CHAPTER 3

RELIABILITY AND PROFIT EVALUATION OF A TWO-UNIT COLD STANDBY SYSTEM WITH INSPECTION AND REPLACEMENT
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RELIABILITY AND PROFIT EVALUATION OF A TWO-UNIT COLD STANDBY SYSTEM WITH INSPECTION AND CHANCES OF REPLACEMENT

INTRODUCTION

In the previous chapter, we have studied the concept of inspection and replacement for a one-unit system. PLCs as two-unit cold standby systems are also used by a number of industries/firms.

Thus, in the present chapter, we discuss a two-unit cold standby system with inspection and replacement. Initially one unit is operative and the other is cold standby. On the failure of operative unit, standby takes some time (called activation time) to become operative. After the completion of activation time, the operative unit is undertaken for inspection. The inspection is carried out to detect the reparability of the unit. Two reliability models have been analysed with the same considerations as those for the models discussed in the previous chapter. Other assumptions are same as taken in the previous chapter.

Following measures of the system effectiveness are obtained.

- Mean time to system failure (MTSF)
- Steady-state availability of the system
- Expected busy period (for repair only) per unit time by ordinary/expert repairman
- Expected busy period (for inspection only) per unit time by ordinary/expert repairman
- Expected busy period (for replacement only) per unit time by ordinary repairman
• Expected number of visits per unit time by ordinary/expert repairman
• Expected number of replacements per unit time
• Expected activation time
• Expected profit incurred to the system

The models have been analysed by making use of semi-Markov processes and regenerative point technique. Profit incurred to the system is evaluated. Graphical study is also made.

NOTATIONS

$\lambda$ : constant failure rate of operative unit
$p_1$ : probability that unit is repairable
$p_2$ : probability that unit is irreparable

$h_1(t), H_1(t)$ : p.d.f. and c.d.f. of time to inspection for detecting reparability of a failed unit

$h_2(t), H_2(t)$ : p.d.f. and c.d.f. of replacement time

$w(t), W(t)$ : p.d.f. and c.d.f. of activation time

$g(t), G(t)$ : p.d.f. and c.d.f. of time to repair of ordinary repairman

$h_e(t), H_e(t)$ : p.d.f. and c.d.f. of time to inspection of expert repairman

$g_e(t), G_e(t)$ : p.d.f. and c.d.f. of time to repair of expert repairman

Symbols for the states of the system are

$o$ : operative unit

cs : cold standby unit

$F_{ui}$ : failed unit under inspection of ordinary repairman

$F_{ur}$ : failed unit under repair of ordinary repairman

$F_{rep}$ : failed unit under replacement of ordinary repairman

$F_{uie}$ : failed unit under inspection of expert repairman

$F_{re}$ : failed unit under repair of expert repairman
failed unit waiting for repair

repair of failed unit is continuing by the ordinary repairman from the previous state.

inspection of the failed unit is continuing by the ordinary/expert repairman from the previous state

repair of the failed unit is continuing by the expert repairman from the previous state.

**MODEL 3.1**

Additional assumption in this model is that if the unit is declared irreparable by the ordinary repairman, it is replaced with a new one. State transition diagram is shown as in Fig. 3.1.

**TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES**

The epochs of entry into states 0, 1, 2, 3, 4, 8 and 9 are regeneration points and thus 0, 1, 2, 3, 4, 8 and 9 are regenerative states. States 5, 6, 7, 8 and 9 are failed states. State 1 is down state.

The transition probabilities are given by:

\[
\begin{align*}
q_{01}(t) &= \lambda e^{-\lambda t} ; \quad q_{12}(t) = w(t) \\
q_{23}(t) &= p_1 h_1(t) e^{-\lambda t} ; \quad q_{24}(t) = p_2 h_1(t) e^{-\lambda t} ; \quad q_{25}(t) = \lambda e^{-\lambda t} \bar{H}_1(t) \\
q_{28}^{(5)}(t) &= p_1 [\lambda e^{-\lambda t} \odot 1] h_1(t) = p_1 (1-e^{-\lambda t}) h_1(t) \\
q_{29}^{(5)}(t) &= p_2 [\lambda e^{-\lambda t} \odot 1] h_1(t) = p_2 [1 - e^{-\lambda t}] h_1(t) \\
q_{30}(t) &= e^{-\lambda t} g(t) ; \quad q_{36}(t) = \lambda e^{-\lambda t} \bar{G}(t) \\
q_{32}^{(6)}(t) &= [\lambda e^{-\lambda t} \odot 1] g(t) = [1 - e^{-\lambda t}] g(t) \\
q_{40}(t) &= h_2(t) e^{-\lambda t} ; \quad q_{47}(t) = \lambda e^{-\lambda t} \bar{H}_2(t) \\
q_{42}^{(7)}(t) &= [\lambda e^{-\lambda t} \odot 1] h_2(t) = [1 - e^{-\lambda t}] h_2(t) \\
q_{82}(t) &= g(t) , \quad q_{92}(t) = h_2(t)
\end{align*}
\]

The non-zero elements \( p_{ij} \) are given as follows:

\( (3.1.1-3.1.15) \)
Fig. 3.1
\( p_{01} = 1 \); \( p_{12} = 1 \); \( p_{23} = p_1 h_1 (\lambda) \); \( p_{24} = p_2 h_1 (\lambda) \)
\( p_{25} = 1 - h_1 (\lambda) \); \( p_{28}^{(5)} = p_1 [1 - h_1 (\lambda)] \); \( p_{29}^{(5)} = p_2 [1 - h_2 (\lambda)] \)
\( p_{30} = \gamma (\lambda) \); \( p_{36} = 1 - \gamma (\lambda) \); \( p_{32}^{(6)} = 1 - \gamma (\lambda) \)
\( p_{40} = h_2 (\lambda) \); \( p_{47} = 1 - h_2 (\lambda) \); \( p_{42}^{(7)} = 1 - h_2 (\lambda) \)
\( p_{82} = 1 \); \( p_{92} = 1 \)  

(3.1.16-3.1.30)

By these transition probabilities, it can be verified that

\( p_{01} = 1 \), \( p_{12} = 1 \), \( p_{23} + p_{24} + p_{25} = 1 \)
\( p_{23} + p_{24} + p_{28}^{(5)} + p_{29}^{(5)} = 1 \), \( p_{30} + p_{36} = 1 \)
\( p_{30} + p_{32}^{(6)} = 1 \), \( p_{40} + p_{47} = 1 \)
\( p_{40} + p_{42}^{(7)} = 1 \), \( p_{82} = 1 \), \( p_{92} = 1 \)  

(3.1.31-3.1.40)

The mean sojourn times \((\mu_i)\) in state ‘i’ are :

\( \mu_0 = \frac{1}{\lambda} \), \( \mu_1 = \int_0^\infty w(t) \, dt \)
\( \mu_2 = \frac{[1 - h_1 (\lambda)]}{\lambda} \), \( \mu_3 = \frac{1 - \gamma (\lambda)}{\lambda} \)
\( \mu_4 = \frac{1 - h_2 (\lambda)}{\lambda} \), \( \mu_8 = \int_0^\infty g(t) \, dt \)
\( \mu_6 = \int_0^\infty t h_2 (t) \, dt \)  

(3.1.41-3.1.47)

The unconditional mean time taken by the system to transit for any state ‘j’ when it is counted from epoch of entrance into state ‘i’ is mathematically stated as :

\( m_{ij} = \int_0^\infty t q_{ij} (t) dt = - q_{ij}^{**} (0) \)  

(3.1.48)

Thus,

\( m_{01} = \mu_0 \), \( m_{12} = \mu_1 \)
\( m_{23} + m_{24} + m_{25} = \mu_2 \)
\[ m_{23} + m_{24} + m_{28}^{(5)} + m_{29}^{(5)} = \int_0^t h_1(t) \, dt = k_1 \text{(say)} \]

\[ m_{30} + m_{36} = \mu_3, \quad m_{30} + m_{32}^{(6)} = \mu_8 \]

\[ m_{40} + p_{47} = \mu_4, \quad m_{40} + m_{42}^{(7)} = \mu_9 \]

\[ m_{82} = \mu_8, \quad m_{92} = \mu_9 \quad (3.1.49-3.1.58) \]

**MEAN TIME TO SYSTEM FAILURE**

By probabilistic arguments, we obtain the following recursive relations for \( \phi_i(t) \):

\[
\begin{align*}
\phi_0(t) &= Q_{01}(t) \phi_1(t) \\
\phi_1(t) &= Q_{12}(t) \phi_2(t) \\
\phi_2(t) &= Q_{23}(t) \phi_3(t) + Q_{24}(t) \phi_4(t) + Q_{25}(t) \\
\phi_3(t) &= Q_{30}(t) \phi_0(t) + Q_{36}(t) \\
\phi_4(t) &= Q_{40}(t) \phi_0(t) + Q_{47}(t) \quad (3.1.59-3.1.63)
\end{align*}
\]

Taking L.S.T. of these relations and solving them for \( \phi_0^{**}(s) \), we obtain

\[
\phi_0^{**}(s) = \frac{N(s)}{D(s)} \quad (3.1.64)
\]

where

\[
N(s) = Q_{01}^{**}(s) Q_{12}^{**}(s) \left[ Q_{23}^{**}(s) Q_{30}^{**}(s) + Q_{24}^{**}(s) Q_{47}^{**}(s) \right] + Q_{25}^{**}(s) \quad (3.1.65)
\]

\[
D(s) = 1 - Q_{01}^{**}(s) Q_{12}^{**}(s) \left[ Q_{23}^{**}(s) Q_{30}^{**}(s) + Q_{24}^{**}(s) Q_{40}^{**}(s) \right] \quad (3.1.66)
\]

