\[
\begin{align*}
    x(t + \delta t) &= x(t) - \beta x z \delta t - \lambda x \delta t \quad (4.1) \\
    y(t + \delta t) &= y(t) + \beta x z \delta t + \lambda x \delta t - \gamma y \delta t \quad (4.2) \\
    z(t + \delta t) &= z(t) + \gamma y \delta t - \alpha z \delta t \quad (4.3) \\
    r(t + \delta t) &= r(t) + \alpha z \delta t \quad (4.4)
\end{align*}
\]

Simplifying we have

\[
\begin{align*}
    \frac{dx}{dt} &= - (\lambda + \beta z)x \quad (4.5) \\
    \frac{dy}{dt} &= (\lambda + \beta z)x - \gamma y \quad (4.6) \\
    \frac{dz}{dt} &= \gamma y - \alpha z \quad (4.7) \\
    \frac{dr}{dt} &= \alpha z \quad (4.8)
\end{align*}
\]

To have a solution of the above equation is not possible so we now study its stochastic analogue.

4.2 Stochastic Model

While dealing with the probabilistic treatment of the model we have to consider random variables \( X(t) \), \( Y(t) \) and \( Z(t) \) representing the number of susceptibles, inactive infectives and active infectives respectively. As usual, we write the
chance of one new inactive infective due to presence of active infectives as $\beta XZ \, dt$ in time $dt$, where $\beta$ is the rate of contact. When this transition occurs $X$ decreases by one unit, $Y$ increases by one unit while $Z$ remains constant. Also $\gamma Y \, dt$ is the chance of converting inactive infective into active infective. Due to this event $Y$ decreases by one unit, $Z$ increases by one unit and $X$ remains silent. But here we have taken into account the possibility of removal as well. Let the chance of one removal in $dt$ be $\alpha Z \, dt$, $\alpha$ being the removal rate. The variable $Z(t)$ decreases by one unit after this transition but $X$ and $Y$ remain unaltered. So, we define the model as follows.

We suppose that at time $t = 0$, there are $n$ susceptibles and $a$ active infectives. We write $P_{x,y,z}(t)$ for the probability that at time $t$ there are $x$ susceptibles, $y$ inactive infectives and $z$ active infectives in circulation. The chance of one infection in the interval $(t, t + dt)$ is taken to be $\beta xz \, dt$ due to the presence of $z$ active infectives. The chance of one new active infective in $(t, t + dt)$ is taken to be $\gamma y \, dt$ and the chance of one removal as $\alpha z \, dt$. Then the possible transitions are given by

$$(x,y,z) \rightarrow (x,y,z)$$

$$(x + 1, y - 1, z) \rightarrow (x,y,z)$$

$$(x,y + 1, z - 1) \rightarrow (x,y,z)$$
\[(x, y, z + 1) \rightarrow (x, y, z)\]

The above transitions lead to the following equations

\[
P_{x,y,z}(t + \delta t) = (1 - \beta x \delta t)(1 - \gamma y \delta t)(1 - \alpha z \delta t)P_{x,y,z}(t)
+ \beta(x+1)z \delta t P_{x+1,y-1,z}(t)
+ \gamma(y+1) \delta t P_{x,y+1,z-1}(t)
+ \alpha(z+1) \delta t P_{x,y,z+1}(t)
\]

\[0 \leq x+y+z \leq n+a, \ 0 \leq x < n, \ 0 < y \leq n+a,
 a < z \leq n+a \] \hspace{1cm} (4.9)

\[
P_{n,0,a}(t + \delta t) = (1 - \beta na \delta t)(1 - \alpha a \delta t)P_{n,0,a}(t) \] \hspace{1cm} (4.10)

with initial condition

\[
P_{n,0,a}(0) = 1 \] \hspace{1cm} (4.11)

