ON SOME PROBLEMS OF HYPERGEOMETRIC POLYNOMIALS

ABSTRACT OF THESIS
SUBMITTED FOR THE AWARD OF THE DEGREE OF

Doctor of Philosophy

IN
APPLIED MATHEMATICS

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ALIGARH-202002, INDIA
2011
Abstract

The present thesis entitled “On some problems of hypergeometric polynomials” is an outcome of the studies made by the author at the Department of Applied Mathematics, Aligarh Muslim University, Aligarh. The thesis consists of eight chapters. Articles, definitions and equations have been numbered chapter wise in such a way that, when read as decimals they stand in their proper order e.g., (6.5.1) refers to equation number 1 of section 5 of chapter 6.

A brief review of some important special functions, some integral transforms, the definitions, notations and miscellaneous results which commonly arise in practice and explore many of their salient properties, are given in the first chapter.

Chapter II introduces modified Hermite polynomials of one and two variables, denoted by $H_n(x; a)$ and $H_n(x, y; a)$. The aim of the present chapter is to investigate these two polynomials and acquire some important results such as generating functions, recurrence relations, Rodrigues formula, orthogonality conditions, expansion formulae, integrals, fractional integrals, fractional derivatives, operator representations and other properties of the above polynomials.

Chapter III deals with a study of a new two variables analogue of modified Hermite polynomials $\tilde{H}_n(x, y; a)$ whose special Hermite Polynomials $\tilde{H}_n(x, y)$ seem more natural than that of Hermite polynomials of two variables defined and studied by Khan, M.A. and Abukhhammad, G.S. [19]. A systematic analysis of the above polynomials gives us various unknown results such as generating functions, recurrence relations, Rodrigues formula, relationship with Hermite and Legendre polynomials of one variable, some special properties and
expansion of Legendre polynomials in terms of modified Hermite polynomials of two variables.

In chapter IV, a study has been made of a new three and \( m \)-variable analogue of Hermite polynomials of first kind whose two variable analogue seems more natural than that of Hermite polynomials of two variables defined and studied by Khan, M.A. and Abukhammash, G.S. [19]. This chapter contains generating functions, recurrence relations, Rodrigues formula, relationship with Hermite polynomials of one variable, some special properties and expansion of Legendre polynomials in terms of Hermite polynomials of three and \( m \)-variables.

Chapter V concerns with the study of a new three and \( m \)-variable analogue of Hermite polynomials of second kind whose two variable analogue seems more natural than that of Hermite polynomials of two variables defined and studied by Khan, M.A. and Abukhammash, G.S. [19] and gives some unknown results such as generating functions, recurrence relations, Rodrigues formula, relationship with Hermite polynomials of one variable, some special properties and expansion of Legendre polynomials in terms of Hermite polynomials of three and \( m \)-variables.

Chapter VI gives an extension of certain results obtained by Burchnall, J.L. for Hermite polynomials to similar results for Hermite polynomials of several variables.

The chapter VII deals with a study of Bedient polynomials and gives us a systematic analysis of various unknown results for Bedient polynomials such as generating integrals, operational representations, fractional integrals, fractional derivatives, hypergeometric representations, Laplace transforms, Mellin transforms and relationship with generalized Rice polynomials.
Finally, in chapter VII we study the Bedient polynomials of two, three and $m$-variables and discover a systematic analysis of various unknown results for Bedient polynomials of two, three and $m$-variables such as generating integrals, generating functions and relationship of Hermite, Legendre and Gegenbauer polynomials with Bedient polynomials.

In the end an exhaustive and up to date list of research papers and books related to the subject matters of this thesis have been provided in the form of a bibliography.