CHAPTER - I

ELASTIC WAVES IN PRESTRESSED MEDIA

1.1. Introduction

The propagation of plane waves in the interior of an infinite solid has been discussed by Ewing et al. (1957), Kolsky (1963), Bullen (1965), Achenbach (1973), Miklowitz (1973), Hudson (1980), Aki and Richards (1980) and Ben-Menahem and Singh (1981). In these works the effect of initial stress has not been considered. The Earth is a highly stressed medium. It is therefore of great interest to study the effect of the initial stress on the propagation of elastic waves.

Biot (1965) derived the basic equations governing wave motion under initial stresses. Using Biot's theory, the propagation of elastic waves in prestressed solids has been discussed, among others, by Anderson (1961), Tolstoy (1982), Norris (1983) and Sidhu and Singh (1983, 84, 85).

This chapter consists of two parts. The first part explains the propagation of P, SV and SH waves in a prestressed homogeneous medium having induced anisotropy with orthotropic symmetry. Using the results of Sidhu and Singh (1984), the velocity equations are derived. It is shown that the velocities of P, SV and SH waves depend on the direction of wave propagation. The second part deals with slowness surfaces and the apparent velocities for these waves in the
aforesaid medium. The apparent velocities are derived and calculated numerically for possible range of initial stress parameters. The slowness surfaces are graphically compared with the surfaces without initial stress.

1.2. Basic Equations

Consider a homogeneous prestressed solid. The material is either isotropic in finite strain or anisotropic with orthotropic symmetry. The principal directions of prestress are chosen to coincide with the directions of elastic symmetry and the coordinate axes. The state of prestress is, therefore, defined by principal components $S_{11}$, $S_{22}$ and $S_{33}$ of the initial stress. The general form of dynamical equations for prestressed solids in the absence of external forces is given by Biot (1965, p. 264):

$$s_{ij,j} + S_{jk}^j \omega_{ik,j} + S_{ik}^j \omega_{jk,j} - e_{jk}^j S_{ik,j} = \rho u_{i,tt}, \quad (1.1)$$

where $\rho$ is the density, $u_i$ are the displacement components and, $i$ indicates differentiation with respect to $x_i$. The incremental stresses $s_{ij}$ are assumed to be linearly related to the incremental strains $e_{ij}$ through the incremental elastic coefficients $B_{ij}$, $q_1$ and $q_2$:

$$s_{11} = B_{11} \frac{\partial u}{\partial x} + B_{12} \left( \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right),$$

$$s_{22} = (B_{12} - p) \frac{\partial u}{\partial x} + B_{22} \frac{\partial v}{\partial y} + B_{23} \frac{\partial w}{\partial z}. $$
\[ s_{33} = (B_{12} - P) \frac{\partial u}{\partial x} + B_{23} \frac{\partial v}{\partial y} + B_{22} \frac{\partial w}{\partial z}, \]
\[ s_{12} = q_2 \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \]
\[ s_{13} = q_2 \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right), \]
\[ s_{23} = q_1 \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right), \]
\[ \text{where, it is assumed that } s_{22} = s_{33}, \]
\[ (x, y, z) = (x_1, x_2, x_3), \]
\[ (u, v, w) = (u_1, u_2, u_3), \]
\[ e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \]
\[ \omega_{ij} = \frac{1}{2} (u_{i,j} - u_{j,i}), \]
\[ u_{i,j} = \frac{\partial^2 u_i}{\partial x_j}, \quad u_{i,t} = \frac{\partial^2 u_i}{\partial t}, \quad \text{etc.,} \]
\[ p = s_{33} - s_{11}. \]

The \( s_{ij} \) are the components of prestress which are assumed to satisfy the equilibrium equations
and are related to prestrains \( \varepsilon_{ij} \) by Hooke's law:

\[
\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} .
\]