On simplification we have

\[
\frac{dP_{x,y,z}(t)}{dt} = - (\beta xz + \gamma y + \alpha z)P_{x,y,z}(t) + \beta(x+1)zP_{x+1,y-1,z}(t)
+ \gamma(y+1)P_{x,y+1,z-1}(t) + \alpha(z+1)P_{x,y,z+1}(t)
\]

\[0 \leq x+y+z \leq n+a, \ 0 \leq x < n, \ 0 < y \leq n+a,
 a < z \leq n+a \] \hspace{1cm} (4.12)
\[
\frac{dn_{n,0,a}}{dt} = - (\beta n a + \alpha a) n_{n,0,a}(t) \tag{4.13}
\]

Applying Laplace transformation, we get

\[
P_n^*(s) = \frac{\beta (x+1)zP^*_{x+1,y-1,z} - \gamma (y+1)P^*_{x,y+1,z-1} + \alpha (z+1)P^*_{x,y,z+1}}{s + \beta xz + \gamma y + \alpha z} \tag{4.14}
\]

\[
0 \leq x+y+z \leq n+a, \quad 0 \leq x < n,
\]

\[
0 < y \leq n+a, \quad a < z \leq n+a \tag{4.15}
\]

\[
P_n^*(0) = \frac{1}{s + \beta na + \alpha a} \tag{4.17}
\]

where \(P_n^*(s)\) denotes the Laplace transform of \(P_{n,0,a}^*(t)\).

Now we fix \(x+y = n, z = a\) to get the probability that \(\text{upto time } t\), there has been no removal i.e. no infective exhibit normal symptoms of disease and there has been no addition in the group of infectives. As a result \(P_{x,y,z}^*(s)\) and \(P_{x,y,z+1}^*(s)\) will vanish and hence we get

\[
P_n^*(s) = \frac{\beta a(x+1)P_{x+1,y-1,a}^*(s)}{s + \beta xa + \gamma y + \alpha a}, \quad 0 < x+y \leq n \tag{4.16}
\]

\[
P_n^*(0) = \frac{1}{s + \beta na + \alpha a} \tag{4.17}
\]

Putting \(x = n-1, y = 1\) in (4.16) we obtain using (4.17)
Putting $x = n-2$, $y = 2$ in (4.16) and then with the help of (4.18) we get

$$p^*_{n-2,2,a}(s) = \frac{n(n-1)\beta^2a^2}{(s+\beta na+\alpha a)(s+\beta(n-1)a+\alpha a+\gamma)(s+\beta(n-2)a+\alpha a+2\gamma)}$$

and so solving recursively we derive

$$p^*_{n-r,r,a}(s) = \frac{n(n-1)(n-2)...(n-(r-1))\beta^r a^r}{(s+\beta na+\alpha a)(s+\beta(n-1)a+\alpha a+\gamma)...[s+\alpha \{\alpha + \beta (n-r)\} + \gamma r]}$$

Taking inversions of above results we get

$$p_{n,0,a}(t) = e^{-\alpha(n+\beta)t}$$

and

$$p_{n-r,r,a}(t) = \frac{\beta^r a^r n(n-1)(n-2)...(n-(r-1))}{(\gamma - \alpha \beta)^r} \sum \left( \prod_{m=0}^{r} \left( -1 \right)^m r c_m e^{-[\alpha \beta (n-m)] + \gamma r t} \right)$$

Now the probability that up to time $t$ epidemic does not break out is defined by
\( P(t) = \sum_{r=0}^{n} \sum_{r} P_{n-r,r,a}(t) \) \hspace{1cm} (4.22)

The numerical values of \( P(t) \) are obtained for different values of the parameters \( \alpha, \beta \) and \( \gamma \) and are given in Tables 4. to 4. . We have plotted the graphs also against these values and they are shown in the figures to.