Now, the mean time to system failure (MTSF) when the system starts from the state ‘0’ is

\[
T_0 = \lim_{s \to 0} \frac{1 - \phi_{0}^{**}(s)}{s} \quad (3.1.67)
\]

Using L’Hospital rule and putting the value of \( \phi_0^{**}(s) \) from equation (3.1.64), we have
\[ T_0 = \frac{N}{D} \quad (3.1.68) \]

where
\[ N = \mu_0 + \mu_1 + \mu_2 + p_{23}\mu_3 + p_{24}\mu_4 \quad (3.1.69) \]
\[ D = 1-(p_{23}p_{30} + p_{24}p_{40}) \quad (3.1.70) \]

**AVAILABILITY ANALYSIS**

Using the arguments of the theory of regenerative processes, the availability \( A_i(t) \) is seen to satisfy the following recursive relations:

\[
A_0(t) = M_0(t) + q_{01}(t) \odot A_1(t) \\
A_1(t) = q_{12}(t) \odot A_2(t) \\
A_2(t) = M_2(t) + q_{23}(t) \odot A_3(t) + q_{24}(t) \odot A_4(t) + q_{28}(5)(t) \odot A_8(t) \\
\hspace{1cm} + q_{29}(5) \odot A_9(t) \\
A_3(t) = M_3(t) + q_{30}(t) \odot A_0(t) + q_{32}(6)(t) \odot A_2(t) \\
A_4(t) = M_4(t) + q_{40}(t) \odot A_0(t) + q_{42}(7)(t) \odot A_2(t) \\
A_8(t) = q_{82}(t) \odot A_2(t) \\
A_9(t) = q_{92}(t) \odot A_2(t) \\
\text{where} \quad (3.1.71-3.1.77)
\]

\[
M_0(t) = e^{-\lambda t}, \quad M_2(t) = e^{-\lambda t} \bar{H}_1(t), \quad M_3(t) = e^{-\lambda t} \bar{G}(t)
\]

and
\[
M_4(t) = e^{-\lambda t} \bar{H}_2(t) \quad (3.1.78-3.1.81)
\]

Taking L.T. of the above equations and solving them for \( A_0^*(s) \) we get

\[
A_0^*(s) = \frac{N_1(s)}{D_1(s)} \quad (3.1.82)
\]

where
\[
N_1(s) = M_0^*(s) \left\{ 1-q_{92}^*(s)q_{29}(5)^*(s) - q_{24}^*(s)q_{42}(7)^*(s)-q_{82}^*(s)q_{28}(5)^*(s) - q_{23}^*(s)q_{32}(6)^*(s) \right\} + q_{01}^*(s)q_{12}^*(s)\{M_4^*(s)q_{24}^*(s)
\]
\[ + M_2^*(s) + q_{23}^*(s)M_3^*(s) \]

and

\[ D_1(s) = 1 - q_{29}^*(s)q_{29}^{(5)}(s) - q_{82}^*(s)q_{28}^{(5)}(s)q_{24}^*(s)q_{42}^{(7)}(s) \]

\[ - q_{23}^*(s)q_{32}^{(6)}(s) - q_{11}^*(s)q_{12}^*(s)q_{23}^*(s)q_{30}^*(s) \]

\[ + q_{24}^*(s)q_{40}^*(s) \]  

(3.1.83-3.1.84)

In steady-state, the availability of the system is given by

\[ A_0 = \lim_{s \to 0} \left\{ sA_0^*(s) \right\} = \frac{N_1}{D_1} \]  

where

\[ N_1 = (p_{23}p_{30} + p_{24}p_{40})\mu_0 + \mu_2 + \mu_3p_{23} + \mu_4p_{24} \]

and

\[ D_1 = (p_{23}p_{30} + p_{24}p_{40})\left( \mu_0 + \mu_1 \right) + (p_{23} + p_{29}^{(5)})\mu_8 + (p_{24} + p_{29}^{(5)})\mu_9 + k_1 \]  

(3.1.86-3.1.87)

### BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN

(Repair Time Only)

By probabilistic arguments, we have the following recursive relations for \( B_j(t) \):

\[ B_0(t) = q_{01}(t) \odot B_1(t) \]

\[ B_1(t) = q_{12}(t) \odot B_2(t) \]

\[ B_2(t) = q_{23}(t)B_3(t) + q_{24}(t)B_4(t) + q_{28}^{(5)}(t)B_8(t) \]

\[ + q_{29}^{(5)}(t)B_9(t) \]

\[ B_3(t) = W_3(t) + q_{30}(t) \odot B_0(t) + q_{32}^{(6)}(t) \odot B_2(t) \]

\[ B_4(t) = q_{40}(t) \odot B_0(t) + q_{42}^{(7)}(t) \odot B_2(t) \]

\[ B_8(t) = W_8(t) + q_{82}(t) \odot B_2(t) \]

\[ B_9(t) = q_{92}(t) \odot B_2(t) \]  

(3.1.88-3.1.94)

where

\[ W_3(t) = e^{-\lambda t} \tilde{G}(t) + [\lambda e^{-\lambda t} \odot 1] \tilde{G}(t) \]
\[ G(t) = G(t) \]
\[ W_8(t) = G(t) \]  \hspace{1cm} (3.1.95-3.1.96)

Taking L.T. of the above equations and solving them for \( B_0^*(s) \), we get

\[ B_0^*(s) = \frac{N_2(s)}{D_1(s)} \]  \hspace{1cm} (3.1.97)

where

\[ N_2(s) = q_{01}^*(s)q_{12}^*(s)[q_{23}^*(s) W_3^*(s) + q_{28}^{(5)}(s) W_8^*(s)] \]  \hspace{1cm} (3.1.98)

and \( D_1(s) \) is already specified in equation (3.1.84).

In steady-state, the total fraction of time for which the system is under repair of ordinary repairman is given by

\[ B_0 = \lim_{s \to 0} \{ s \, B_0^*(s) \} = \frac{N_2}{D_1} \]  \hspace{1cm} (3.1.99)

where

\[ N_2 = (p_{23} + p_{28}^{(5)})\mu_8 \]  \hspace{1cm} (3.1.100)

**BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN**

*(Inspection Time Only)*

By probabilistic arguments, we have the following recursive relations:

\[ B_{10}(t) = q_{01}(t) \odot B_{11}(t) \]
\[ B_{11}(t) = q_{12}(t) \odot B_{12}(t) \]
\[ B_{12}(t) = W_2(t) + q_{23}(t) \odot B_{13}(t) + q_{24}(t) \odot B_{14}(t) + q_{28}^{(5)}(t) \odot B_{18}(t) \]
\[ + q_{29}^{(5)}(t) \odot B_{19}(t) \]
\[ B_{13}(t) = q_{30}(t) \odot B_{16}(t) + q_{32}^{(6)}(t) \odot B_{12}(t) \]
\[ B_{14}(t) = q_{40}(t) \odot B_{10}(t) + q_{42}^{(7)}(t) \odot B_{12}(t) \]
\[ B_{15}(t) = q_{82}(t) \odot B_{12}(t) \]
\[ B_{10}(t) = q_{92}(t) \odot B_{12}(t) \]  
\[ W_2(t) = \bar{H}_1(t) \]  
Taking L.T. of the above equations and solving them for \( B_{10}*(s) \), we get
\[ B_{10}*(s) = \frac{N_3(s)}{D_1(s)} \]
where
\[ N_3(s) = q_{01}*(s)q_{12}*(s)W_2*(s) \]  
and \( D_1(s) \) is already specified.

In steady-state, the total fraction of the time for which the system is under inspection of ordinary repairman is given by
\[ B_{10} = \lim_{s \to 0} \{s B_{10}*(s)\} = \frac{N_3}{D_1} \]
where
\[ N_3 = k_1 \]

**BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN**  
(Replacement Time Only)

\[ B_{R0}(t) = q_{01}(t) \odot B_{R1}(t) \]  
\[ B_{R1}(t) = q_{12}(t) \odot B_{R2}(t) \]  
\[ B_{R2}(t) = q_{23}(t) \odot B_{R3}(t) + q_{24}(t) \odot B_{R4}(t) + q_{28}^{(5)}(t) \odot B_{R6}(t) \]
\[ + q_{29}^{(5)}(t) \odot B_{R9}(t) \]  
\[ B_{R3}(t) = q_{30}(t) \odot B_{R0}(t) + q_{32}^{(6)}(t) \odot B_{R2}(t) \]  
\[ B_{R4}(t) = W_4(t) + q_{40}(t) \odot B_{R0}(t) + q_{42}^{(7)}(t) \odot B_{R2}(t) \]  
\[ B_{R6}(t) = q_{62}(t) \odot B_{R2}(t) \]  
\[ B_{R9}(t) = W_9(t) + q_{92}(t) \odot B_{R2}(t) \]
where
\[ W_4(t) = W_9(t) \bar{H}_2(t) \]
Taking L.T. of the above equations and solving them for $BR_0^*(s)$, we get

$$BR_0^*(s) = \frac{N_4(s)}{D_1(s)} \quad (3.1.121)$$

where

$$N_4(s) = q_{01}^*(s)q_{12}^*(s) \left[ q_{24}^*(s)W_4^*(s) + q_{29}^{(5)}(s)W_9^*(s) \right] \quad (3.1.122)$$

and $D_1(s)$ is already specified.

In steady-state, the total fraction of the time for which the system is under replacement of ordinary repairman is given by

$$BR_0 = \lim_{s \to 0} \{ s \ BR_0^*(s) \} = \frac{N_4}{D_1} \quad (3.1.123)$$

where

$$N_4 = (p_{24} + p_{29}^{(5)})\mu_9 \quad (3.1.124)$$

and $D_1$ is already specified.