We assume that \( \lambda \) and \( \mu \) are constants. On using equations (1.2) to (1.6) in equation (1.1), we get

\[
B_{11} \frac{\partial^2 u}{\partial x^2} + A_3 \left( \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + A_1 \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \rho \frac{\partial^2 u}{\partial t^2} ,
\]

\[
B_{22} \frac{\partial^2 v}{\partial y^2} + A_3 \frac{\partial^2 u}{\partial y^2} + A_2 \frac{\partial^2 v}{\partial z^2} + A_4 \frac{\partial^2 w}{\partial y^2} + q_1 \frac{\partial^2 v}{\partial z^2} = \rho \frac{\partial^2 v}{\partial t^2} ,
\]

\[
B_{22} \frac{\partial^2 w}{\partial z^2} + A_3 \frac{\partial^2 u}{\partial z^2} + A_2 \frac{\partial^2 w}{\partial x^2} + A_4 \frac{\partial^2 v}{\partial y^2} + q_1 \frac{\partial^2 w}{\partial z^2} = \rho \frac{\partial^2 w}{\partial t^2} ,
\]

where

\[
A_1 = q_2 + P/2 , \quad A_2 = q_2 - P/2 ,
\]

\[
A_3 = B_{12} + q_2 - P/2 , \quad A_4 = B_{23} + q_1 .
\]

1.3. Plane Waves in Infinite Media

We wish to investigate the behaviour of plane waves in \( xz \)-plane with displacements \( u \) and \( w \) in the \( x \) and \( z \) directions, respectively. The principal stress \( S_{33} \) does not enter explicitly into the equation of motion but its influence is included indirectly in the values of the incremental elastic
coefficients which appear in the equations of motion. Letting $v = 0$ and $\frac{\partial^3}{\partial y^3}$ in the first and last members of equation (1.7), we have

$$B_{11} \frac{\partial^2 u}{\partial x^2} + A_1 \frac{\partial^2 u}{\partial z^2} + A_1 \frac{\partial^2 u}{\partial z^2} = \rho \frac{\partial^2 u}{\partial t^2},$$

$$B_{22} \frac{\partial^2 w}{\partial z^2} + A_2 \frac{\partial^2 w}{\partial x^2} + A_2 \frac{\partial^2 w}{\partial x^2} = \rho \frac{\partial^2 w}{\partial t^2}. \quad (1.9)$$

For plane waves of circular frequency $\omega$, wave number $k$ and phase velocity $c$, its direction of propagation making an angle $\theta$ with z-axis (Fig. 1.1), we may assume

$$u = U_1 \exp(iP_1), \quad w = W_1 \exp(iP_1), \quad (1.10)$$

where $U_1$ and $W_1$ are the amplitude factors and

$$P_1 = k \left\{ \omega t - \frac{x \sin \theta - z \cos \theta}{c} \right\} \quad (1.11)$$

is the phase factor associated with the incident wave. Substituting equation (1.10) in equation (1.9), we have, as in Sidhu and Singh (1984)

$$- (D_1 - \rho c^2) U_1 + A_3 \sin \theta \cos \theta W_1 = 0,$$

$$A_3 \sin \theta \cos \theta U_1 - (D_2 - \rho c^2) W_1 = 0 \quad (1.12)$$
D₁ and D₂ are given by

\[ D₁(θ) = B₁ \sin^2 θ + A₁ \cos^2 θ, \]
\[ D₂(θ) = B₂ \cos^2 θ + A₂ \sin^2 θ. \]  \hspace{1cm} (1.13)

The set of homogeneous equations (1.12) in \( U₁, W₁ \) has a non-trivial solution only if

\[
\begin{vmatrix}
-(D₁ - \rho C^2) & A_3 \sin θ \cos θ \\
A_3 \sin θ \cos θ & -D₂ - \rho C^2 \\
\end{vmatrix} = 0. \hspace{1cm} (1.14)
\]

The two solutions of equation (1.14) are given by

\[
2 \rho C^2(θ) = (D₁ + D₂) \pm \left[ (D₁ - D₂)^2 + 4A_3^2 \sin^2 θ \cos^2 θ \right]^{1/2}. \hspace{1cm} (1.15)
\]

