CHAPTER 2

ANALYSIS OF TWO-UNIT REDUNDANT SYSTEM WITH IMPERFECT SWITCHING AND CONNECTION TIME
CHAPTER-II

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2.1 Introduction

When the operating component breaks down, the switch will be able to detect the failure using the sensor and replace the defective component with a functional spare, so the system can keep operating. Therefore, the switch and the sensor have direct impact on normal operations of the switching systems.

Switching probably is the most overlooked and undervalued part of a system design. Great attention is spent selecting the measurement and stimulus instruments. But more often than not, the chosen switching does not complement the instruments. It doesn't matter how accurate the instruments are if the signals pass through a poor switch design to get to them.

Engineers are familiar with test instruments because they have used them during their school years and at work testing products or debugging new designs. So it's easy for them to select instruments for an automated test system. Unfortunately, switching often is overlooked.

We begin with assuming that the switching device is perfect in the sense that it does not fail. However, there are practical situations where the switching device can even fail. Such a situation is termed as system with imperfect switching. E.g.1). Imperfect switching causes huge traffic jam in a metropolitan cities. Eg. 2). Boiler feed pump in thermal power plant etc.

The purpose of the chapter is to incorporate the concept of imperfect switching device and connection time with single repairman facility.

In the literature of reliability, many authors including Gupta and Kumar [1997], Kumar and Render [ 1993 ], Tuteja and Taneja [1992], Goyal and Kumar [2006 ], Kumar and Sani [1994], Gupta and Goel[1989], Kumar and Kumar[2007], Kumar, et al [2008] and others have been analysed two and more units repairable
system models assuming continuous distribution of time to failure and time to repair of a unit.

Taking the above facts in view, the present chapter is devoted to analyse two reliability models have been discussed i.e model-I and model II, useful in every day life.

Model-I consists of two-units, the main unit and the standby unit. It is assumed that the main unit can work in normal mode i.e. with full efficiency. The single repairman facility is available with the system to repair the totally failed unit and switch, the distribution of time for failure is taken as negative exponential while the distribution of time to repair is taken as general.

Model-II consists of two units, the main unit and the standby unit. In this model we consider the concept of disconnecting time “the time taken by the repairman in disconnecting the damaged switch from the unit”, connecting time “the time taken by the repairman in connecting the new switch to the unit” and time taken by the repairman to replace the damaged switch with a new one. The single repairman facility is available with the system to repair the totally failed unit, disconnect the damaged switch and to connect the new switch. For both the units, the distribution of time for failure is taken as negative exponential while the repair time distribution is taken to be general. The distribution of time taken to connect or disconnect the switch from the unit is assumed to be negative exponential and the distribution of time taken in replacement of the damaged switch with a new switch is general.

Using regenerative point technique, the following reliability characteristics of interest have been obtained.

(i) Distribution of time to system failure (TSF) and its mean.
(ii) Pointwise and steady state availability of the system
(iii) Expected busy period of the repairman in \((0, t]\) and in steady state.
(iv) Expected number of visits by the repairman in \((0, t]\) and in steady state.
(v) Expected profit incurred in \((0, t]\) and in steady state.

Few characteristics such as MTSF, system availability and profit in steady state have also been studied through graphs.


2.2 Mathematical Treatment for Model-I

(i) A cold standby system consists of two-identical units-operative and standby. Each unit has two modes: normal (N), and total failure (F). The standby unit cannot fail.

(ii) Failure is self-announcing.

(iii) A single repairman is available to repair a failed unit and switch.

(iv) Switching is imperfect in the transition from standby state to operating state.

(v) First priority of the repairman is to repair the failed switch.

(vi) The failure time distribution of each unit is negative exponential while repair time distribution of the repairman is arbitrary.

(vii) The repair time distribution of the switch is general.

(viii) The detection of a failed unit is instantaneous and perfect.

(ix) The unit works as a new one after repaired.

(x) All the random variables are independent.

Symbols and notation used for the system states:

Various symbols used to represent the states of the system are

\[ N_0, N_s \]: Unit normal mode and operative/unit in normal mode and standby.

\[ F_r \]: Unit in failure mode and under repair.

\[ F_R \]: Unit in failure mode and repair continued from the earlier state.

\[ F_{wr} \]: Unit in failure mode and waiting for repair.

\[ SFN_s \]: Switch failed and unit in standby state.

\[ S_r \]: Switch in failure mode and under repair.