**EXPECTED NUMBER OF VISITS BY THE REPAIRMAN**

By probabilistic arguments, we have the following recursive relations

$$V_0(t) = Q_{01}(t) \ 1 + V_1(t)$$

$$V_1(t) = Q_{12}(t) \ V_2(t)$$

$$V_2(t) = Q_{23}(t) \ V_3(t) + Q_{24}(t) \ V_4(t) + Q_{29}^{(5)}(t) \ V_5(t)$$

$$+ Q_{29}^{(5)}(t) \ V_6(t)$$

$$V_3(t) = Q_{30}(t) \ V_0(t) + Q_{32}^{(6)}(t) \ V_2(t)$$

$$V_4(t) = Q_{40}(t) \ V_0(t) + Q_{42}^{(7)}(t) \ V_2(t)$$

$$V_5(t) = Q_{52}(t) \ V_2(t)$$

$$V_6(t) = Q_{62}(t) \ V_2(t) \quad (3.1.125-3.1.131)$$

Taking L.S.T. of the above equations and solving them for $V_0^{**}(s)$, we get
\[ V_0^{**}(s) = \frac{N_5(s)}{D_1(s)} \]  \hspace{1cm} (3.1.132)

where

\[ N_5(s) = Q_{11}^{**}(s) \{ 1 - Q_{23}^{**}(s)Q_{32}^{(6)^{**}(s)} - Q_{24}^{**}(s)Q_{42}^{(7)^{**}(s)} \}
- Q_{82}^{**}(s)Q_{28}^{(5)^{**}(s)} - Q_{92}^{**}(s)Q_{29}^{(5)^{**}(s)} \} \]  \hspace{1cm} (3.1.133)

and \( D_1(s) \) is specified earlier.

In steady-state, the expected number of visits of the repairman on the system is given by

\[ V_0 = \frac{N_5}{D_1} \]  \hspace{1cm} (3.1.134)

where

\[ N_5 = p_{23}p_{30} + p_{24}p_{40} \]  \hspace{1cm} (3.1.135)

and \( D_1 \) is already specified.

**EXPECTED NUMBER OF REPLACEMENTS**

By probabilistic arguments, we have the following recursive relations:

\[ R_0(t) = Q_{01}(t) R_1(t) \]
\[ R_1(t) = Q_{12}(t) R_2(t) \]
\[ R_2(t) = Q_{23}(t) R_3(t) + Q_{24}(t) [1 + R_4(t)] + Q_{28}(5)(t) R_8(t) + Q_{29}(5)(t) [1 + R_9(t)] \]
\[ R_3(t) = Q_{30}(t) R_0(t) + Q_{32}(6)(t) R_2(t) \]
\[ R_4(t) = Q_{40}(t) R_0(t) + Q_{42}(7)(t) R_2(t) \]
\[ R_8(t) = Q_{82}(t) R_2(t) \]
\[ R_9(t) = Q_{92}(t) R_2(t) \]  \hspace{1cm} (3.1.136-3.1.142)

Taking L.S.T. of the above equations and solving them for \( R_0^{**}(s) \), we get

\[ R_0^{**}(s) = \frac{N_6(s)}{D_1(s)} \]  \hspace{1cm} (3.1.143)
where
\[ N_6(s) = (Q_{24}^{**}(s) + Q_{29}^{(5)}**)Q_{01}**(s)Q_{12}**(s) \]  
(3.1.144)

and \( D_1(s) \) is already specified.

In steady-state, the expected number of replacements of the failed units by the repairman is given by

\[ R_0 = \frac{N_b}{D_1} \]  
(3.1.145)

where

\[ N_b = P_{24} + P_{29}^{(5)} \]  
(3.1.146)

and \( D_1 \) is already specified.

**ANALYSIS OF ACTIVATION TIME**

The following recursive relations for \( AT_i(t) \):

\[ AT_0(t) = q_{01}(t) \otimes AT_1(t) \]
\[ AT_1(t) = W_1(t) + q_{12}(t) \otimes AT_2(t) \]
\[ AT_2(t) = q_{23}(t) \otimes AT_3(t) + q_{24}(t) \otimes AT_4(t) + q_{28}^{(5)}(t) \otimes AT_8(t) \]
\[ + q_{29}^{(5)}(t) \otimes AT_9(t) \]
\[ AT_3(t) = q_{30}(t) \otimes AT_0(t) + q_{32}^{(6)}(t) \otimes AT_2(t) \]
\[ AT_4(t) = q_{40}(t) \otimes AT_0(t) + q_{42}^{(7)}(t) \otimes AT_2(t) \]
\[ AT_8(t) = q_{82}(t) \otimes AT_2(t) \]
\[ AT_9(t) = q_{92}(t) \otimes AT_2(t) \]
\[ W_1(s) = \mu_1 \]  
(3.1.147-3.1.154)

Taking L.T. of the above equations and solving them for \( AT_0^*(s) \), we get

\[ AT_0^*(s) = \frac{N_7(s)}{D_1(s)} \]  
(3.1.155)
where

$$\text{N}_7(s) = W_1 \times (s)[q_{01} \times (s)[1-q_{23} \times (s)q_{32}^{(6)} \times (s) - q_{24} \times (s)q_{42}^{(7)} \times (s) - q_{28}^{(5)} \times (s)q_{82} \times (s) - q_{29}^{(5)} \times (s)q_{92} \times (s)]]$$

(3.1.156)

$D_i(s)$ is already specified.

In steady-state, the total activation time of the system is given by

$$\text{AT}_0 = \frac{\text{N}_7}{D_1}$$

(3.1.157)

where

$$\text{N}_7 = [p_{23}p_{30} + p_{24}p_{40}] \mu_1$$

(3.1.158)

**PROFIT ANALYSIS**

The expected total profit incurred to the system in steady state is given by

$$P_{31} = C_0A_0 - C_1B_0 - C_2B_1 - C_3B_0 - C_4V_0 - C_5R_0 - C_9A_0$$

(3.1.159)

where

- $C_0 =$ revenue per unit up time of the system
- $C_1 =$ cost per unit time for which the repairman is busy in repair
- $C_2 =$ cost per unit time for which the repairman is busy in inspection of the failed unit
- $C_3 =$ cost per unit time for which the repairman is busy in replacing the failed unit.
- $C_4 =$ cost per visit of the repairman
- $C_5 =$ cost per replacement of the failed unit with new one
- $C_9 =$ cost per unit activation time.
PARTICULAR CASE
For the graphical interpretation, the following particular case is assumed:

\[ g(t) = \alpha_1 e^{-\alpha_1 t}, \quad w(t) = \beta e^{-\beta_1 t}, \]

\[ h_1(t) = \gamma_1 e^{-\gamma_1 t}, \quad h_2(t) = \gamma e^{-\gamma t} \]

and the remaining distributions are same as in the general case.

Therefore, we get

\[ p_{01} = 1, \quad p_{12} = 1, \quad p_{23} = \frac{p_1 \gamma_1}{\gamma_1 + \lambda}, \quad p_{24} = \frac{p_2 \gamma_1}{\gamma_1 + \lambda}, \quad p_{25} = \frac{\lambda}{\gamma_1 + \lambda} \]

\[ p_{26}^{(5)} = \frac{p_1 \lambda}{\gamma_1 + \lambda}, \quad p_{29}^{(5)} = \frac{p_2 \lambda}{\gamma_1 + \lambda}, \quad p_{30} = \frac{\alpha_1}{\alpha_1 + \lambda}, \quad p_{36} = \frac{\lambda}{\alpha_1 + \lambda} \]

\[ p_{32}^{(6)} = \frac{\lambda}{\alpha_1 + \lambda}, \quad p_{40} = \frac{\gamma}{\gamma + \lambda}, \quad p_{47} = \frac{\lambda}{\gamma + \lambda}, \quad p_{42}^{(7)} = \frac{\lambda}{\gamma + \lambda} \]

\[ p_{32} = 1, \quad p_{92} = 1, \quad \mu_0 = \frac{1}{\lambda}, \quad \mu_1 = \frac{1}{\beta}, \quad \mu_2 = \frac{1}{\gamma + \lambda} \]

\[ k_1 = \frac{1}{\gamma_1}, \quad \mu_3 = \frac{1}{\alpha_1 + \lambda}, \quad \mu_4 = \frac{1}{\gamma + \lambda}, \quad \mu_8 = \frac{1}{\alpha_1} \]

\[ \mu_0 = \frac{1}{\gamma} \quad (3.1.160-3.1.186) \]

Using the above equations and the equations (3.1.68), (3.1.85), (3.1.99), (3.1.111), (3.1.123), (3.1.134), (3.1.145), (3.1.157) and (3.1.159). We can have the expressions for MTSF, availability and profit for this particular case.

MODEL 3.2

Fig. 3.2 gives the transition diagram showing the various states of transition of the system. In this model, expert opinion is taken
Fig. 3.2
regarding reparability if the inspection by ordinary repairman declares that unit is not repairable.

**TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES**

The epochs of entry into states 0, 1, 2, 3, 4, 7, 8, 9, 10, 13 and 14 are regeneration points and thus are the regenerative states. States 5, 6, 8, 9, 10, 11, 12, 13, 14 and 15 are failed states. State 1 is down state. The transition probabilities are given by:

\[
\begin{align*}
q_{01}(t) &= \lambda e^{-\lambda t}; \\
q_{12}(t) &= w(t); \\
q_{23}(t) &= p_1 e^{-\lambda t} h_1(t) \\
q_{24}(t) &= p_2 e^{-\lambda t} h_1(t); \\
q_{25}(t) &= \lambda e^{-\lambda t} \overline{H}_1(t) \\
q_{26}^{(5)}(t) &= p_1 [\lambda e^{-\lambda t} \odot 1] h_1(t) = p_1 (1 - e^{-\lambda t}) h_1(t) \\
q_{2,10}^{(5)}(t) &= p_2 [\lambda e^{-\lambda t} \odot 1] h_1(t) = p_2 (1 - e^{-\lambda t}) h_1(t) \\
q_{3,0}(t) &= e^{-\lambda t} g(t); q_{36}(t) = \lambda e^{-\lambda t} \overline{G}(t); q_{32}^{(6)}(t) = [\lambda e^{-\lambda t} \odot 1] g(t) \\
q_{4,15}(t) &= \lambda e^{-\lambda t} \overline{H}_c(t) \\
q_{4,14}^{(15)}(t) &= p_2 [\lambda e^{-\lambda t} \odot 1] h_c(t) \\
q_{4,1}\cdot 11 (t) &= p_1 [\lambda e^{-\lambda t} \odot 1] h_c(t) \\
q_{4,14}^{(15)}(t) &= p_2 [\lambda e^{-\lambda t} \odot 1] h_c(t) \\
q_{4,8}(t) &= p_2 e^{-\lambda t} h_c(t); q_{70}(t) = e^{-\lambda t} g_c(t); q_{72}^{(11)}(t) = [\lambda e^{-\lambda t} \odot 1] g_c(t) \\
&= [1 - e^{-\lambda t} \odot 1] g_c(t) \\
q_{7,11}(t) &= \lambda e^{-\lambda t} \overline{G}_c(t); q_{80}(t) = e^{-\lambda t} h_2(t); q_{82}^{(12)}(t) = [\lambda e^{-\lambda t} \odot 1] h_2(t) \\
q_{92}(t) &= g(t); q_{10,13}(t) = p_1 h_c(t) \\
q_{14,3}(t) &= h_2(t); q_{10,14}(t) = p_2 h_c(t) \\
q_{14,3}(t) &= h_2(t); q_{10,14}(t) = p_2 h_c(t)
\end{align*}
\]

The non zero elements \(p_{ij}\) are given as follows:

\[
\begin{align*}
p_{01} &= 1, \\
p_{12} &= 1, \\
p_{23} &= p_1 h_1^*(\lambda), \\
p_{24} &= p_2 h_1^*(\lambda), \\
p_{25} &= (1 - h_1^*(\lambda)), \\
p_{29}^{(5)} &= p_1 \{1 - h_1^*(\lambda)\}, \\
p_{2,10}^{(5)} &= p_2 \{1 - h_1^*(\lambda)\}
\end{align*}
\]

(3.2.1-3.2.25)
\[ p_{30} = g^*(\lambda), \quad p_{36} = 1 - g^*(\lambda), \quad p_{32}^{(6)} = 1 - g^*(\lambda), \quad p_{47} = p_{1} h_c^*(\lambda), \]
\[ p_{48} = p_{2} h_c^*(\lambda), \quad p_{4,15} = 1 - h_c^*(\lambda), \quad p_{4,13}^{(15)} = p_{1}\{1 - h_c^*(\lambda)\} \]
\[ p_{4,14}^{(15)} = p_{2}\{1 - h_c^*(\lambda)\}, \quad p_{70} = g_c^*(\lambda), p_{72}^{(11)} = 1 - g_c^*(\lambda) \]
\[ p_{80} = h_2^*(\lambda), p_{82}^{(12)} = 1 - h_2^*(\lambda), p_{92} = 1, \quad p_{10,13} = p_{1}, \]
\[ p_{10,14} = p_{2}, \quad p_{13,2} = 1, \quad p_{14,2} = 1, \quad p_{7,11} = 1 - g_c^*(\lambda) \]

(3.2.26 - 3.2.50)

By these transition probabilities, it can be verified that

\[ p_{01} = 1, \quad p_{12} = 1, \quad p_{23} + p_{24} + p_{25} = 1, \quad p_{23} + p_{24} + p_{29}^{(5)} + p_{2,10}^{(5)} = 1 \]

\[ p_{30} + p_{36} = 1, \quad p_{30} + p_{32}^{(6)} = 1, \quad p_{47} + p_{48} + p_{4,15} = 1, \]
\[ p_{47} + p_{48} + p_{4,13}^{(15)} + p_{4,14}^{(15)} = 1, \quad p_{70} + p_{7,11} = 1, \quad p_{70} + p_{72}^{(11)} = 1, \]
\[ p_{80} + p_{82}^{(11)} = 1, \quad p_{10,13} + p_{10,14} = 1, \quad p_{13,2} = 1, \quad p_{14,2} = 1 \]

(3.2.51-3.2.65)

The mean sojourn times (\( \mu_i \)) in state (i) are:

\[ \mu_0 = \frac{1}{\lambda}, \quad \mu_1 = \int_0^\infty \overline{W}(t) \, dt = \int_0^\infty tw(t) \, dt \]
\[ \mu_2 = \int_0^\infty e^{-\lambda t} \overline{H_1}(t) \, dt = \frac{1 - h_1^*(\lambda)}{\lambda}, \quad \mu_3 = \int_0^\infty e^{-\lambda t} \overline{G}(t) \, dt = \frac{1 - g^*(\lambda)}{\lambda}, \]
\[ \mu_4 = \int_0^\infty e^{-\lambda t} \overline{H_c}(t) \, dt = \frac{1 - h_c^*(\lambda)}{\lambda}, \quad \mu_7 = \int_0^\infty e^{-\lambda t} \overline{G_d}(t) \, dt = \frac{1 - g_c^*(\lambda)}{\lambda} \]
\[ \mu_8 = \int_0^\infty e^{-\lambda t} \overline{H_2}(t) \, dt = \frac{1 - h_2^*(\lambda)}{\lambda}, \quad \mu_9 = \int_0^\infty t g(t) \, dt \]
\[ \mu_{10} = \int_0^\infty t h_c(t) \, dt, \quad \mu_{13} = \int_0^\infty t g_c(t) \, dt, \quad \mu_{14} = \int_0^\infty t h_2(t) \, dt \]

(3.2.66-3.2.76)

The unconditional mean time taken by the system to transit for any state \( \text{\textquoteleft} j \text{\textquoteright} \) when it is counted from epoch of entrance into state \( \text{\textquoteleft} i \text{\textquoteright} \) is mathematically stated as:

\[ m_{ij} = \int_0^\infty t q_{ij}(t) \, dt = -q_j q_i' \, (0) \quad \text{(3.2.77)} \]

Thus,

\begin{align*}
& \quad m_{01} = \mu_0, \quad m_{12} = \mu_1; \quad \mu_{23} + \mu_{24} + \mu_{25} = \mu_2, \\
& m_{23} + m_{24} + m_{29}^{(5)} + m_{2,10}^{(5)} = k_1 \text{(say)}; \quad \mu_{30} + \mu_{36} = \mu_3, \\
& m_{30} + m_{32}^{(6)} = \mu_9; \quad m_{47} + m_{48} + m_{4,15} = \mu_4, \\
& m_{47} + m_{48} + m_{4,13}^{(15)} + m_{4,14}^{(15)} = \mu_{10}; \quad m_{70} + m_{7,11} = \mu_7, \\
& m_{70} + m_{72}^{(11)} = \mu_{13}; \quad m_{80} + m_{8,12} = \mu_8, \\
& m_{80} + m_{8,12}^{(12)} = \mu_{14}; \quad m_{92} = \mu_9, \\
& m_{10,13} + m_{10,14} = \mu_{10}; \quad m_{13,2} = \mu_{13}, \\
& m_{14,2} = \mu_{14} \quad \text{(3.2.78-3.2.93)}
\]

**MEAN TIME TO SYSTEM FAILURE**

By probabilistic arguments, we obtain the following recursive relations for \( \phi_i(t) \):

\[ \phi_0(t) = Q_{01}(t) \, \phi_1(t) \]

\[ \phi_1(t) = Q_{12}(t) \, \phi_2(t) \]

\[ \phi_2(t) = Q_{23}(t) \, \phi_3(t) + Q_{24}(t) \, \phi_4(t) + Q_{25}(t) \]

\[ \phi_3(t) = Q_{30}(t) \, \phi_0(t) + Q_{36}(t) \]

\[ \phi_4(t) = Q_{47}(t) \, \phi_7(t) + Q_{48}(t) \, \phi_8(t) + Q_{4,15}(t) \]

\[ \phi_7(t) = Q_{70}(t) \, \phi_0(t) + Q_{7,11}(t) \]

\[ \phi_8(t) = Q_{80}(t) \, \phi_0(t) + Q_{8,12}(t) \quad \text{(3.2.94-3.2.100)} \]
Taking L.S.T. of the above relations and solving them for $\phi_0 \ast\ast(s)$, we get

$$\phi_0 \ast\ast(s) = \frac{N(s)}{D(s)}$$ \hspace{1cm} (3.1.101)

where

$$N(s) = Q_{01} \ast\ast(s) Q_{12} \ast\ast(s) \{ Q_{23} \ast\ast(s) + Q_{23} \ast\ast(s) Q_{36} \ast\ast(s) + Q_{24} \ast\ast(s) (Q_{47} \ast\ast(s) + Q_{48} \ast\ast(s) Q_{80} \ast\ast(s)) \}$$

$$D(s) = 1 - Q_{01} \ast\ast(s) Q_{12} \ast\ast(s) \{ Q_{23} \ast\ast(s) Q_{30} \ast\ast(s) + Q_{24} \ast\ast(s) (Q_{47} Q_{70} \ast\ast(s) + Q_{48} \ast\ast(s) Q_{80} \ast\ast(s)) \}$$ \hspace{1cm} (3.2.102-3.2.103)

Now, the mean time to system failure (MTSF) when the system starts from the state '0' is

$$T_0 = \lim_{s \to 0} \frac{1 - \phi \ast\ast(s)}{s}$$ \hspace{1cm} (3.2.104)