Thus, in general, in this two-dimensional model of the prestressed solid, there are two types of plane waves whose velocities depend on the initial stresses \( S_{33} \) and \( S_{11} \) and also on the direction of wave propagation, \( θ \). The two values of \( C^2(θ) \) denoted by \( C_P^2(θ) \) and \( C_{SV}^2(θ) \) correspond to quasi- P waves and quasi- SV waves, respectively. The explicit expressions for these velocities are given by

\[
C_P(θ) = \frac{1}{2 \rho} \left[ (D₁ + D₂) + \left\{ (D₁ - D₂)^2 + 4A_3^2 \sin^2 θ \cos^2 θ \right\}^{1/2} \right],
\]
\[
C_{SV}(θ) = \frac{1}{2 \rho} \left[ (D₁ + D₂) - \left\{ (D₁ - D₂)^2 + 4A_3^2 \sin^2 θ \cos^2 θ \right\}^{1/2} \right]. \hspace{1cm} (1.16)
\]
Putting \( \frac{\partial^2 v}{\partial y^2} = 0 \), \( v = v(x, z) \) and \( u = w = 0 \) in the second member of equation (1.7), we have

\[
q_1 \frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 v}{\partial x^2} = \rho \frac{\partial^2 v}{\partial t^2}.
\]  

(1.17)

This represents the equation of motion for pure SH waves.

We may assume

\[
v = V_1 \exp (iP_1), \tag{1.18}
\]

where \( P_1 \) is defined in equation (1.11). Using equation (1.18) in equation (1.17), we get the velocity for pure SH waves propagating in the direction \( \theta \), as

\[
C_{SH}(\theta) = \frac{1}{\sqrt{\beta}} \left[ (A_2 - q_1) \sin^2 \theta + q_1 \right]. \tag{1.19}
\]

The velocities of quasi-P waves, quasi-SV waves and pure SH waves in non-dimensional form can be written from equations (1.16) and (1.19) as

\[
\hat{C}_P(\theta) = \frac{C(\theta)}{\beta} = \sqrt{\frac{1}{2\mu} \left[ (D_1 + D_2) + \left( (D_1 - D_2)^2 + 4A_3^2 \sin^2 \theta \cos^2 \theta \right) \right]^{1/2}},
\]

\[
\hat{C}_{SV}(\theta) = \frac{C_{SV}(\theta)}{\beta} = \sqrt{\frac{1}{2\mu} \left[ (D_1 + D_2) - \left( (D_1 - D_2)^2 + 4A_3^2 \sin^2 \theta \cos^2 \theta \right) \right]^{1/2}},
\]

\[
\hat{C}_{SH}(\theta) = \frac{C_{SH}(\theta)}{\beta} = \sqrt{\frac{1}{\mu} \left[ (A_2 - q_1) \sin^2 \theta + q_1 \right]} \tag{1.20}
\]
where
\[ \beta^2 = \nu/\rho \quad (1.21) \]

1.4. Normal and Grazing Incidence

Assuming \( B_{11} > A_2 \) and \( B_{22} \geq A_1 \), it follows from equations (1.16), (1.19) and (1.13) that

\[
C_p(\theta) = \begin{cases} 
B_{11}/\rho & \theta = \pi/2, \\
B_{22}/\rho & \theta = 0,
\end{cases}
\quad (1.22)
\]

\[
C_{SV}^2 = \begin{cases} 
A_2/\rho & \theta = \pi/2, \\
A_1/\rho & \theta = 0,
\end{cases}
\quad (1.23)
\]

and

\[
C_{SH}^2(\theta) = \begin{cases} 
A_2/\rho & \theta = \pi/2, \\
q_1/\rho & \theta = 0.
\end{cases}
\quad (1.24)
\]

Thus, in these two special cases, the phase velocities are

\[ C_p(\theta = \pi/2) = C_p(\pi/2) = B_{11}/\rho, \]
\[ C_{SV}(\theta = \pi/2) = C_{SV}(\pi/2) = A_2/\rho = C_{SH}(\theta = \pi/2) = C_{SH}(\pi/2), \quad (1.25) \]

along x-axis, and
\[ C_p(\theta = 0) = C_p(0) = \frac{B_{22}}{\rho}, \quad C_{SV}(\theta = 0) = C_{SV}(0) = A_1 \rho, \]
\[ C_{SH}(\theta = 0) = C_{SH}(\theta) = \frac{q_1}{\rho}, \quad (1.26) \]
along y-axis.