We have been studying the model in which infection spreads only by physical contact with an infectious individual. If we consider that the susceptibles may get infection from any other source also i.e. direct source e.g. by eating contaminated food, by drinking water of polluted streams etc. Then the corresponding difference differential equation is given by

\[
\frac{dP_{x,y,z}(t)}{dt} = - (\beta xz + \gamma yz + \alpha x) P_{x,y,z}(t) \\
+ (\lambda + \beta z)(x+1) P_{x+1,y-1,z}(t) \\
+ \gamma (y+1) P_{x,y+1,z-1}(t) + \alpha (z+1) P_{x,y,z+1}(t),
\]

\( 0 \leq x+y+z \leq n+a, \quad 0 < x < n, \quad 0 < y < n+a, \)

\( a < z \leq n+a \) \hspace{1cm} \ldots (4.23)

\[
\frac{dP_{n,0,a}(t)}{dt} = - (\beta na + \alpha a + \lambda n) P_{n,0,a}(t) \hspace{1cm} (4.24)
\]

the initial condition being the same \( P_{n,0,a}(0) = 1 \).
Taking Laplace transform of (4.23) and (4.24) and fixing \(x+y = n, z = a\), as already done we get

\[
\mathcal{L}\{p_{x+y, a}(t)\} = \frac{(\lambda+\beta a)(x+1)p_{x+1, y-1, a}(s)}{(s + \rho x a + y a + \alpha a + \lambda x)}, \quad 0 < x+y \leq n \tag{4.25}
\]

\[
\mathcal{L}\{p_{n, 0, a}(s)\} = \frac{1}{s + \rho na + \alpha a + \lambda n} \tag{4.26}
\]

Taking inversion and solving recursively we get

\[
p_{n, 0, a}(t) = e^{-(\alpha+\beta n) + \lambda n}t \tag{4.27}
\]

\[
p_{n-r, r, a}(t) = \frac{n(n-1)(n-2) \ldots (n-r-1)(\lambda + \beta a)^r}{r!(\gamma - \alpha - \lambda)^r} \tag{4.28}
\]

\[
x \sum_{m=0}^{r} (-1)^m r! \frac{\gamma + (n-m}\lambda}{m} e^{-[\alpha+\beta(n-m)] + \gamma(n-m)\lambda}t \tag{4.29}
\]

The effect of direct contact is also studied numerically by calculating \(P(T)\) for different values of \(\alpha, \beta, \gamma\), and are studied graphically also. The numerical results are shown in Table 4.1 to Table 4.8 while the graphs are shown in the figure 4.1 to figure 4.8.

Now we define the probability generating function as

\[
P(u,v,w,t) = \sum_{x,y,z} p_{x,y,z}(t) u^x v^y w^z \tag{4.29}
\]

Multiplying (4.23) by \(u^x v^y w^z\) and summing over \(x,y,z\),
we obtain

\[
\frac{d}{dt} P_{x,y,z}(t) u^x v^y w^z = - \Sigma (\beta x z + \gamma y + \alpha z + \lambda x) P_{x,y,z}(t) u^x v^y w^z \\
+ \Sigma (\lambda + \beta z)(x+1)P_{x+1,y-1,z}(t) u^x v^y w^z \\
+ \Sigma (\gamma + \lambda z)(y+1)P_{x,y+1,z-1}(t) u^x v^y w^z \\
+ \Sigma (\alpha + \gamma z)(z+1)P_{x,y,z+1}(t) u^x v^y w^z
\]

Simplifying we obtain

\[
\frac{\partial P}{\partial t} = \beta w(v-u) \frac{\partial^2 P}{\partial u \partial w} + \lambda (u+v) \frac{\partial P}{\partial u} + \gamma (w-v) \frac{\partial P}{\partial v} + \alpha (1-w) \frac{\partial P}{\partial w}
\]

(4.30)

with initial condition

\[
P(x,y,z,0) = u^n w^a
\]

(4.31)
The values of $P(T)$ at different times for $n = 10$, $a = 1$, $\lambda' = 2$, $\gamma' = 2$