Using L'Hospital rule and putting the value of $\phi_0 \ast\ast(s)$ from equation (3.2.101), we have

$$T_0 = \frac{N}{D}$$ \hspace{1cm} (3.2.105)

where

$$N = \mu_0 + \mu_1 + \mu_2 + p_{23}\mu_3 + p_{24}\mu_4$$ \hspace{1cm} (3.2.106)

$$D = 1 - p_{23}p_{30} + p_{24}(p_{17}p_{70} + p_{48}p_{80})$$ \hspace{1cm} (3.2.107)

**AVAILABILITY ANALYSIS**

Using the arguments of the theory of regenerative processes, the availability $A_i(t)$ is seen to satisfy the following recursive relations:

$$A_0(t) = M_0(t) + q_{01}(t) \odot A_1(t)$$

$$A_1(t) = q_{12}(t) \odot A_2(t)$$

$$A_2(t) = M_2(t) + q_{23}(t) \odot A_3(t) + q_{24}(t) \odot A_4(t) + q_{29}^{(5)}(t) \odot A_9(t) + q_{2,10}^{(5)}(t) \odot A_{10}(t)$$

$$A_3(t) = M_3(t) + q_{30}(t) \odot A_0(t) + q_{32}^{(6)}(t) \odot A_2(t)$$
\[ A_4(t) = M_4(t) + q_{47}(t) A_7(t) + q_{48}(t) A_8(t) + q_{4,13}(15)(t) A_{13}(t) + q_{4,4}(15)(t) A_{14}(t) \]
\[ A_7(t) = M_7(t) + q_{70}(t) A_6(t) + q_{72}(11)(t) A_2(t) \]
\[ A_8(t) = M_8(t) + q_{80}(t) A_6(t) + q_{82}(12)(t) A_2(t) \]
\[ A_9(t) = q_{92}(t) A_2(t) \]
\[ A_{10}(t) = q_{10,13}(t) A_{13}(t) \]
\[ A_{13}(t) = q_{13,2}(t) A_2(t) \]
\[ A_{14}(t) = q_{14,2}(t) A_2(t) \]

where

\[ M_0(t) = e^{-iat} \]
\[ M_2(t) = e^{-iat} H_1(t) \]
\[ M_3(t) = e^{-iat} G(t) \]
\[ M_4(t) = e^{-iat} H_3(t) \]
\[ M_7(t) = e^{-iat} G_3(t) \]
\[ M_8(t) = e^{-iat} H_2(t) \]

Taking L. T. of the above equations and solving them for \( A_0^*(s) \), we get

\[ A_0^*(s) = \frac{N_1(s)}{D_1(s)} \]

where

\[ N_1(s) = M_0^*(s) \{ 1 - q_{23}^*(s)q_{32} (6)^*(s) - q_{2,9} (15)^*(s)q_{9,2}^*(s) \}
+ q_{2,10} (15)^*(s)q_{10,13}^*(s)q_{13,2}^*(s) - q_{24}^*(s) \{ q_{47}^*(s)q_{72} (11)^*(s) \}
+ q_{48}^*(s)q_{8,2} (11)^*(s) + q_{4,13} (15)^*(s)q_{13,2}^*(s) + q_{4,14} (15)^*(s)q_{14,2}^*(s) \} \]
+ q_{91}^*(s)q_{12}^*(s) \{ M_2^*(s) + q_{23}^*(s)M_3^*(s) + q_{24}^*(s)M_4^*(s) + q_{47}^*(s)M_7^*(s) + q_{48}^*(s)M_8^*(s) \}
+ q_{47}^*(s)M_7^*(s) + q_{48}^*(s)M_8^*(s) \]

\[ D_1(s) = 1 - q_{29} (5)^*(s)q_{92}^*(s) - q_{2,10} (6)^*(s)q_{10,13}^*(s)q_{13,2}^*(s) \]
- q_{24}^*(s)q_{4,13}^*(s)q_{13,2}^*(s) - q_{91}^*(s)q_{12}^*(s)q_{24}^*(s)q_{47}^*(s)q_{70}^*(s) \]
+ q_{48}^*(s)q_{8,0}^*(s)) - q_{91}^*(s)q_{12}^*(s)q_{23}^*(s)q_{30}^*(s) \]
- q_{24}^*(s)q_{4,14}^*(s)q_{14,2}^*(s) - q_1 (2,10) (6)^*q_{10,14}^*(s)q_{14,2}^*(s) \]
- \( q_{23}^*(s)q_{32}^*(s) - q_{24}^*(s)q_{47}^*(s)q_{72}^*(s) \)
- \( q_{24}^*(s)q_{48}^*(s)q_{82}^*(s) \) \hspace{1cm} (3.2.121)

In steady state, the availability of the system is given by

\[
A_0 = \lim_{s \to 0} \{ s A_0^*(s) \} = \frac{N_1}{D_1} \hspace{1cm} (3.2.122)
\]

where

\[
N_1 = \mu_0(p_{23}p_{30} + p_{24}p_{47}p_{70} + p_{24}p_{48}p_{80}) + \mu_2 + \mu_{23}p_3 + p_{24}(\mu_4 \\
+ p_{47}\mu_7 + p_{48}\mu_8)
\]

and

\[
D_1 = (\mu_0 + \mu_1)(p_{23}p_{30} + p_{24}p_{47}p_{70} + p_{24}p_{48}p_{80}) + k_1 + (p_{24} + p_{2.10}^{(5)})\mu_{10} \\
+ (p_{24}p_{47} + p_{2.10}^{(5)}p_{10.13} + p_{24}p_{4.13}^{(15)})\mu_{13} + (p_{23} + p_{29}^{(5)})\mu_9 \\
+ (p_{24}p_{48} + p_{24}p_{4.14}^{(15)} + p_{2.10}^{(15)} + p_{10.14})\mu_{14} \hspace{1cm} (3.2.123-3.2.124)
\]

**BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN**

(Repair Time Only)

By probabilistic arguments, we have the following recursive relations for \( B_i(t) \):

\[
B_0(t) = q_{01}(t) \odot B_1(t)
\]

\[
B_1(t) = q_{12}(t) \odot B_2(t)
\]

\[
B_2(t) = q_{23}(t) \odot B_3(t) + q_{24}(t) \odot B_4(t) + q_{29}(t) \odot B_9(t) \\
+ q_{2.10}^{(5)}(t) \odot B_{10}(t)
\]

\[
B_3(t) = W_3(t) + q_{30}(t) \odot B_0(t) + q_{32}^{(6)}(t) \odot B_2(t)
\]

\[
B_4(t) = q_{47}(t) \odot B_7(t) + q_{48}(t) \odot B_8(t) + q_{4.13}^{(15)}(t) \odot B_{13}(t) \\
+ q_{4.14}^{(15)}(t) \odot B_{14}(t)
\]

\[
B_7(t) = q_{70}(t) \odot B_0(t) + q_{72}^{(11)}(t) \odot B_2(t)
\]

\[
B_9(t) = q_{80}(t) \odot B_0(t) + q_{82}^{(12)}(t) \odot B_2(t)
\]
\[ B_0(t) = W_0(t) + q_{92}(t) \odot B_2(t) \]
\[ B_{10}(t) = q_{10,13}(t) \odot B_{13}(t) + q_{10,14}(t) \odot B_{14}(t) \]
\[ B_{13}(t) = q_{13,2}(t) \odot B_2(t) \]
\[ B_{14}(t) = q_{14,2}(t) \odot B_2(t) \]

where

\[ W_3(t) = G(t) = W_9(t) \quad (3.2.125-3.2.136) \]

Taking L.T. of the above equations and solving them for \( B_0^*(s) \), we get

\[ B_0^*(s) = \frac{N_2(s)}{D_1(s)} \quad (3.2.137) \]

where

\[ N_2(s) = q_{01}^*(s)q_{12}^*(s)\{ W_3^*(s)q_{23}^*(s) + W_9^*(s)q_{29}^{(5)}(s)\} \quad (3.2.138) \]

and \( D_1(s) \) is already specified.

In steady state, the total fraction of time for which the system is under repair of ordinary repairman is given by

\[ B_{11} = \lim_{s \to 0} \{ s B_0^*(s) \} = \frac{N_2}{D_1} \quad (3.2.139) \]

where

\[ N_2 = (p_{23} + p_{29}^{(5)}) \mu_9 \quad (3.2.140) \]

\( D_1 \) is already specified.

**BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN (Inspection Time Only)**

By probabilistic arguments, we have the following recursive relations:

\[ BI_0(t) = q_{01}(t) \odot BI_1(t) \]
\[ BI_1(t) = q_{12}(t) \odot BI_2(t) \]
\[ BI_2(t) = W_2(t) + q_{23}(t) \odot BI_3(t) + q_{24}(t) \odot BI_4(t) + q_{29}^{(5)}(t) \odot BI_9(t) \]
\[ + q_{2,10}^{(15)}(t) \odot B_{10}(t) \]

\[ B_{13}(t) = q_{30}(t) \odot B_{1}(t) + q_{32}^{(6)}(t) \odot B_{2}(t) \]

\[ B_{14}(t) = q_{47}(t) \odot B_{7}(t) + q_{48}(t) \odot B_{8}(t) + q_{4,13}^{(15)}(t) \odot B_{13}(t) \]

\[ + q_{4,14}^{(15)}(t) \odot B_{14}(t) \]

\[ B_{17}(t) = q_{70}(t) \odot B_{10}(t) + q_{72}^{(11)}(t) \odot B_{2}(t) \]

\[ B_{18}(t) = q_{80}(t) \odot B_{10}(t) + q_{82}^{(12)}(t) \odot B_{2}(t) \]

\[ B_{19}(t) = q_{92}(t) \odot B_{2}(t) \]

\[ B_{10}(t) = q_{10,13}(t) \odot B_{13}(t) + q_{10,14}(t) \odot B_{14}(t) \]

\[ B_{13}(t) = q_{13,2}(t) \odot B_{2}(t) \]

\[ B_{14}(t) = q_{14,2}(t) \odot B_{2}(t). \] (3.2.141-3.2.151)

where

\[ W_{2}(t) = \overline{H}_{1}(t) \] (3.2.152)

Taking L.T. of the above equations and solving them for \( B_{10}^{*}(s) \), we get

\[ B_{10}^{*}(s) = \frac{N_{3}(s)}{D_{1}(s)} \] (3.2.153)

where

\[ N_{3}(s) = q_{01}^{*}(s)q_{12}^{*}(s) W_{2}^{*}(s) \] (3.2.154)

and \( D_{1}(s) \) is already specified.