We assume that
\[ q_1, A_1, A_2, B_{11}, B_{22} > 0, \quad (1.27) \]
so that the phase velocities in equations (1.25) and (1.26) are all real. It is obvious from equations (1.15) and (1.13) that \( C_p(\theta) \) is always real. In order that \( C_{SV}(\theta) \) should be real and non-zero, we have, from equation (1.15),
\[ A_3^2 \sin^2 \theta \cos^2 \theta - D_1 D_2 < 0. \quad (1.28) \]
Using the values of \( D_1, D_2 \) from equation (1.13) and dividing by \( \cos^4 \theta \), the condition (1.28) may be written as
\[ ar^2 - br + c > 0, \quad (1.29) \]
where
\[ a = B_{11} A_2 > 0, \quad b = A_3^2 - B_{11} B_{22} - A_1 A_2, \]
\[ c = B_{22} A_1 > 0, \quad r = \tan^2 \theta \geq 0. \quad (1.30) \]
Beginning with the condition (1.29), Sidhu and Singh (1985) have shown that quasi-SV waves will exist for all values of the angle of propagation if
\[ |A_3| \leq (B_{11} B_{22})^{1/2} + (A_1 A_2)^{1/2} \]  
\[ (1.31) \]

If, however,
\[ |A_3| > (B_{11} B_{22})^{1/2} + (A_1 A_2)^{1/2} \]  
\[ (1.32) \]

quasi-SV waves do not exist for angles of propagation such that \( \theta_1 \leq \theta \leq \theta_2 \), where
\[ \theta_1 = \tan^{-1} \left( \frac{1}{\sqrt{r_1}} \right), \quad \theta_2 = \tan^{-1} \left( \frac{1}{\sqrt{r_2}} \right), \]  
\[ (1.33) \]
\[ r_1 = \left[ b - (b^2 - 4ac)^{1/2} \right] / 2a > 0, \]
\[ r_2 = \left[ b + (b^2 - 4ac)^{1/2} \right] / 2a > 0. \]  
\[ (1.34) \]

1.5. Particular Cases

In case the initial state is unstressed, the medium is isotropic and the first order theory of classical elasticity is assumed, then it may be shown that (Biot 1965, pp. 111-112): 
\[ B_{11} = B_{22} = \lambda + 2\mu, \quad B_{12} = \lambda, \quad q_1 = q_2 = \mu, \]
\[ A_1 = A_2 = \mu, \quad A_3 = \lambda + \mu. \]  
\[ (1.35) \]

Equations (1.25) and (1.26) now become
For the isotropic stress-free case of classical elasticity, the condition (1.31) becomes

$$\lambda + \mu \leq (\lambda + 2\mu) + \mu,$$

which is satisfied.

1.6. Numerical Calculations and Discussions

Equation (1.35) gives the values of the elastic constants $B_{11}$, $B_{22}$ etc. for initially unstressed media.

To study the effect of the initial stress, we assume that for an initially stressed medium.

$$B_{11} = \lambda + 2\mu + P,$$

$$B_{22} = \lambda + 2\mu,$$

$$B_{12} = \lambda + P,$$

$$A_1 = \mu + P/2,$$

$$A_2 = \mu - P/2,$$

$$A_3 = \lambda + \mu + P/2,$$

where $P = S_{33} - S_{11}$.

Equations (1.16), (1.19) and (1.20) reduce to

$$C_p(\theta) = \frac{1}{2^p} \left[ (\lambda + 3\mu + P/2) + \left( (\lambda + \mu + P/2)^2 + 4P(\lambda + \mu + P/2)\sin^2 \theta \right)^{1/2} \right].$$
\[
C_{SV}(\theta) = \sqrt{\frac{1}{2\rho} \left[ \frac{(\lambda + 3\mu + P/2) - \left( (\lambda + \mu + P/2)^2 + 4\mu (\lambda + \mu + P/2) \sin^2 \frac{\theta}{2} \right)^{1/2}}{\mu} \right]},
\]
\[
C_{SH}(\theta) = \sqrt{\frac{1}{\rho} \left[ - \frac{P}{2} \sin^2 \theta + \mu \right]},
\]
and
\[
\hat{C}_P(\theta) = \frac{C_P(\theta)}{\beta} = \sqrt{\frac{1}{2} \left[ (3 + 3\mu + P) - \left( (3 + 1 - P)^2 + 8\mu (3 + 1 + P) \sin^2 \frac{\theta}{2} \right)^{1/2} \right]},
\]
\[
\hat{C}_{SV}(\theta) = \frac{C_{SV}(\theta)}{\beta} = \sqrt{\frac{1}{2} \left[ (3 + 3\mu) - \left( (3 + 1 - P)^2 + 8\mu (3 + 1 + P) \sin^2 \frac{\theta}{2} \right)^{1/2} \right]},
\]
\[
\hat{C}_{SH}(\theta) = \frac{C_{SH}(\theta)}{\beta} = \sqrt{\left[ - \frac{P}{2} \sin^2 \theta + 1 \right]},
\]
where
\[
s = \lambda\mu, \quad p = P/2\mu, \quad \beta^2 = \mu/\rho.
\]