Following the above symbols, the possible states of the system model are

**Up States**

\[ S_0 = (N_0, N_s), \quad S_1 = (F_r, N_0) \]

**Down States**

\[ S_3 = (F_R, F_{wr}), \quad S_2 = (F_{wr}, SFN_s) \]

Fig. 2.1. represents the states transition diagram of the system model. Further let,

\[ P \]: The probability that switch is perfect

\[ q \]: The probability that switch is imperfect.
\( \alpha \) : Constant failure rate of operative unit from its normal to total failure mode.

\( g_1(t), G_1(t) \) : p.d.f. and c.d.f. is the repair time of the unit by the repairman.

\( g_2(t), G_2(t) \) : p.d.f. and c.d.f. is the repair time of the switch by the repairman.

**Transition Diagram**

2.3 Transition Probabilities and Sojourn Times

The state transition diagram as in Fig. 2.1. States 3 and 2 are failed states. The epochs of entry into states 0, 1 are regenerative points and thus S0 and S1 states are regenerative states. The transition probabilities from the state \( S_i \) to \( S_j \) are given as follows:

\[
Q_{01}(t) = \int_0^t p \alpha e^{-\alpha u} \, du
\]

\[
Q_{02}(t) = \int_0^t q \alpha e^{-\alpha u} \, du
\]

\[
Q_{10}(t) = \int_0^t g_1(u) e^{-\alpha u} \, du
\]
\[ Q_{13}(t) = \int_0^t \alpha e^{-\alpha u} G_1(u) du \]
\[ Q_{11}^3(t) = \int_0^t \{ \alpha e^{-\alpha u} \} g_1(u) du \]
\[ Q_{21}(t) = \int_0^t g_2(u) du \quad \text{...(2.1-2.6)} \]

Considering \( t \to \infty \) in (2.1-2.6), the expression for steady state transition probabilities can be obtained as follows:

\[
\begin{align*}
P_0 &= \lim_{s \to 0} q_0^*(s) \\
P_{01} &= p, \quad P_{02} = q \\
P_{10} &= g_1^*(\alpha), \quad P_{13} = 1 - g_1^*(\alpha) = P_{11}^3 \\
P_{21} &= 10 \\
\end{align*}
\]

The following relation hold good among the probabilities in (2.7-2.11)

\[
\begin{align*}
P_{01} + P_{02} &= 1 \\
P_{10} + P_{13} &= 1 \\
P_{10} + P_{11}^3 &= 1 \\
P_{21} &= 1 \\
\end{align*}
\]

Also \( \mu_i \), the mean sojourn time in state \( i \) are:

\[
\begin{align*}
\mu_0 &= \frac{1}{\alpha} \\
\mu_1 &= \frac{1}{\alpha} [1 - g_1^*(\alpha)] \\
\mu_2 &= -g_2^*(0) \\
\end{align*}
\]

Defining \( m_{ij} \) as the mean sojourn time of the system in state \( S_i \) when the system is to transit into state \( S_j \), i.e.

\[ m_{ij} = \int_0^\infty t \ dQ_{ij}(t) = \left[ \frac{d}{ds} q_{ij}^*(s) \right]_{s=0} \]

Thus

\[ m_{01} + m_{02} = \mu_0 \]
\[ m_{10} + m_{13} = \mu_1 \]
\[ m_{11}^3 + m_{10} = K_1 \] ... (2.19-2.21)

2.4 Mean Time To System Failure

By probabilistic arguments, we obtain the following recursive relation for \( \phi_t(t) \):

\[ \phi_0(t) = Q_0(t) \cdot S \cdot \phi_1(t) + Q_{02}(t), \]
\[ \phi_1(t) = Q_{10}(t) \cdot S \cdot \phi_0(t) + Q_{13}(t) \] ... (2.22-2.23)

Taking Laplace-Stieltjes transforms (L.S.T.) of above relation and solving for \( \phi_0^{**}(s) \), the mean time to system failure when the system starts from the state ‘0’ is given by

\[
\text{MTSF} = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{N_1}{D_1}
\] ... (2.24)

where

\[ N_1 = \mu_0 + \mu_1 \cdot P_{01} \]
\[ D_1 = 1 - P_{01} \cdot P_{10} \]