In steady state, the total fraction of the time for which the system is under inspection of ordinary repairman is given by

\[ B_{10} = \frac{N_{3}}{D_{1}} \] (3.2.155)

where \( N_{3} = k_{1} \) (3.2.156)

and \( D_{1} \) is already specified.
BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN
(Replacement Time Only)

\[ \text{BR}_0(t) = q_{01}(t) \odot \text{BR}_1(t) \]
\[ \text{BR}_1(t) = q_{12}(t) \odot \text{BR}_2(t) \]
\[ \text{BR}_2(t) = q_{23}(t) \odot \text{BR}_3(t) + q_{24}(t) \odot \text{BR}_4(t) + q_{29}^{(5)}(t) \odot \text{BR}_9(t) + q_{210}^{(5)}(t) \odot \text{BR}_2(t) \]
\[ \text{BR}_3(t) = q_{30}(t) \odot \text{BR}_0(t) + q_{32}^{(6)}(t) \odot \text{BR}_2(t) \]
\[ \text{BR}_4(t) = q_{47}(t) \odot \text{BR}_7(t) + q_{48}(t) \odot \text{BR}_8(t) + q_{413}^{(15)}(t) \odot \text{BR}_{13}(t) + q_{414}^{(15)}(t) \odot \text{BR}_4(t) \]
\[ \text{BR}_7(t) = q_{70}(t) \odot \text{BR}_0(t) + q_{72}^{(11)}(t) \odot \text{BR}_2(t) \]
\[ \text{BR}_8(t) = W_8(t) + q_{80}(t) \odot \text{BR}_0(t) + q_{82}^{(12)}(t) \odot \text{BR}_2(t) \]
\[ \text{BR}_9(t) = q_{92}(t) \odot \text{BR}_2(t) \]
\[ \text{BR}_{10}(t) = q_{10.13}(t) \odot \text{BR}_{13}(t) + q_{10.14}(t) \odot \text{BR}_{14}(t) \]
\[ \text{BR}_{13}(t) = q_{13.2}(t) \odot \text{BR}_2(t) \]
\[ \text{BR}_{14}(t) = W_{14}(t) + q_{14.2}(t) \odot \text{BR}_2(t). \]

where

\[ W_8(t) = W_{14}(t) = \bar{H}_2(t) \quad (3.2.157-3.2.168) \]

Taking L.T. of the above equations and solving them for \( \text{BR}_0^*(s) \), we get

\[ \text{BR}_0^*(s) = \frac{N_4(s)}{D_1(s)} \quad (3.2.169) \]

where

\[ N_4(s) = q_{01}^*(s)q_{12}^*(s) \{ W_8^*(s) + q_{24}^*(s)q_{48}^*(s) \]
\[ + W_{14}^*(s) (q_{24}^*(s) q_{414}^{(15)}*(s) + q_{2.10}^{(5)}(s) q_{10.14}^*(s)) \} \]
\[ (3.2.170) \]

and \( D_1(s) \) is already specified.

In steady state, the total fraction of the time for which the system is under replacement of ordinary repairman is given by
\[ \text{BR}_0 = \frac{N_4}{D_1} \]  

(3.2.171)

where

\[ N_4 = (p_{24} p_{48} + p_{24} p_{4.14}^{(15)} + p_{2.10}^{(5)} p_{10.14}) \mu_{14} \]  

(3.2.172)

and \( D_1 \) is already specified.

**EXPECTED NUMBER OF VISITS BY THE REPAIRMAN**

The following recursive relations for \( V_i(t) \) are obtained:

\[
V_0(t) = Q_{01}(t) \otimes [1 + V_1(t)]
\]

\[
V_1(t) = Q_{12}(t) \otimes V_2(t)
\]

\[
V_2(t) = Q_{23}(t) \otimes V_3(t) + Q_{24}(t) \otimes V_4(t) + Q_{29}^{(5)}(t) \otimes V_9(t) + Q_{2.10}^{(5)}(t) \otimes V_{10}(t)
\]

\[
V_3(t) = Q_{30}(t) \otimes V_0(t) + Q_{32}^{(6)}(t) \otimes V_2(t)
\]

\[
V_4(t) = Q_{47}(t) \otimes V_7(t) + Q_{48}(t) \otimes [1 + V_8(t)] + Q_{4,13}^{(15)}(t) \otimes V_{13}(t) + Q_{4,14}^{(15)}(t) \otimes [1 + V_4(t)]
\]

\[
V_7(t) = Q_{70}(t) \otimes V_0(t) + Q_{72}^{(11)}(t) \otimes [1 + V_2(t)]
\]

\[
V_8(t) = Q_{80}(t) \otimes V_0(t) + Q_{82}^{(12)}(t) \otimes V_2(t)
\]

\[
V_9(t) = Q_{92}(t) \otimes V_2(t)
\]

\[
V_{10}(t) = Q_{10,13}(t) \otimes V_{13}(t) + Q_{10,14}(t) \otimes [1 + V_{14}(t)]
\]

\[
V_{13}(t) = Q_{13.2}(t) \otimes [1 + V_2(t)]
\]

\[
V_{14}(t) = Q_{14.2}(t) \otimes V_2(t).
\]

(3.2.173-3.2.183)

Taking L.S.T. of the above equations and solving them for \( V_0^{**}(s) \), we get

\[
V_0^{**}(s) = \frac{N_5(s)}{D_1(s)}
\]

(3.2.184)

where

\[
N_5(s) = Q_{01}^{**}(s) Q_{12}^{**}(s) \{ Q_{24}^{**}(s) Q_{48}^{**}(s) + Q_{24}^{**}(s) Q_{4,14}^{(15)}^{**}(s) + Q_{2,10}^{(5)^*}(s) \}
\]

(3.2.185)
In steady-state, the number of visits per unit time by the ordinary repairman is given by

\[ v_o = \frac{N_5}{D_1} \]  \hspace{1cm} (3.2.186)

where

\[ N_5 = p_{24}p_{48} + p_{24}p_{4,14}^{(15)} + p_{2,10}^{(5)}p_{10,14} \]  \hspace{1cm} (3.2.187)

**EXPECTED NUMBER OF REPLACEMENTS**

The following recursive relations are obtained for \( R_i(t) \):

\[
\begin{align*}
R_0(t) &= Q_{01}(t) \otimes R_1(t) \\
R_1(t) &= Q_{12}(t) \otimes R_2(t) \\
R_2(t) &= Q_{23}(t) \otimes R_3(t) + Q_{24}(t) \otimes R_4(t) + Q_{29}^{(5)}(t) \otimes R_9(t) \\
&\quad + Q_{2,10}^{(5)}(t) \otimes R_{10}(t) \\
R_3(t) &= Q_{30}(t) \otimes R_0(t) + Q_{32}^{(6)}(t) \otimes R_2(t) \\
R_4(t) &= Q_{47}(t) \otimes R_7(t) + Q_{48}(t) \otimes [1 + R_8(t)] + Q_{4,13}^{(15)}(t) \otimes R_{13}(t) \\
&\quad + Q_{4,14}^{(15)}(t) \otimes [1 + R_{14}(t)] \\
R_7(t) &= Q_{70}(t) \otimes R_0(t) + Q_{72}^{(11)}(t) \otimes [1 + R_2(t)] \\
R_8(t) &= Q_{80}(t) \otimes R_0(t) + Q_{82}^{(12)}(t) \otimes R_2(t) \\
R_9(t) &= Q_{92}(t) \otimes R_2(t) \\
R_{10}(t) &= Q_{10,13}(t) \otimes R_{13}(t) + Q_{10,14}(t) \otimes [1 + R_{14}(t)] \\
R_{13}(t) &= Q_{13,2}(t) \otimes R_2(t) \\
R_{14}(t) &= Q_{14,2}(t) \otimes R_2(t). \hspace{1cm} (3.2.188-3.2.198)
\end{align*}
\]