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\alpha' = 1$</th>
<th>$\alpha' = 2$</th>
<th>$\alpha' = 3$</th>
<th>$\alpha' = 4$</th>
<th>$\alpha' = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.02616</td>
<td>0.74754</td>
<td>0.67640</td>
<td>0.61203</td>
<td>0.55379</td>
</tr>
<tr>
<td>0.2</td>
<td>0.58319</td>
<td>0.47993</td>
<td>0.39794</td>
<td>0.32171</td>
<td>0.26339</td>
</tr>
<tr>
<td>0.3</td>
<td>0.36958</td>
<td>0.27379</td>
<td>0.20283</td>
<td>0.15026</td>
<td>0.11132</td>
</tr>
<tr>
<td>0.4</td>
<td>0.21212</td>
<td>0.14219</td>
<td>0.09531</td>
<td>0.06382</td>
<td>0.04283</td>
</tr>
<tr>
<td>0.5</td>
<td>0.11281</td>
<td>0.06842</td>
<td>0.04150</td>
<td>0.02517</td>
<td>0.01527</td>
</tr>
<tr>
<td>0.6</td>
<td>0.05635</td>
<td>0.03093</td>
<td>0.01697</td>
<td>0.00921</td>
<td>0.00511</td>
</tr>
<tr>
<td>0.7</td>
<td>0.02671</td>
<td>0.01327</td>
<td>0.00659</td>
<td>0.00327</td>
<td>0.00162</td>
</tr>
<tr>
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<td>0.01212</td>
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<td>0.00245</td>
<td>0.00118</td>
<td>0.00049</td>
</tr>
<tr>
<td>0.9</td>
<td>0.00529</td>
<td>0.00215</td>
<td>0.00088</td>
<td>0.00036</td>
<td>0.00014</td>
</tr>
</tbody>
</table>

where $\lambda' = \frac{A}{\rho}$, $\gamma' = \gamma$ and $\alpha' = \frac{a}{\rho}$, $T = \gamma t$.

Table - 4.1
The values of $P(T)$ at different times for $r = 10$, $a = 1$, $\gamma' = 0$,

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\alpha' = 1, \gamma' = 2$</th>
<th>$\alpha' = 2, \gamma' = 3$</th>
<th>$\alpha' = 3, \gamma' = 4$</th>
<th>$\alpha' = 4, \gamma' = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.82616</td>
<td>0.71720</td>
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</tr>
<tr>
<td>0.2</td>
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<tr>
<td>0.3</td>
<td>0.36958</td>
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<td>0.07503</td>
</tr>
<tr>
<td>0.4</td>
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<td>0.09368</td>
<td>0.04478</td>
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</tr>
<tr>
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<td>0.01504</td>
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</tr>
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</tr>
<tr>
<td>0.8</td>
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<td>0.00043</td>
<td>0.00012</td>
</tr>
<tr>
<td>0.9</td>
<td>0.00529</td>
<td>0.00067</td>
<td>0.00012</td>
<td>0.00003</td>
</tr>
</tbody>
</table>

where $\lambda' = \frac{\lambda}{\beta}$, $\gamma' = \frac{\gamma}{\beta}$ and $\alpha' = \frac{\alpha}{\beta}$, $T = \beta t$.

Table - 4.2
The values of $P(T)$ at different times for $n = 10$, $a = 1$.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\alpha' = 1, \gamma' = 2$</th>
<th>$\alpha' = 2, \gamma' = 3$</th>
<th>$\alpha' = 3, \gamma' = 4$</th>
<th>$\alpha' = 4, \gamma' = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda' = 3$</td>
<td>$\lambda' = 2$</td>
<td>$\lambda' = 1$</td>
<td>$\lambda' = 0$</td>
</tr>
<tr>
<td>0.1</td>
<td>0.02616</td>
<td>0.63272</td>
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<td>0.30899</td>
</tr>
<tr>
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<td>0.10461</td>
<td>0.03518</td>
</tr>
<tr>
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<td>0.38860</td>
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</tr>
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</tr>
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<td>0.05635</td>
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</tr>
<tr>
<td>0.7</td>
<td>0.02671</td>
<td>0.00022</td>
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<tr>
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</tr>
<tr>
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<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

where $\lambda' = \frac{\lambda}{\beta}$, $\gamma' = \frac{\gamma}{\beta}$ and $\alpha' = \frac{\alpha}{\beta}$, $T = \beta t$.

