CHAPTER 7

COMPARATIVE STUDY AND CONCLUSIONS
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In the preceding chapters, reliability models have been discussed for one-unit system, two-unit cold standby system and two-unit hot standby system under various assumptions including the concepts of inspection and replacement. All the models studied in the thesis cannot be good in all the situations. Some of them may be better in some of the situations and the others may be better in some other situations. To know which of the models is better in what situation, the comparative study of the models discussed within the chapters and those between the chapters has been made on the basis of their profits. The conclusions have been drawn accordingly.

COMPARATIVE STUDY OF THE MODELS STUDIED WITHIN THE CHAPTERS

Comparison Between Model 2.1 and 2.2

Comparative study is made for the particular cases assuming all the general distributions as exponential and have already been mentioned in the concerned models. The numerical values assumed and given to various rates/costs have been mentioned along with the graphs.

Comparison Between Model 2.1 and 2.2

Fig. 7.1 shows the behaviour of difference between profits $P_{22}$ (Model 2.2) and $P_{21}$ (Model 2.1) with respect to repair rate ($\alpha_2$) for different values of probability that unit is not repairable. Following conclusions are drawn:

(i) If $p_2 = 0.1$, then $P_{22} - P_{21} > \text{ or } = \text{ or } < 0$ according as $\alpha_2 > \text{ or } = \text{ or } < 5.45$. Hence the Model 2.2 is better or worse than Model 2.1
according as $\alpha_2 > or < 5.45$. Both the models are equally good if $\alpha_2 = 5.45$.

(ii) If $p_2 = 0.5$, then $P_{22} - P_{21} > or = or < 0$ according as $\alpha_2 > or = or < 16.05$. Hence Model 2.2 is better or worse than Model 2.1 according as $\alpha_2 > or < 16.05$. Both the models are equally good if $\alpha_2 = 16.05$.

(iii) If $p_2 = 0.9$, then $P_{22} - P_{21} < 0$ irrespective of the values of repair rate. Hence, it is concluded that Model 2.1 is better than Model 2.2 for $p_2 = 0.9$.

(iv) It can also be concluded that if the chances of non-reparability become more, Model 2.1 becomes more profitable than Model 2.2.

Comparison Between Model 3.1 and 3.2

Behaviour of the difference between profits $P_{32}$ (Model 3.2) and $P_{31}$ (Model 3.1) with respect to repair rate ($\alpha_2$) for different values of probability ($p_2$) that unit is not repairable. Following conclusions are drawn:

(i) If $p_2 = 0.1$, then $P_{32} - P_{31} > or = or < 0$ according as $\alpha_2 > or = or < 1.92$. Hence, Model 3.2 is better or worse than Model 3.1 according as $\alpha_2 > or < 1.92$. Both the models are equally good if $\alpha_2 = 1.92$.

(ii) If $p_2 = 0.5$, then $P_{32} - P_{31} > or = or < 0$ according as $\alpha_2 > or < 5.65$. Hence, Model 3.2 is better or worse than Model 3.1 according as $\alpha_2 > or < 5.65$. Both the models are equally good if $\alpha_2 = 5.65$.

(iii) If $p_2 = 0.9$, then $P_{32} - P_{31} < 0$ irrespective of the values of repair rate. Hence, it is concluded that Model 3.1 is better than Model 3.2 for $p_2 = 0.9$. 
(iv) It can also be concluded that if the chances of non-reparability become more, Model 3.1 becomes better than Model 3.2.

**Comparison Between Model 4.1 and 4.2**

(i) If $p_2 = 0.1$, then $P_{32} - P_{31} > \text{ or } = \text{ or } < 0$ according as $\alpha_2 > \text{ or } = \text{ or } < 1.92$. Hence, Model 4.2 is better or worse than Model 4.1 according as $\alpha_2 > \text{ or } < 1.58$. Both the models are equally good if $\alpha_2 = 1.58$.

(ii) If $p_2 = 0.5$, then $P_{32} - P_{31} > \text{ or } = \text{ or } < 0$ according as $\alpha_2 > \text{ or } < 4$. Hence, Model 4.2 is better or worse than Model 4.1 according as $\alpha_2 > \text{ or } < 5.65$. Both the models are equally good if $\alpha_2 = 4$.