Taking L.S.T. of the above equations and solving them for \( R_0^{**}(s) \), we get

\[ R_0^{**}(s) = \frac{N_6(s)}{D_1(s)} \]  \hspace{1cm} (3.2.199)

where

\[ N_6(s) = Q_{01}^{**}(s)Q_{12}^{**}(s) \left[ (Q_{48}^{**}(s) + Q_{4,14}^{(15)} Q_{24}^{**}(s) \\
&\quad + Q_{10,14}(s)Q_{2,10}^{(5)\bullet\bullet}(s) \right] \]  \hspace{1cm} (3.2.200)
In steady-state, the expected the number of replacements is the system is given by

$$R_0 = \frac{N_b}{D_1} \quad (3.2.201)$$

where

$$N_b = p_{24} (p_{48} + p_{4,14}) + p_{2,10} + p_{10,14} \quad (3.2.202)$$

**BUSY PERIOD ANALYSIS OF EXPERT REPAIRMAN (Repair Time Only)**

By probabilistic arguments, we have the following recursive relations for $B^c_i(t)$:

$$B_0^c(t) = q_{01}(t) \circ B_1^c(t)$$
$$B_1^c(t) = q_{12}(t) \circ B_2^c(t)$$
$$B_2^c(t) = q_{23}(t) \circ B_3^c(t) + q_{24}(t) \circ B_4^c(t) + q_{29}(t) \circ B_9^c(t)$$
$$+ q_{2,10}(t) \circ B_{10}^c(t)$$
$$B_3^c(t) = q_{30}(t) \circ B_0^c(t) + q_{32}(t) \circ B_2^c(t)$$
$$B_4^c(t) = q_{47}(t) \circ B_7^c(t) + q_{48}(t) \circ B_8^c(t) + q_{4,13}(t) \circ B_{13}^c(t)$$
$$+ q_{4,14}(t) \circ B_{14}^c(t)$$
$$B_7^c(t) = W_7(t) + q_{70}(t) \circ B_0^c(t) + q_{72}(t) \circ B_2^c(t)$$
$$B_8^c(t) = q_{80}(t) \circ B_0^c(t) + q_{82}(t) \circ B_2^c(t)$$
$$B_9^c(t) = q_{92}(t) \circ B_2^c(t)$$
$$B_{10}^c(t) = q_{10,13}(t) \circ B_{13}^c(t) + q_{10,14}(t) \circ B_{14}^c(t)$$
$$B_{13}^c(t) = W_{13}(t) + q_{13,3}(t) \circ B_2^c(t)$$
$$B_{14}^c(t) = q_{14,2}(t) \circ B_2^c(t)$$

where

$$W_{13}(t) = \bar{G}(t) = W_7(t) \quad (3.2.203-3.2.214)$$

Taking L.T. of the above equations and solving them for $B_0^c*(s)$, we get
\[
B_0^e(s) = \frac{N_7(s)}{D_1(s)} \quad (3.2.215)
\]

where
\[
N_7(s) = q_{01}(s)q_{12}(s)[W_7(s)q_{24}(s)(q_{47}(s) + W_{13}(s)(q_{24}(s) + q_{4,13}(5)(s)q_{2,10}(5)(s)q_{10,13}(s)] \quad (3.2.216)
\]

In steady-state, the total fraction of time for which the system is under repair of expert repairman is given by

\[
B_0^e = \frac{N_7}{D_1} \quad (3.2.217)
\]

where
\[
N_7 = p_{24}p_{47}m_7 + (p_{24}p_{4,13}(15) + p_{2,10}(5)p_{10,13})m_{13}. \quad (3.2.218)
\]

**BUSY PERIOD ANALYSIS OF EXPERT REPAIRMAN**
(Inspection Time Only)

The following recursive relations are obtained for \(B_{i_e}(t)\):

\[
\begin{align*}
B_{i_0}(t) &= q_{01}(t) \odot B_{i_0}(t) \\
B_{i_1}(t) &= q_{12}(t) \odot B_{i_1}(t) \\
B_{i_2}(t) &= q_{23}(t) \odot B_{i_2}(t) + q_{24}(t) \odot B_{i_4}(t) + q_{29}(5)(t) \odot B_{i_5}(t) \\
&\quad + q_{2,10}(5)(t) \odot B_{i_7}(t) \\
B_{i_3}(t) &= q_{30}(t) \odot B_{i_0}(t) + q_{32}(13)(t) \odot B_{i_2}(t) \\
B_{i_4}(t) &= W_d(t) + q_{47}(t) \odot B_{i_7}(t) + q_{48}(t) \odot B_{i_8}(t) + q_{4,13}(15)(t) \odot B_{i_13}(t) \\
&\quad + q_{4,14}(15)(t) \odot B_{i_4}(t) \\
B_{i_7}(t) &= q_{70}(t) \odot B_{i_0}(t) + q_{72}(11)(t) \odot B_{i_2}(t) \\
B_{i_8}(t) &= q_{80}(t) \odot B_{i_0}(t) + q_{82}(12)(t) \odot B_{i_2}(t) \\
B_{i_9}(t) &= q_{92}(t) \odot B_{i_2}(t) \\
B_{i_{10}}(t) &= W_{10}(t) + q_{10,13}(t) \odot B_{i_{13}}(t) + q_{10,14}(t) \odot B_{i_{14}}(t) \\
B_{i_{11}}(t) &= q_{13,2}(t) \odot B_{i_2}(t) \\
B_{i_{14}}(t) &= q_{14,2}(t) \odot B_{i_2}(t). \quad (3.2.219-3.2.230)
\end{align*}
\]
where
\[ W_2(t) = H_c(t) = W_{10}(t) \]  
(3.2.231)

Taking L.T. of the above equations and solving them for \( BI_{i0}^c*(s) \), we get
\[ BI_{i0}^c*(s) = \frac{N_8(s)}{D_1(s)} \]  
(3.2.232)

where
\[ N_8(s) = q_{01}*(s)q_{12}*(s) [W_4*(s)q_{24}*(s) + W_{10}*(s)q_{2.10}(s)] \]  
(3.2.233)

In steady-state, the total fraction of the time for which the system is under inspection of expert repairman is given by
\[ BI_{i0}^c = \frac{N_8}{D_1} \]  
(3.2.234)

where \( N_8 = (p_{24} + p_{2.10}(s))\mu_{10} \)  
(3.2.235)

**EXPECTED NUMBER OF VISITS BY THE EXPERT REPAIRMAN**

The following recursive relations for \( V_i^c(t) \) are obtained:
\[ V_0^c(t) = Q_{01}(t)\bigcirc V_1^c(t) \]
\[ V_1^c(t) = Q_{12}(t)\bigcirc V_2^c(t) \]
\[ V_2^c(t) = Q_{23}(t)\bigcirc V_3^c(t) + Q_{24}(t)\bigcirc [1 + V_4^c(t)] + Q_{29}(s)\bigcirc V_9^c(t) \]
\[ + Q_{2.10}(s)\bigcirc [1 + V_{10}^c(t)] \]
\[ V_3^c(t) = Q_{30}(t)\bigcirc V_0^c(t) + Q_{32}(s)\bigcirc V_2^c(t) \]
\[ V_4^c(t) = Q_{47}(t)\bigcirc V_7^c(t) + Q_{48}(s)\bigcirc V_8^c(t) + Q_{4.13}(s)\bigcirc V_{13}^c(t) \]
\[ + Q_{4.14}(s)\bigcirc V_{4}^c(t) \]
\[ V_7^c(t) = Q_{70}(t)\bigcirc V_0^c(t) + Q_{72}(s)\bigcirc V_2^c(t) \]
\[ V_8^c(t) = Q_{80}(t)\bigcirc V_0^c(t) + Q_{82}(s)\bigcirc V_2^c(t) \]
\[ V_9^c(t) = Q_{92}(t)\bigcirc V_2^c(t) \]
\[ V_{10}^c(t) = Q_{10,13}(t) \circ V_{13}^c(t) + Q_{10,14}(t) \circ V_{14}^c(t) \]
\[ V_{13}^c(t) = Q_{13,2}(t) \circ V_2^c(t) \]
\[ V_{14}^c(t) = Q_{14,2}(t) \circ V_2^c(t). \]  
(3.2.236-3.2.246)

Taking L.S.T. of the above equations and solving them for \( V_0^c(s) \), we get
\[ V_0^c(s) = N_9(s)/D_1(s) \]  
(3.2.247)

where
\[ N_9(s) = Q_{01}(s)Q_{12}(s) [Q_{24}(s) + Q_{2,10}^{(5)}(s)] \]  
(3.2.248)

In steady-state, the total number of visits of the expert on the system is given by
\[ V_0^c = \frac{N_o}{D_1} \]  
(3.2.249)

where
\[ N_o = p_{24} + p_{2,10}^{(5)} \]  
(3.2.250)

**ANALYSIS OF ACTIVATION TIME**

The following recursive relations are obtained for \( AT_i(t) \):
\[ AT_0(t) = q_{01}(t) \circ AT_1(t) \]
\[ AT_1(t) = W_1(t) + q_{12}(t) \circ AT_2(t) \]
\[ AT_2(t) = q_{23}(t) \circ AT_3(t) + q_{24}(t) \circ AT_4(t) + q_{29}(t) \circ AT_9(t) \]
\[ + q_{2,10}^{(5)}(t) \circ AT_{10}(t) \]
\[ AT_3(t) = q_{30}(t) \circ AT_0(t) + q_{32}^{(6)}(t) \circ AT_2(t) \]
\[ AT_4(t) = q_{47}(t) \circ AT_7(t) + q_{48}(t) \circ AT_8(t) + q_{4,13}^{(15)}(t) \circ AT_{13}(t) \]
\[ + q_{4,14}^{(15)}(t) \circ AT_{14}(t) \]
\[ AT_7(t) = q_{70}(t) \circ AT_0(t) + q_{72}^{(11)}(t) \circ AT_2(t) \]
\[ AT_8(t) = q_{80}(t) \circ AT_0(t) + q_{82}^{(12)}(t) \circ AT_2(t) \]
\[ AT_9(t) = q_{92}(t) \circ AT_2(t) \]
\[ AT_{10}(t) = q_{10,13}(t) \circ AT_{13}(t) + q_{10,14}(t) \circ AT_{14}(t) \]
\[ AT_{13}(t) = q_{13.2}(t) \odot AT_2(t) \]
\[ AT_{14}(t) = q_{14.2}(t) \odot AT_2(t) \]

where
\[ W_1(t) = \bar{W}(t) = \mu_1 \]  
(3.2.251-3.2.262)

Taking L.S.T. of the above equations and solving them for \( R_0^{**}(s) \), we get

\[ AT_0^{**}(s) = \frac{N_{10}(s)}{D_1(s)} \]  
(3.2.263)

where

\[ N_{10}(s) = W_1(s) q_{23}^{***(s)} [1 - q_{23}^{***(s)} q_{32}^{(6)*}(s) - q_{92}^{***(s)} q_{29}^{(5)*}(s) \]
\[ - q_{24}^{(5)*}(s) (q_{13.2}^{*(s)} q_{10.13}^{*(s)} + q_{14.2}^{*(s)} q_{10.14}^{*(s)}) \]
\[ - q_{47}^{*(s)} q_{72}^{*(s)} + q_{32}^{*(s)} q_{48}^{*(s)} \]
\[ + q_{13.2}^{*(s)} q_{4.13}^{(15)*}(s) - q_{14.2}^{*(s)} q_{4.14}^{(15)*}(s)] \]  
(3.2.264)

In steady-state, the total activation time of the system is given by

\[ AT_0 = \frac{N_{10}}{D_1} \]  
(3.2.265)

where

\[ N_{10} = [p_{23} p_{30} + p_{24} (p_{47} p_{70} + p_{48} p_{80})] \mu_1 \]  
(3.2.266)

and \( D_1 \) is already specified.