2.5 Availability Analysis

The availability \( A_i(t) \) is seen to satisfy the following recursive relations:

\[ A_0(t) = M_0(t) + q_{01}(t) \cdot A_1(t) + q_{02}(t) \cdot A_2(t) \]
\[ A_1(t) = M_1(t) + q_{10}(t) \cdot A_0(t) + q_{11}(t) \cdot A_1(t) \]
\[ A_2(t) = q_{21}(t) \cdot A_1(t) \] ... (2.25-2.27)

where,

\[ M_0(t) = e^{-\alpha t}, \quad M_1(t) = e^{-\alpha t} \cdot G_1(t) \]

Taking the Laplace Transform of relations in (2.25-2.27). When the system starts from \( S_0 \), is given by

\[ A_0^{*}(s) = \frac{N_2(s)}{D_2(s)} \]

where

\[ N_2(s) = M_0^*(s)(1 - q_{11}^*(s)) + q_{01}^*(s)M_1^*(s) + q_{02}^*(s)q_{21}^*(s)M_1^*(s) \]
\[ D_2(s) = (1 - q_{11}^*(s)) - q_{01}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{10}^*(s) - q_{21}^*(s) \]

In steady state, the availability of the system is given by

\[ A_0 = \lim_{s \to 0} s A_0^*(s) = \frac{N_2}{D_2} \quad \text{(2.28)} \]

Where,

\[ N_2 = \mu_0 P_{10} + \mu_1 \]
\[ D_2 = \mu_0 P_{10} + K_1 + P_{02} P_{10} K_2 \]

2.6 Busy Period Analysis of the Repairman

Let \( B_i(t) \) be the busy period of a repairman starting from a regenerative state \( S_i \) at \( t_0 \) given by

\[ B_0(t) = q_{01}(t) B_1(t) + q_{02}(t) B_2(t) \]
\[ B_1(t) = W_1(t) + q_{10}(t) B_0(t) + q_{11}(t) B_1(t) \]
\[ B_2(t) = W_2(t) + q_{21}(t) B_1(t) \quad \text{(2.29-2.31)} \]

where

\[ W_1(t) = e^{-\alpha t} G_1(t) \]
\[ W_2(t) = G_2(t) \]

Taking Laplace Transforms of the above equations and solving them for \( B_0^*(s) \), we get

\[ B_0^*(s) = \frac{N_3(s)}{D_2(s)} \]

where,

\[ N_3(s) = q_{01}(s)W_1^*(s) + q_{02}(s)q_{21}(s)W_1^*(s) + q_{02}(s)W_2^*(s) - q_{02}(s)q_{11}(s)W_2^*(s) \]

In steady state, the total fraction of time which the system is under repair of the repairman, is given by

\[ B_0 = \lim_{s \to 0} s B_0^*(s) = \frac{N_3}{D_2} \quad \text{(2.32)} \]

where

\[ N_3 = \mu_1 + P_{10} P_{02} \mu_2 \]
and $D_2$ is already specified.

2.7 Expected Number of Visits By The Repairman

We have the following recursive relation

$V_0(t) = Q_{01}(t) \cdot \mathbb{S} [1 + V_1(t)] + Q_{02}(t) \cdot \mathbb{S} [1 + V_2(t)]$

$V_1(t) = Q_{10}(t) \cdot \mathbb{S} \cdot V_0(t) + Q_1(t) \cdot \mathbb{S} \cdot V_1(t) \quad \cdots (2.33-2.35)$

$V_2(t) = Q_{21}(t) \cdot \mathbb{S} \cdot V_1(t)$

Taking Laplace Stieltjes Transforms (L.S.T.) of the above equation and solving them for $V_0**(s)$, we get

$$V_0**(s) = \frac{N_4(s)}{D_2(s)}$$

where

$$N_4(s) = [Q_{01}**(s) + Q_{02}**(s)] [1 - Q_1**(s)]$$

In steady-state, the total number of visits by the repairman per unit time are given by

$$V_0 = \lim_{t \to \infty} \left[ \frac{V_0(t)}{t} \right]$$

$$= \lim_{s \to 0} [s \cdot V_0**(s)] = \frac{N_4}{D_2} \quad \cdots (2.36)$$

where

$$N_4 = P_{10}$$

and $D_2$ is already specified.