(iii) If $p_2 = 0.9$, then $P_{32} - P_{31} < 0$ irrespective of the values of repair rate. Hence, it is concluded that Model 4.1 is better than Model 4.2 for $p_2 = 0.9$.

(iv) It can also be concluded that if the chances of non-reparability become more, Model 4.1 becomes better than Model 4.2.

**Comparison Between Model 4.1 and 4.2**

Fig. 7.4 shows the behaviour of difference between profits $P_{52}$ (Model 2.2) and $P_{51}$ (Model 2.1) with respect to repair rate ($\alpha_2$) for different values of probability that unit is not repairable. Following conclusions are drawn:

(i) If $p_2 = 0.1$, then $P_{22} - P_{21} > \text{ or } = \text{ or } < 0$ according as $\alpha_2 > \text{ or } = \text{ or } < 1.59$. Hence the Model 2.2 is better or worse than Model 2.1 according as $\alpha_2 > \text{ or } < 1.59$. Both the models are equally good if $\alpha_2 = 1.59$.

(ii) If $p_2 = 0.5$, then $P_{52} - P_{51} > \text{ or } = \text{ or } < 0$ according as $\alpha_2 > \text{ or } = \text{ or } < 2.18$. Hence Model 2.2 is better or worse than Model 2.1.
according as $\alpha_2 >$ or $< 2.18$. Both the models are equally good if $\alpha_2 = 2.18$.

(iii) If $p_2 = 0.9$, then $P_{52} - P_{51} < 0$ irrespective of the values of repair rate. Hence, it is concluded that Model 2.1 is better than Model 2.2 for $p_2 = 0.9$.

(iv) It can also be concluded that if the chances of non-repairability become more, Model 2.1 becomes more profitable than Model 2.2.

Comparison Between Model 3.1 and 3.2

Behaviour of the difference between profits $P_{62}$ (Model 3.2) and $P_{61}$ (Model 3.1) with respect to repair rate ($\alpha_2$) for different values of probability ($p_2$) that unit is not repairable. Following conclusions are drawn:

(i) If $p_2 = 0.1$, then $P_{62} - P_{61}$ is always greater than 0 for $\alpha_2 \geq 4$. Hence, Model 3.2 is better than Model 3.1 for as $\alpha_2 \geq 4$.

(ii) If $p_2 = 0.5$, then $P_{62} - P_{61} >$ or $= 0$ according as $\alpha_2 >$ or $< 42.5$. Hence, Model 3.2 is better or worse than Model 3.1 according as $\alpha_2 >$ or $< 42.5$. Both the models are equally good if $\alpha_2 = 42.5$.

(iii) If $p_2 = 0.9$, then $P_{32} - P_{31} < 0$ irrespective of the values of repair rate. Hence, it is concluded that Model 3.1 is better than Model 3.2 for $p_2 = 0.9$.

(iv) It can also be concluded that if the chances of non-repairability become more, Model 3.1 becomes better than Model 3.2.

COMPARISON OF THE MODELS STUDIED BETWEEN THE CHAPTERS

Comparative study of the models is done on the basis of profits incurred to the system for the particular cases mentioned in the concerned models. Assumed numerical values have been shown along with the graphs.
Comparison Between Model 2.1 and Model 3.1

Fig. 7.6 shows the behaviour of the difference between $P_{31} - K$ and $P_{21}$ with respect to cost ($K$) of installing an additional unit for different values of activation rate ($\beta$), where $P_{21}$ and $P_{31}$ are the profits for Model 2.1 and Model 3.1 respectively and $K$ is the cost for installing an additional unit. The following conclusions are drawn:

(i) If $\beta = 5$, $(P_{31} - K) - P_{21} \geq 0$ according as $K \leq 75$. So, Model 3.1 is better or worse than Model 2.1 if $K \leq 75$. Both the models are equally good if $K = 75$.

(ii) If $\beta = 5$, $(P_{31} - K) - P_{21} \geq 0$ according as $K \leq 105$. So, Model 3.1 is better or worse than Model 2.1 if $K \leq 105$. Both the models are equally good if $K = 105$.