Table - 4.3
The values of $P(T)$ at different times for $n = 10$.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\alpha' = 1, \gamma' = 2$</th>
<th>$\alpha' = 2, \gamma' = 3$</th>
<th>$\alpha' = 3, \gamma' = 4$</th>
<th>$\alpha' = 4, \gamma' = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda' = 0, \lambda = 1$</td>
<td>0.82616</td>
<td>0.41122</td>
<td>0.16423</td>
<td>0.04932</td>
</tr>
<tr>
<td>$\lambda' = 2, \lambda = 2$</td>
<td>0.58619</td>
<td>0.08565</td>
<td>0.00878</td>
<td>0.00080</td>
</tr>
<tr>
<td>$\lambda' = 3, \lambda = 3$</td>
<td>0.36958</td>
<td>0.01171</td>
<td>0.00026</td>
<td>0.00000</td>
</tr>
<tr>
<td>$\lambda' = 4, \lambda = 4$</td>
<td>0.21212</td>
<td>0.00120</td>
<td>0.00001</td>
<td>0.00000</td>
</tr>
<tr>
<td>$\lambda' = 5, \lambda = 5$</td>
<td>0.11281</td>
<td>0.00010</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

where $\lambda' = \frac{\lambda}{\beta}$, $\gamma' = \frac{\gamma}{\beta}$ and $\alpha' = \frac{\alpha}{\beta}$, $T = \beta t$.
The values of \( P(T) \) at different times for \( n = 5, a = 1, \lambda' = 2, \gamma' = 2 \)

<table>
<thead>
<tr>
<th>( T )</th>
<th>( a' = 1 )</th>
<th>( a' = 2 )</th>
<th>( a' = 3 )</th>
<th>( a' = 4 )</th>
<th>( a' = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.01410</td>
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<td>0.01886</td>
<td>0.00767</td>
<td>0.00312</td>
<td>0.00127</td>
</tr>
</tbody>
</table>

where \( \lambda' = \frac{\lambda}{\beta} \), \( \gamma' = \frac{\gamma}{\beta} \) and \( a' = \frac{a}{\beta} \), \( T = \beta t \).

Table - 4.5
The values of $P(T)$ at different times for $n = 5$, $a = 1$, $\lambda' = 0$

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\alpha' = 1, \gamma' = 2$</th>
<th>$\alpha' = 2, \gamma' = 3$</th>
<th>$\alpha' = 3, \gamma' = 4$</th>
<th>$\alpha' = 4, \gamma' = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.86460</td>
<td>0.76628</td>
<td>0.67999</td>
<td>0.60411</td>
</tr>
<tr>
<td>0.2</td>
<td>0.69277</td>
<td>0.52883</td>
<td>0.40679</td>
<td>0.31503</td>
</tr>
<tr>
<td>0.3</td>
<td>0.52325</td>
<td>0.33862</td>
<td>0.22368</td>
<td>0.15033</td>
</tr>
<tr>
<td>0.4</td>
<td>0.37708</td>
<td>0.20516</td>
<td>0.11613</td>
<td>0.06786</td>
</tr>
<tr>
<td>0.5</td>
<td>0.26158</td>
<td>0.11922</td>
<td>0.05793</td>
<td>0.02956</td>
</tr>
<tr>
<td>0.6</td>
<td>0.17586</td>
<td>0.06709</td>
<td>0.02808</td>
<td>0.01258</td>
</tr>
<tr>
<td>0.7</td>
<td>0.11517</td>
<td>0.03683</td>
<td>0.01334</td>
<td>0.00527</td>
</tr>
<tr>
<td>0.8</td>
<td>0.07319</td>
<td>0.01982</td>
<td>0.00624</td>
<td>0.00219</td>
</tr>
<tr>
<td>0.9</td>
<td>0.04640</td>
<td>0.01050</td>
<td>0.00289</td>
<td>0.00090</td>
</tr>
</tbody>
</table>

where $\lambda' = \frac{\lambda}{\beta}$, $\gamma' = \frac{\gamma}{\beta}$ and $\alpha' = \frac{a}{\beta}$, $T = \beta t$.