The apparent velocities for quasi-P waves, quasi-SV waves and pure SH waves in non-dimensional form can be obtained from equation (1.39):
\[
\hat{C}_{Pa}(\theta) = \frac{\hat{C}_P(\theta)}{\sin \theta},
\]
\[
\hat{C}_{SVa}(\theta) = \frac{\hat{C}_{SV}(\theta)}{\sin \theta},
\]
\[
\hat{C}_{SHA}(\theta) = \frac{\hat{C}_{SH}(\theta)}{\sin \theta}.
\]

The numerical values of the dimensionless slowness \(1/C_p(\theta), 1/C_{SV}(\theta)\) and \(1/C_{SH}(\theta)\) have been calculated from
equation (1.39), assuming $\beta = 1$, for different values of $p$ in the range $-0.8$ to $0.8$ and for different values of $\theta$. Tables (1.1) - (1.3) give the values of the dimensionless slowness as function of the angle of incidence for P, SV and SH waves. Tables (1.4) - (1.6) give the values of dimensionless apparent velocity as function of the angle of incidence of these waves. The variation of the dimensionless slowness for the P, SV and SH waves with the angle of incidence is shown with the help of figures (1.2) to (1.8) for $p = -0.6, -0.4, -0.2, 0.2, 0.4, 0.6, 0.8$. In each figure the slowness surfaces for quasi-P, quasi-SV and SH waves for a given values of $p = P/2\mu$ are compared with the corresponding slowness surfaces for an initially unstressed medium ($p = 0$).

1.7. Conclusions

The effect of prestress on the propagation of P, SV and SH waves has been studied on the basis of mathematical analysis and numerical calculation. The expressions for the velocities of P, SV and SH waves are given by equation (1.38). The apparent velocities have been calculated using equation (1.41). The study shows that the velocities of these waves are highly affected by the initial stresses present in the medium as well as by the direction of wave propagation.
different values of the angle of incidence $\theta$ and dimensionless $p$ initial stress $p = \frac{\sigma}{\mu}$. Table 1.1: Values of the dimensionless slowness for the quasi-P waves for different values of the angle of incidence $\theta$ and dimensionless $p$ initial stress $p = \frac{\sigma}{\mu}$.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2\pi}{\theta}$</td>
<td>0.0</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Note: The table entries are not legible due to the image quality.
<table>
<thead>
<tr>
<th>$\frac{\sigma_0}{d}$</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>0</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
</tr>
</tbody>
</table>

\[ \frac{\sigma_0}{d} = p \text{ dimensionless initial stress} \]

For different values of the angle of incidence and

\[ \theta = \text{values of the dimensionless stresses for the quasi-\textit{SD}} \text{ waves} \]

**Table I.2**: Values of the dimensionless stresses for the quasi-SD waves.
<table>
<thead>
<tr>
<th>Angle of Incidence (°)</th>
<th>Dimensionless Slowness</th>
<th>Initial Stress (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.745</td>
<td>0.90</td>
</tr>
<tr>
<td>0.2</td>
<td>0.765</td>
<td>0.75</td>
</tr>
<tr>
<td>0.4</td>
<td>0.806</td>
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</tr>
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<td>0.845</td>
<td>0.65</td>
</tr>
<tr>
<td>0.8</td>
<td>0.886</td>
<td>0.58</td>
</tr>
<tr>
<td>1.0</td>
<td>0.922</td>
<td>0.51</td>
</tr>
</tbody>
</table>