**PROFIT ANALYSIS**

The expected total profit incurred to the system is steady-state is given by

\[ P_{32} = C_0 A_0 - C_1 B_0 - C_2 B_1 - C_3 B R_0 - C_4 V_0 - C_5 R_0 - C_6 B_0^e \]
\[ - C_7 B_1^e - C_8 V_0^e - C_9 AT_0. \]  
(3.2.267)

where

\[ C_0 = \text{revenue per unit up time of the system} \]
\( C_1 = \) cost per unit time for which ordinary repairman is busy in repair
\( C_2 = \) cost per unit time for which ordinary repairman is busy in inspection
\( C_3 = \) cost per unit time for which ordinary repairman is busy in replacement
\( C_4 = \) cost per visit of ordinary repairman
\( C_5 = \) Cost per replacement of the unit
\( C_6 = \) Cost per unit time for which expert repairman is busy in repair
\( C_7 = \) Cost per unit time for which expert repairman is busy in inspection
\( C_8 = \) Cost per visit of the expert repairman
\( C_9 = \) Cost per unit activation time.

**PARTICULAR CASE**

Let us assume and the remaining distribution are same as in general case. Therefore, we get

\[
g(t) = \alpha_1 e^{-\lambda t} \quad ; \quad g_x(t) = \alpha_2 e^{-\lambda x t} \quad ; \quad W(t) = \beta e^{-\mu t}
\]

\[
h_1(t) = \gamma_1 e^{-\gamma_1 t} \quad ; \quad h_x(t) = \gamma_2 e^{-\gamma_2 t} \quad ; \quad h_2(t) = \gamma e^{-\gamma t}
\]

and the remaining distribution are same as in general case. Therefore, we get

\[
p_{01} = p_{12} = 1 \quad ; \quad p_{23} = \frac{p_1 \gamma_1}{\lambda + \gamma_1} \quad ; \quad p_{24} = \frac{p_2 \gamma_1}{\lambda + \gamma_1}
\]

\[
p_{25} = \frac{\lambda}{\lambda + \gamma_1} \quad ; \quad p_{29}^{(5)} = \frac{p_1 \lambda}{\lambda + \gamma_1} \quad ; \quad p_{2,10}^{(5)} = \frac{p_2 \lambda}{\lambda + \gamma_1}
\]

\[
p_{30} = \frac{\alpha_1}{\lambda + \alpha_1} \quad ; \quad p_{36} = \frac{\lambda}{\lambda + \alpha_1} \quad ; \quad p_{32}^{(6)} = \frac{\lambda}{\lambda + \alpha_1}
\]

\[
p_{47} = \frac{p_1 \gamma_2}{\lambda + \gamma_2} \quad ; \quad p_{48} = \frac{p_2 \gamma_2}{\lambda + \gamma_2} \quad ; \quad p_{4,15} = \frac{\lambda}{\lambda + \gamma_2}
\]
Using the above equations and the equations (3.2.105), (3.2.122), (3.2.139), (3.2.155), (3.2.171), (3.2.177), (3.2.186), (3.2.201), (3.2.217), (3.2.234), (3.2.249), (3.2.265) and (3.2.267). We can have MTSF, availability and profit for this particular case.

**GRAPHICAL INTERPRETATION FOR MODEL 3.1**

For the graphical interpretation, the mentioned particular case is considered. The behaviour of MTSF and the availability \((A_0)\) with respect to failure rate \((\lambda)\) for different values of repair rate \((\alpha_1)\) is shown as in Figs. 3.3 and 3.4 respectively. It is clear from the graphs that the MTSF and availability decrease with increase in the values of failure rate. However, their values become higher for higher values of repair rate \((\alpha_1)\).
Fig. 3.5 shows the behaviour of profit ($P_{31}$) with respect to cost per replacement ($C_5$) for different values of probability ($p_2$) that the unit is not repairable. Following conclusions can be drawn:

(i) If $p_2 = 0.1$, then profit will become negative for a very high value of cost ($C_5$) and hence it can be concluded that if there are less chances of replacement, then the system is more profitable.

(ii) If $p_2 = 0.5$, then $P_{31} > or = or < 0$ according as $C_5 < or = or > 3883$. i.e. The system is profitable if $C_5 \leq 3883$. Hence the cost per replacement should not exceed.

(iii) If $p_2 = 0.9$ then $P_{31} > or = or <$ according as $C_5 < or = or > 2161$ i.e. the system is profitable if $C_5 \leq 2161$

Fig. 3.6. depicts the behaviour of profit ($P_{31}$) with respect to cost per down time ($C_9$) for different values of activation rate ($\beta$). It is interpreted as follows:

(i) Initially the profit has higher values for lower values of $\beta$, but a stage comes when the values of profit become lower for higher values of $\beta$.

(ii) Profit decreases on increasing the cost ($C_9$)

(iii) $P_{31} > or = < 0$ according as $C_9 < or = or > 2376$ irrespective of the values of $\beta(1 \leq \beta \leq 3)$. It is, therefore, concluded that the down time cost per unit time should not exceed 2376 for profitability of the system. If it is so, then activation time should be reduced.

GRAPHICAL INTERPRETATION FOR MODEL 3.2

For the graphical interpretation, the mentioned particular case is considered. The behaviour of MTSF and the availability ($A_0$) with respect to failure rate ($\lambda$) for different values of repair rate ($\alpha_1$) is
shown as in Figs. 3.7 and 3.8 respectively. It is clear from the graphs that the MTSF and availability decrease with increase in the values of failure rate. However, their values become higher for higher values of repair rate ($\alpha_1$).

Fig. 3.9 shows the behaviour of profit ($P_{32}$) with respect to cost per replacement ($C_3$) for different values of probability ($p_2$) that the unit is not repairable. Following conclusions can be drawn:

(i) If $p_2 = 0.1, p_2 = 0.5$, then profit will become negative for a very high value of cost ($C_3$) and hence it can be concluded that if there are less chances of replacement. Then the system is more profitable.

(ii) If $p_2 = 0.9$ then $P_{32} > \text{or} = \text{or} < 0$ according as $C_3 < \text{or} = \text{or} > 2355$ i.e. the system is profitable if $C_3 \leq 2355$. Hence the cost per replacement should not exceed.

Fig. 3.10 shows the behaviour of profit ($P_{32}$) with respect to cost per down time ($C_g$) for different values of activation rate ($\beta$). It is interpreted as follows:

(i) If $\beta = 0.1$, then $P_{32} > \text{or} = \text{or} < 0$ according as $C_g < \text{or} = \text{or} > 161$ i.e. the system is profitable if $C_g \leq 161$. Hence the cost per down time should not exceed.

(ii) If $\beta = 2.1$ then $P_{32} > \text{or} = \text{or} < 0$ according as $C_g < \text{or} = \text{or} > 3050$ i.e. the system is profitable if $C_g \leq 3050$.

(iii) For $\beta = 4.1$. The profit becomes negative at a higher value of cost ($C_g$) as compared that for lower values of $\beta$. 

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Failure Rate

Fig. 3.3

MTSF VERSUS FAILURE RATE (\( \alpha \)) FOR DIFFERENT VALUES OF REPAIR RATE (\( \alpha \))

\[ p_1 = 0.5, p_2 = 0.5, y = 5, y = 5, \quad \alpha = 1, \alpha = 2, \alpha = 3 \]
Failure Rate (X)

Fig. 3.4

Availability versus Failure Rate (X) for different values of repair rate (a-).

Repair Rate (a-)

A

0.1  0.15  0.2  0.25  0.3  0.35  0.4  0.45  0.5  0.55  0.6  0.65  0.7  0.75  0.8  0.85  0.9  0.95

0  0.25  0.5  0.75  1

p = 0.5, p = 1, p = 5, p = 8, p = 10

A = 0.5, A = 0.75, A = 1

a = 1, a = 2, a = 3, a = 4

Failure Rate (X)
Figure 3.6

Profit $V$ versus Cost $C^9$ for different values of activation rate $B$. 

Graphical representation showing the relationship between profit and cost for different values of activation rate $B$. The axes are labeled as follows:

- X-axis: Cost $C^9$
- Y-axis: Profit

Legend:
- $B=3$
- $B=2$
- $B=1$
Profit versus cost ($C^p$) for different values of activation rate ($\gamma$).
MTSF VERSUS FAILURE RATE ($\lambda$) FOR DIFFERENT VALUES OF REPAIR RATE ($\alpha$)
Fig. 3.8

Failure Rate (A)

Availability versus failure rate (A) for different values of repair rate (\( \alpha \)).
Profit versus cost (C<sub>6</sub>) for different values of probability (p<sub>2</sub>).
PROFIT VERSUS COST ($C_9$) FOR DIFFERENT VALUES OF ACTIVATION RATE ($\beta$)

\[ y = 5, y_1 = 8, z = 10, \alpha_1 = 2, \alpha_2 = 3, \lambda = 0.5, \gamma = 1000, C_1 = 500, C_2 = 30, C_3 = 75, C_4 = 25, C_5 = 100, C_6 = 40 \]