2.8 Profit Analysis

The expected total profit earned by the system in steady-state is given by

$$P_{11} = C_0 A_0 - C_1 B_0 - C_2 V_0 \quad \cdots (2.37)$$

where

$C_0$ : is the revenue per-unit up-time

$C_1$ : is the cost per-unit time for which the repairman is busy

$C_2$ : is the cost per-visit by the repairman

2.9 Particular Case

For graphical interpretation, the following particular case is considered:

$$g_1(t) = \beta e^{-\beta t}, \ g_2(t) = \gamma e^{-\gamma t}$$
Thus, we can easily obtain the following:

\[
\begin{align*}
P_{01} &= p, & P_{10} &= \frac{\beta}{\alpha + \beta} \\
P_{02} &= q, & P_{21} &= 1 \\
\mu_0 &= \frac{1}{\alpha}, & \mu_1 &= \frac{\beta}{\alpha + \beta}, & \mu_2 &= \frac{1}{\gamma} \\
K_1 &= \frac{1}{\beta}, & K_2 &= \frac{1}{\gamma}
\end{align*}
\]

Using the above equation (2.24), (2.28), (2.32), (2.36) and (2.37) we can have the expression for MTSF, availability etc. for this particular case.

On the basis of the numerical values taken as:

\[
\begin{align*}
\alpha &= 0.1, & \beta &= 0.25, & \gamma &= 0.95, & p &= 0.65, & q &= 0.35
\end{align*}
\]

The values of various measures of system effectiveness are obtained as:

**Mean time to system failure (MTSF) = 19.533**

**Availability \( (A_0) \) = 0.68886.**

**Busy period of analysis of repairman \( (B_0) \) = 0.105471**

**Expected number of visits by the repairman \( (V_0) \) = 0.062624**

For the graphical interpretation, the mentioned particular case is considered.

Figs. 2.2 and 2.3 show the behaviour of MTSF and availability respectively with respect to failure rate \( (\alpha) \). It is clear from the graph that the MTSF and the availability both get decrease with increase in the value of failure rate.

Fig. 2.4 and 2.5 shows the behaviour of MTSF and availability respectively with respect to repair rate \( (\beta) \). It is clear from the graph that MTSF and the availability gets increase with increase in the value of repair rate.
MTSF Vs FAILURE RATE ($\alpha$)

\[ \gamma = 0.95, \ p = 0.65, \ q = 0.35 \]

Fig 2.2
AVAILABILITY Vs FAILURE RATE ($\alpha$)

\[ \gamma = 0.95, \ p = 0.65, \ q = 0.35 \]

Fig 2.3
MTSF Vs REPAIR RATE ($\beta$)

- $\alpha=0.25$
- $\alpha=0.3$
- $\alpha=0.4$

$\gamma = 0.95, p = 0.65, q = 0.35$

Fig 2.4
Figure 2.5

AVAILABILITY Vs REPAIR RATE ($\beta$)

- $\alpha=0.25$
- $\alpha=0.3$
- $\alpha=0.4$

$\gamma = 0.95$, $p = 0.65$, $q = 0.35$
Fig. 2.6 reveals the pattern of the profit with respect to failure rate ($\alpha$) for different values of repair rate ($\beta$). The profit decreases with the increase in the value of failure rate ($\alpha$) and is higher for higher values of repair rate ($\beta$).

PROFIT Vs FAILURE RATE ($\alpha$)

$\beta=0.25$  $\beta=0.3$  $\beta=0.4$

$\gamma = 0.95, p = 0.65, q = 0.35,$
$C_0 = 600, C_1 = 550, C_2 = 450.$
Fig. 2.7 shows the pattern of the profit with respect to repair rate ($\beta$) for different values of failure rate ($\alpha$). The profit increases with the increase in the value of repair rate ($\beta$) and is lower for higher values of failure rate ($\alpha$).

\[ \gamma = 0.95, \quad p = 0.65, \quad q = 0.35, \quad C_0 = 600, \quad C_1 = 550, \quad C_2 = 450. \]
2.10 Mathematical Treatment For Model-II

(i) A cold standby system consists of two-identical units-operative and standby. Each unit has two modes: normal (N) and total failure (F). The standby unit cannot fail.

(ii) Failure is self-announcing.

(iii) A single repairman is available to repair a failed unit and to connect/disconnect the switch to/from the unit.

(iv) Switching is imperfect in the transition from standby state to operating state.

(v) First priority of the repairman is to replace the damaged switch.

(vi) The failure time distribution of each unit is negative exponential while repair time distribution of the repairman is arbitrary.

(vii) The distribution of time taken to connect or disconnect the switch from the unit is assumed to be negative exponential and the distribution of time taken in replacement of the damaged switch with a new one is general.