Table - 4.6
The values of $P(T)$ at different times for $n = 5$, $a = 1$,

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\lambda' = 0$</th>
<th>$\lambda' = 1$</th>
<th>$\lambda' = 2$</th>
<th>$\lambda' = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.86460</td>
<td>0.71974</td>
<td>0.58024</td>
<td>0.45511</td>
</tr>
<tr>
<td>0.2</td>
<td>0.69277</td>
<td>0.42603</td>
<td>0.23961</td>
<td>0.12573</td>
</tr>
<tr>
<td>0.3</td>
<td>0.52325</td>
<td>0.22051</td>
<td>0.08015</td>
<td>0.02617</td>
</tr>
<tr>
<td>0.4</td>
<td>0.37708</td>
<td>0.10354</td>
<td>0.02324</td>
<td>0.00454</td>
</tr>
<tr>
<td>0.5</td>
<td>0.26158</td>
<td>0.04516</td>
<td>0.00608</td>
<td>0.00069</td>
</tr>
<tr>
<td>0.6</td>
<td>0.17586</td>
<td>0.01860</td>
<td>0.00147</td>
<td>0.00010</td>
</tr>
<tr>
<td>0.7</td>
<td>0.11517</td>
<td>0.00732</td>
<td>0.00034</td>
<td>0.00001</td>
</tr>
<tr>
<td>0.8</td>
<td>0.07319</td>
<td>0.00277</td>
<td>0.00007</td>
<td>0.00000</td>
</tr>
<tr>
<td>0.9</td>
<td>0.04640</td>
<td>0.00102</td>
<td>0.00002</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

where $\lambda' = \frac{\lambda}{\beta}$, $\gamma' = \frac{\gamma}{\beta}$ and $\alpha' = \frac{\alpha}{\beta}$, $T = \beta t$.

Table - 4.7
The values of $P(t)$ at different times for $n = 5$.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\alpha' = 1, \gamma' = 2$</th>
<th>$\alpha' = 2, \gamma' = 3$</th>
<th>$\alpha' = 4, \gamma' = 4$</th>
<th>$\alpha' = 5, \gamma' = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda' = 0, a = 1$</td>
<td>0.86460</td>
<td>0.52503</td>
<td>0.19143</td>
<td>0.06689</td>
</tr>
<tr>
<td>$\lambda' = 2, a = 2$</td>
<td>0.69277</td>
<td>0.19618</td>
<td>0.02090</td>
<td>0.00204</td>
</tr>
<tr>
<td>$\lambda' = 3, a = 3$</td>
<td>0.52325</td>
<td>0.05938</td>
<td>0.00169</td>
<td>0.00004</td>
</tr>
<tr>
<td>$\lambda' = 4, a = 4$</td>
<td>0.37708</td>
<td>0.01558</td>
<td>0.00011</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

where $\lambda' = \frac{\lambda}{\beta}$, $\gamma' = \frac{\gamma}{\beta}$ and $\alpha' = \frac{\alpha}{\beta}$, $T = \beta t$.

Table - 4.8
$N=10 \quad a=1$

\[ x = \lambda / \beta \]
\[ y = \gamma / \beta \]
\[ \alpha = \alpha / \beta \]
\[ T = \beta t \]

RELATION BETWEEN T & P(T)
RELATION BETWEEN T & P(T)

N = 10

\[ \lambda' = \lambda / \beta \]

\[ \gamma' = \gamma / \beta \]

\[ \alpha' = \alpha / \beta \]

\[ T = \beta t \]
RELATION BETWEEN T&P(T)

N = 5, a = 1
\chi = 0

\chi' = \lambda / \beta
\gamma' = \gamma / \beta
\delta' = \delta / \beta
T = \beta t
$\lambda = \lambda / \beta$
$\gamma = \gamma / \beta$
$\alpha = \alpha / \beta$
$\beta = \beta / \beta$
$N = 5$

Diagram: Curves representing the relation between $T$ and $P(T)$.