(iii) If $\beta = 5$, $(P_{31} - K) - P_{21} \geq 0$ according as $K \leq 115$. So, Model 3.1 is better or worse than Model 2.1 if $K \leq 115$. Both the models are equally good if $K = 115$.

(iv) The difference becomes higher for higher values of activation rate.

Comparison Between Model 2.2 and 3.2

Fig. 7.7 shows the behaviour of the difference between $P_{32} - K$ and $P_{22}$ with respect to cost ($K$) of installing an additional unit for different values of activation rate ($\beta$), where $P_{22}$ and $P_{32}$ are the profits for Model 2.2 and Model 3.2 respectively and $K$ is the cost for installing an additional unit. The following conclusions are drawn:

(i) If $\beta = 5$, $(P_{31} - K) - P_{21} \geq 0$ according as $K \leq 100$. So, Model 3.1 is better or worse than Model 2.1 if $K \leq 100$. Both the models are equally good if $K = 100$.
(ii) If \( \beta = 5 \), \((P_{3i} - K) - P_{2i} > or < 0\) according as \( K < or > 129 \). So, Model 3.1 is better or worse than Model 2.1 if \( K < or > 129 \). Both the models are equally good if \( K = 129 \).

(iii) If \( \beta = 5 \), \((P_{3i} - K) - P_{2i} > or < 0\) according as \( K < or = or > 138 \). So, Model 3.1 is better or worse than Model 2.1 if \( K < or > 138 \). Both the models are equally good if \( K = 138 \).

(iv) The difference becomes higher for higher values of activation rate.

**Comparison Between Model 3.1 and 4.1**

Behaviour of the difference between \( P_{4i} \) and \( P_{3i} \) with respect to failure rate (\( \alpha \)) for different values of activation rate (\( \beta \)) and cost per unit down time (\( C_9 \)) is shown as in Fig. 7.8; where \( P_{3i} \) and \( P_{4i} \) are the profits for Model 3.1 and Model 4.1 respectively. Following conclusions have been drawn:

(i) For \( \beta = 20 \), \( C_9 = 10 \) or 20 ; \( P_{4i} - P_{3i} > 0 \) irrespective of the values of failure rate (\( \alpha \)). Therefore, for this particular case Model 4.1 is better than Model 3.1.

(ii) For \( \beta = 180 \), \( C_9 = 10 \); \( P_{4i} - P_{3i} > or = or < 0 \) according as \( \alpha < or = or > 3.35 \). Hence, for this particular case, Model 4.1 is better or worse than Model 3.1 according as \( \alpha < or > 3.35 \). Both the models are equally good if \( \alpha = 3.35 \).

(iii) For \( \beta = 180 \), \( C_9 = 20 \); \( P_{4i} - P_{3i} > or = or < 0 \) according as \( \alpha < or = or > 9.2 \) Hence, for this particular case, Model 4.1 is better or worse than Model 3.1 according as \( \alpha < or > 9.2 \). Both the models are equally good if \( \alpha = 9.2 \).

(v) It can also be noticed from the graphs that the difference decreases for higher values of \( \beta \) but increases for higher values of \( C_9 \) i.e. Model 4.1
may become better than Model 3.1 if values of $C_9$ become higher and values of $\beta$ become lower.

**Comparison Between 3.2 and 4.2**

Fig. 7.9 depicts the behaviour of the difference $(P_{42} - P_{32})$ with respect to failure rate ($\alpha$) and different values of activation rate ($\beta$) and cost per unit down time ($C_9$) where $P_{32}$ and $P_{42}$ are the profits for Model 3.2 and 4.2 respectively. Following can be interpreted:

(i) For $\beta = 1$, $C_9 = 50$; $P_{42} - P_{32} > 0$ or $< 0$ according as $\alpha < or = or > 1.14$. So, for this particular case, Model 4.2 is better or worse than Model 3.2 according as $\alpha < or > 1.14$. Both the models are equally good if $\alpha = 114$.