(viii) The detection of a failed unit is instantaneous and perfect.

(ix) The unit works as a new one after repaired.

(x) All the random variables are independent.

Symbol and notation used for the system states:

Various symbols used to represent the states of the system are:

\[ N_0, N_s \] : Unit normal mode and operative/unit in normal mode and standby.

\[ F_r \] : Unit in failure mode and under repair.

\[ F_R \] : Unit in failure mode and repair continued from the earlier state.

\[ F_{wr} \] : Unit in failure mode and waiting for repair.

\[ SFN_s \] : Switch failed and unit in standby state.

\[ S_D \] : Switch is damaged.

\[ S_N \] : Switch is new.

Following the above symbols, the possible states of the system model are
2.11 Transition Probabilities and Sojourn Times

The state transition diagram as in Fig 2.8. States 2, 3, 4 and 5 are failed stats. The epochs of entry into states 0, 1 are regenerative points and thus. S₀ and S₁ states are regenerative states. The transition probabilities from the state Sᵢ to Sⱼ are given as follows:

\[ Q_{01}(t) = \int_{0}^{1} p \alpha e^{-\alpha u} \, du \]
\[ Q_{02}(t) = \int_{0}^{1} q \alpha e^{-\alpha u} \, du \]
\[ Q_{10}(t) = \int_{0}^{1} g_1(u) e^{-\alpha u} \, du \]
\[ Q_{13}(t) = \int_{0}^{1} \alpha e^{-\alpha u} G_1(u) \, du \]
\[ Q_{11}(t) = \int_{0}^{1} \{\alpha e^{-\alpha u} \otimes 1\} g_1(u) \, du \]
The steady state transition probabilities

\[ P_{ij} = \lim_{t \to \infty} Q_{ij}(t) = \lim_{s \to 0} q_{ij}^*(s) \]

\[ \therefore P_{01} = p \quad P_{10} = g_1^*(\alpha) \]

\[ P_{02} = q \quad P_{13} = P_{11} = 1 - g_1^*(\alpha) \]

\[ P_{45} = g^*(0) = 1 \]

\[ P_{51} = P_{24} = 1 \quad \ldots (2.45-2.50) \]

By these probabilities, it can be verified that

\[ P_{01} + P_{02} = 1 \]

\[ P_{10} + P_{13} = 1 \]

\[ P_{10} + P_{11}^3 = 1 \quad \ldots (2.51-2.53) \]

The mean sojourn times (\( \mu_i \)) in state \( i \) are:
\[ \mu_i = \lim_{t \to \infty} \int_{0}^{t} p(t) \, dt \]

\[ \mu_0 = \frac{1}{\alpha} \]

\[ \mu_1 = \frac{1}{\alpha} \left[ 1 - g_i^*(\alpha) \right] \]

\[ \mu_2 = \int_{0}^{\infty} G_2(t) = -g_2^*(0) = \int_{0}^{\infty} t \, g_2(t) \, dt \]

\[ \mu_4 = \frac{1}{\eta} \]

The conditional mean time taken by the system to transit for any state \( j \) when it has taken form epoch of entrance into regenerative state \( i \) is mathematically stated as

\[ m_{ij} = \int_{0}^{\infty} t \, d \, Q_{ij}(t) = -\left[ \frac{d}{ds} \left[ q_{ij}^* \right] \right]_{s=0} = \int_{0}^{\infty} t \, q_{ij}(t) \, dt \]

Thus,

\[ m_{01} + m_{02} = \mu_0 \]

\[ m_{10} + m_{13} = \mu_1 \]

\[ m_{11} + m_{10} = K_1 \]

\[ m_{41} = \mu_4 = \frac{1}{\eta} \]

\[ m_{24} = K_2 \]

2.12 Mean Time to System Failure

By probabilistic arguments, we obtain the following recursive relation for \( \phi_i(t) \)

\[ \phi_0(t) = Q_{01}(t) \cdot \phi_1(t) + Q_{02}(t) \]

\[ \phi_1(t) = Q_{10}(t) \cdot \phi_0(t) + Q_{13}(t) \]

Taking Laplace-Stieltjes transforms (L. S.T.) of above relation and solving \( \phi_0**(s) \),

the mean time to system failure when the system starts from the state ‘0’ is given by

\[ \text{MTSF} = \lim_{s \to 0} \frac{1-\phi_0**(s)}{s} = \frac{N_1}{D_1} \]

where
\[ N_1 = \mu_0 + \mu_1 P_{01} \]
\[ D_1 = 1 - P_{01} P_{10} \]