*Greek letter: \( \sigma \) for absolute stress, \( \epsilon \) for strain, \( \chi \) for frequency ratio, \( \theta \) for angle of incidence, \( \eta \) for dimensionless slowness.*

Table 1.3: Values of the dimensionless slowness for the quasi-SH waves for different values of the angle of incidence \( \theta \) and dimensionless initial stress \( p = \frac{\sigma}{\sigma_{ref}} \).
Table 1.4: Values of the dimensionless apparent velocity for the quasi-P waves at different values of the angle of incidence and the dimensionless initial stress $\frac{\sigma_0}{\sigma}$.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\sigma_0}{\sigma}$</td>
<td>0.0</td>
<td>1.0</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
<td>6.0</td>
<td>7.0</td>
<td>8.0</td>
</tr>
<tr>
<td>0.0</td>
<td>1.0</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
<td>6.0</td>
<td>7.0</td>
<td>8.0</td>
<td>9.0</td>
</tr>
<tr>
<td>0.1</td>
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<td>2.2</td>
<td>3.3</td>
<td>4.4</td>
<td>5.5</td>
<td>6.6</td>
<td>7.7</td>
<td>8.8</td>
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</tr>
<tr>
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<td>8.4</td>
<td>9.6</td>
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</tr>
<tr>
<td>0.3</td>
<td>1.3</td>
<td>2.6</td>
<td>3.9</td>
<td>5.2</td>
<td>6.5</td>
<td>7.8</td>
<td>9.1</td>
<td>10.4</td>
<td>11.7</td>
</tr>
<tr>
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<td>1.4</td>
<td>2.8</td>
<td>4.1</td>
<td>5.4</td>
<td>6.7</td>
<td>8.0</td>
<td>9.3</td>
<td>10.6</td>
<td>11.9</td>
</tr>
</tbody>
</table>

Initial stress $\frac{\sigma_0}{\sigma}$ = 0.0
Table 1.5: Values of the dimensionless apparent velocity for the quasi-waves for different values of the angle of incidence $\theta$ and the dimensionless initial stress $p$. 

<table>
<thead>
<tr>
<th>$\theta$ (°)</th>
<th>0.8</th>
<th>0.6</th>
<th>0.4</th>
<th>0.2</th>
<th>0</th>
<th>-0.2</th>
<th>-0.4</th>
<th>-0.6</th>
<th>-0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>$r_1$</td>
<td>$r_2$</td>
<td>$r_3$</td>
<td>$r_4$</td>
<td>$r_5$</td>
<td>$r_6$</td>
<td>$r_7$</td>
<td>$r_8$</td>
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Table 1.6: Values of the dimensionless apparent velocity for pure SH waves.
Fig. 1.2. The variation of the dimensionless slowness with the angle of incidence for $p=0.6$, $\theta=\theta'$. The dotted curves correspond to the initially stress free state ($\theta=0$).
The dotted curves correspond to the initially stress free state (p = 0, s(p) = 0). Figure 1.3: The variation of the dimensionless slowness with the angle of incidence for p = 0.4.
Fig. 1.4. The variation of the dimensionless slowness with the angle of incidence $\theta$ for $b = \mu = 0.2, \sigma = 1$ ($\phi = 0$). The dotted curves correspond to the initially stress-free state ($\phi = 0$).
Fig. 1.5. The variation of the dimensionless slowness with the angle of incidence $\theta$ for $p = p_0$. The dotted curves correspond to the initially stress free state $p = 0$. The solid curves indicate $p = p_0$. The angle of incidence $0$.
Fig. 1.6. The variation of the dimensionless slowness with the angle of incidence \( \theta \) for \( \phi = \theta = 0, \gamma = 0 \). The dotted curves correspond to the initially stress-free state (\( d = 0 \)).
Fig. 1.7 The variation of the dimensionless slowness with the angle incidence $\theta$ for $t \geq \pi/2$, $l = l_0(X_{syb})$. The dotted curves correspond to the initially stress free state ($p = 0$).
The variation of the dimensionless slowness with the angle of incidence $\beta$ for $\delta = \delta_1 = 0.8$.

The dotted curves correspond to the initially stress-free state ($\beta = 0$).
Fig. 2.1