PREFACE

Algebraic graph theory is a branch of mathematics in which algebraic methods are applied to problems in graphs. There are three main branches of algebraic graph theory - using linear algebra, using group theory and the study of graph invariants.

For the last two decades, the connection between commutative algebra and graph theory is growing very enormously and is studied by many researchers. For such kind of study, researchers define a graph whose vertices are a set of elements in a group or a ring or a set of ideals in a ring and edges are defined with respect to a condition on the elements of the vertex set.

The focus of this thesis is the association between commutative rings and graph theory. This thesis is concerned with the use of algebraic techniques in the study of graphs. We aim to translate properties of graphs into algebraic properties and then, using the results and methods of algebra we deduce theorems about graphs.

There are many ways to associate graphs to a ring. In this thesis we try to establish some connections between ring theory and graph theory. Some of the graphs defined out of rings are the nilpotent graph, cozero-divisor graph, reduced cozero-divisor graph, Cayley graph and some of the graphs defined out of ideals of a ring are comaximal ideal graph and annihilating ideal graph. Our goal has been to present and illustrate the main tools
and ideas of algebraic graph theory, with an emphasis on current rather than classical topics.

This thesis consists of eight chapters. The details are as below:

Chapter 1: Introduction
Chapter 2: Preliminaries
Chapter 3: Nilpotent graph of commutative rings
Chapter 4: Zero-divisor graph with respect to the nilpotent elements
Chapter 5: Cozero-divisor graph of commutative rings
Chapter 6: Comaximal ideal graph of commutative rings
Chapter 7: Sum annihilating ideal graph of commutative rings
Chapter 8: Some more graphs from rings

In Chapter 2, we include some basic definitions and results which are needed for the subsequent chapters. Unless otherwise noted, all definitions relating to algebra are from I. Kaplansky [37], and all definitions relating to graph theory are from G. Chartrand et al. [27]. The interested reader can find information on commutative algebra in M. F. Atiyah and I. G. Macdonald [20] and, for topological graph-theoretic perspective in D. Archdeacon [18] and A. T. White [51], while additional information about algebraic graph theory can be found in N. L. Biggs [27].

In Chapter 3, we characterize all finite commutative rings for which $\Gamma_N(R)$ is a split graph. Also we determine all isomorphism classes of
finite commutative rings with identity whose $\Gamma_N(R)$ is planar. Further, we characterize all finite commutative rings $R$ with identity for which $\Gamma_N(R)$ has genus one. The contents of this chapter have been published in *Discrete Mathematics, Algorithms and Applications*.

In Chapter 4, we construct a graph called zero-divisor graph of a commutative ring $R$ with respect to nilpotent elements as a simple undirected graph $\Gamma^*_N(R)$ with vertex set $\mathcal{Z}_N(R)^*$, and two vertices $x$ and $y$ are adjacent if and only if $xy$ is nilpotent and $xy \neq 0$. Here $\mathcal{Z}_N(R) = \{x \in R : xy$ is nilpotent, for some $y \in R^*\}$. We investigate the interplay between the graph theoretic properties of $\Gamma^*_N(R)$ and the ring theoretic properties of $R$. Also we characterize the class of rings for which $\Gamma^*_N(R)$ is planar. Finally, we determine all isomorphism classes of finite commutative rings with identity whose $\Gamma^*_N(R)$ has genus one. The contents of this chapter have been published in *Transactions on Combinatorics*.

In Chapter 5, we characterize all finite commutative non-local rings for which the cozero-divisor graph $\Gamma'(R)$ has genus one. Further, we characterize all finite commutative non-local rings $R$ for which the reduced cozero-divisor graph $\Gamma_r(R)$ is planar.

In Chapter 6, a dominating set of the comaximal ideal graph $\mathcal{I}(R)$ is constructed using elements of the center when $R$ is a finite commutative ring. We find the radius, center and median of the comaximal ideal graph.
$\mathcal{I}(R)$ and we prove that the domination number of $\mathcal{I}(R)$ is equal to the number of factors in the Artinian decomposition of $R$. Also, we characterize all finite commutative rings (non-local rings) with identity for which $\mathcal{I}(R)$ is planar. The results obtained in this direction are communicated to *Discrete Mathematics, Algorithms and Applications*.

In **Chapter 7**, we study some fundamental properties of $\Omega(R)$. Especially we identify when the annihilating ideal graph is isomorphic to some well-known graphs. Further, we discuss about the Hamiltonian property of $\Omega(R)$. Finally, we characterize all commutative rings $R$ for which $\Omega(R)$ is planar. Also we determine all isomorphism classes of finite commutative rings with identity whose $\Omega(R)$ has genus one.

In **Chapter 8**, we characterize all finite commutative rings for which the Cayley graph $\text{CAY}(R)$ has genus one. Further, we study some fundamental properties of $\Omega(R)$. Especially we identify when the inclusion ideal graph is isomorphic to some well-known graphs. Finally, we characterize all finite commutative non-local rings $R$ for which the inclusion ideal graph $\mathcal{I}(R)$ is planar. Finally, we determine all isomorphism classes of finite commutative non-local rings with identity whose $\text{CAY}(R)$ has genus one.