2.13 Availability Analysis

The availability \( A_j(t) \) is seen to satisfy the following recursive relations:

\[ A_0(t) = M_0(t) + q_{01}(t) A_1(t) + q_{02}(t) A_2(t) \]
\[ A_1(t) = M_1(t) + q_{10}(t) A_0(t) + q_{11}(t) A_1(t) \]
\[ A_2(t) = q_{21}(t) A_1(t) \]
\[ A_4(t) = q_{42}(t) A_3(t) \]
\[ A_5(t) = q_{51}(t) A_4(t) \] ...(2.66-2.70)

where

\[ M(t) = e^{-at} \]
\[ M_1(t) = e^{-at} G_1(t) \]

Taking the Laplace transform of the above equation. When the system starts from \( S_0 \) is given by

\[ A_0^*(s) = \frac{N_2(s)}{D_2(s)} \]
\[ N_2(s) = M^*_0(s) \{1 - q_{11}^*(s)\} + M^*_1(s) q_{01}^*(s) + M^*_1(s) q_{02}^*(s) q_{24}^*(s) q_{51}^*(s) \]
\[ D_2(s) = 1 - q_{11}^*(s) - q_{01}^*(s) q_{02}^*(s) q_{10}^*(s) - q_{02}^*(s) q_{10}^*(s) q_{24}^*(s) q_{51}^*(s) \]

In steady state, the availability of the system is given by

\[ A_0 = \lim_{s \to 0} s A_0^*(s) = \frac{N_2}{D_2} \] ...(2.71)

where,

\[ N_2 = \mu_0 P_{10} + \mu_1 \]
\[ D_2 = \mu P_{10} + (K_2 + \mu_4) P_{02} P_{10} + K_1 \]

2.14 Busy Period Analysis of the Repairman

By probabilistic arguments, we have the following recursive relation for \( B_i(t) \)

\[ B_0(t) = q_{01}(t) B_1(t) + q_{02}(t) B_2(t) \]
\[ B_1(t) = W_1(t) + q_{10}(t) B_0(t) + q_{11}(t) B_1(t) \]
\[ B_2(t) = W_2(t) + q_{21}(t) B_1(t) \]
\[ B_4(t) = W_4(t) + q_{45}(t)B_5(t) \]
\[ B_5(t) = W_5(t) + q_{51}(t)B_1(t) \]  \( \ldots (2.72-2.76) \)

where,
\[ W_1(t) = e^{-at}G_1(t) \]
\[ W_2(t) = W_5(t) = e^{-nt} \]
\[ W_4(t) = G_2(t) \]

Taking Laplace Transform of the above equation and solving them for \( B_0^*(s) \), we get
\[ B_0^*(s) = \frac{N_3(s)}{D_2(s)} \]

where,
\[ N_3(s) = q_{01}^*(s)W_1^*(s) + q_{02}^*(s)q_{41}^*(s)q_{24}^*(s)W_1^*(s) + q_{02}^*(s)W_2^*(s) + q_{02}^*(s)q_{24}^*(s) \]
\[ W_4^*(s) - q_{02}^*q_{11}^*(s)W_2^*(s) - q_{02}^*q_{11}^*(s)q_{24}^*(s)W_4^*(s) \]

and \( D_2(s) \) already specified.

In steady state, the total fraction of time which the system is under repair of the repairman, is given by
\[ B_0 = \lim_{s \to 0} B_0^*(s) = \frac{N_3}{D_2} \]  \( \ldots (2.77) \)

where
\[ N_3 = \mu_1 + P_{10}P_{02} \mu_2 + P_{10}P_{02} \mu_4 \]

and \( D_2 \) is already specified.

**2.15 Expected Number of Visits by the Repairman**

By the probabilistic arguments, we have the following recursive relations:
\[ V(t) = Q_1(t) \cdot [1 + V_1(t)] + Q_{02}(t) \cdot [1 + V_2(t)] \]
\[ V_1(t) = Q_{10}(t) \cdot V_0(t) + Q_{11}(t) \cdot V_1(t) \]
\[ V_2(t) = Q_{24}(t) \cdot V_4(t) \]
\[ V_4(t) = Q_{45}(t) \cdot V_5(t) \]
\[ V_5(t) = Q_{51}(t) \cdot V_1(t) \]  \( \ldots (2.78-2.82) \)
Taking S. L. T.

\[ V_0^{**}(s) = \frac{N_4(s)}{D_2(s)} \]

where

\[ N_4(s) = [Q_{01}^{**}(s) + Q_{02}^{**}(s)] [1 - Q_{11}^{**}(s)] \]

In steady state, expected number of visits by the repairman per-unit time are given by

\[ V_0 = \lim_{t \to \infty} \left[ \frac{V_0(t)}{t} \right] \]

\[ = \lim_{s \to 0} s V_0^{**}(s) = \frac{N_4(0)}{D_2(0)} \] \hspace{1cm} \text{(2.83)}

\[ N_4 = P_{10} \]

and \( D_2 \) is already specified.

2.16 Profit Analysis

The expected total profit earned by the system in steady-state is given by

\[ P_{12} = C_0A_0 - C_1B_0 - C_2V_0 \] \hspace{1cm} \text{(2.84)}

where,

\( C_0 \) : is the revenue per-unit up time.

\( C_1 \) : is the cost per-unit up time for which the repairman is busy

\( C_2 \) : is the cost per-visit by the repairman

2.17 Particular Case

For graphical interpretation, the following particular case is considered:

\[ g_1(t) = \beta e^{-\beta t}, \quad g_2(t) = \gamma e^{-\gamma t} \]

Thus, we can easily obtained the following:

\[ P_{01} = p, \quad P_{10} = \frac{\beta}{\alpha + \beta} \]

\[ P_{02} = q \]

\[ P_{24} = P_{51} = 1 \]

\[ P_{45} = 1 \]
Using the above equation (2.65), (2.71), (2.77), (2.83) and (2.84), we can have the expression MTSF, availability etc. for this particular case.

On the basis of the numerical values taken as:
\[ \alpha = 0.1, \quad \beta = 0.25, \quad \gamma = 0.95, \quad p = 0.65, \quad q = 0.35, \quad \eta = 0.85 \]

The values of various measures of system effectiveness are obtained as:

- **Mean time to system failure (MTSF)** = 19.533
- **Availability** \((A_0)\) = 0.671543.
- **Busy period of analysis of repairman** \((B_0)\) = 0.098804
- **Expected number of visits by the repairman** \((V_0)\) = 0.061049

For the graphical interpretation, the mentioned particular case is considered.

Figs. 2.9 and 2.10 show the behaviour of MTSF and availability respectively with respect to failure rate \((\alpha)\). It is clear from the graph that the MTSF and the availability both get decrease with increase in the value of failure rate.

Fig. 2.11 and 2.12 shows the behaviour of MTSF and availability respectively with respect to repair rate \((\beta)\). It is clear from the graph that MTSF and the availability gets increase with increase in the value of repair rate.
MTSF vs Failure Rate ($\alpha$)

- $\beta=0.25$
- $\beta=0.3$
- $\beta=0.4$

$\gamma = 0.95, \ p = 0.65, \ q = 0.35$

Fig 2.9
AVAILABILITY VS FAILURE RATE ($\alpha$)

- $\beta = 0.25$
- $\beta = 0.3$
- $\beta = 0.4$

$\gamma = 0.95, \eta = 0.85, \rho = 0.65, q = 0.35$

Fig 2.10.
Fig 2.11
AVAILABILITY VS REPAIR RATE ($\beta$)

- $\alpha=0.25$
- $\alpha=0.3$
- $\alpha=0.4$

$\gamma = 0.95$, $\eta = 0.85$, $p = 0.65$, $q = 0.35$

Fig 2.12
Fig. 2.13 reveals the pattern of the profit with respect to failure rate ($\alpha$) for different values of repair rate ($\beta$). The profit decreases with the increase in the value of failure rate ($\alpha$) and is higher for higher values of repair rate ($\beta$).

\[ \gamma = 0.95, \; \eta = 0.85, \; p = 0.65, \; q = 0.35, \]
\[ C_0 = 600, \; C_1 = 550, \; C_2 = 450. \]
Fig. 2.14 shows the pattern of the profit with respect to repair rate ($\beta$) for different values of failure rate ($\alpha$). The profit increases with the increase in the value of repair rate ($\beta$) and is lower for higher values of failure rate ($\alpha$).

\[
\gamma = 0.95, \eta = 0.85, p = 0.65, q = 0.35, \\
C_0 = 600, C_1 = 550, C_2 = 450.
\]