(ii) For $\beta = 1$, $C_9 = 450$; $P_{42} - P_{32} > 0$ or $< 0$ according as $\alpha < or = or > 0.36$. So, for this particular case, Model 4.2 is better or worse than Model 3.2 according as $\alpha < or > 0.36$. Both the models are equally good if $\alpha = 0.36$.

(iii) For $\beta = 1$, $C_9 = 50$; $P_{42} - P_{32} > 0$ or $< 0$ according as $\alpha < or = or > 0.18$. So, for this particular case, Model 4.2 is better or worse than Model 3.2 according as $\alpha < or > 0.18$. Both the models are equally good if $\alpha = 0.18$.

(iv) For $\beta = 2$, $C_9 = 450$; Model 3.2 is better than Model 4.2 irrespective of the values of failure rate ($\alpha$).

**Comparison Between Model 4.2 and Model 5.1**

Fig. 7.10 shows the behaviour of the difference between $P_{4i} - P_{5i}$ with respect to cost ($C_4$) for different values of $C_0$ where $P_{4i}$ and $P_{5i}$ are the profits for Model 4.1 and Model 5.1 respectively. The following conclusions are drawn:
(i) If \( C_0 = 1000 \), then \( P_{41} - P_{51} > \) or \( = \) or \( < 0 \) according as \( C_4 < \) or \( = \) or \( > 192 \). So, Model 4.1 is better or worse than Model 5.1 if \( C_4 < 192 > 192 \). Both the Models are equally good if \( C_4 = 191.6 \).

(ii) If \( C_0 = 4000 \), then \( P_{41} - P_{51} > \) or \( = \) or \( < 0 \) according as \( C_4 < \) or \( = \) or \( > 367 \). So, Model 4.1 is better or worse than Model 5.1 if \( C_4 < 0 > 367 \). Both the models are equally good if \( C_4 = 367 \).

(iii) For \( C_0 = 7000 \) or \( C_0 = 10000 \); the difference becomes negative at a very high cost (\( C_4 \)). Therefore, Model 5.1 is better than Model 4.1 till \( C_4 \) becomes very high.

**COMPARISON BETWEEN MODEL 4.2 AND MODEL 5.2**

Fig. 7.11 shows the behaviour of the difference between \( P_{42} - P_{52} \) with respect to cost \( C_4 \) for different values of \( C_0 \), where \( P_{42} \) and \( P_{52} \) are the profits for Model 4.2 and Model 5.2 respectively. The following conclusions are drawn:

(i) If \( C_0 = 1000 \), then \( P_{42} - P_{52} > \) or \( = \) or \( < 0 \) according as \( C_4 < \) or \( = \) or \( > 70 \). So, Model 4.2 is better or worse than Model 5.2 if \( C_4 < 0 > 70 \). Both the models are equally good if \( C_4 = 70 \).

(ii) If \( C_0 = 4000 \), then \( P_{42} - P_{52} > \) or \( = \) or \( < 0 \) according as \( C_4 < \) or \( = \) or \( > 295 \). So Model 4.2 is better or worse than Model 5.2 if \( C_4 < 0 > 295 \). Both models are equally good if \( C_4 = 295 \).

(iii) If \( C_0 = 7000 \) then \( P_{42} - P_{52} > \) or \( = \) or \( < 0 \) according as \( C_4 < \) or \( = \) or \( > 468 \). So model 4.2 is better or worse than Model 5.2 according as \( C_4 < 0 > 468 \). Both the models are equally good if \( C_4 = 468 \).

(iv) If \( C_0 = 10000 \), the difference becomes negative at a very high cost (\( C_4 \)). Therefore, Model 5.1 is better.

**Comparison Between Model 5.1 and Model 6.1**
Pattern of the difference \((P_{6i} - P_{5i})\) with respect to cost per random inspection \((C_{10})\) for different values of failure rate \((\alpha)\) is shown as in Fig. 7.12: where \(P_{5i}\) and \(P_{6i}\) are the profits for Model 5.1 and Model 6.1 respectively. Following conclusions are drawn:

(i) For \(\alpha = 0.1\), \(P_{6i} - P_{5i} > \) or = or < 0 according as \(C_{10} < \) or = or > 128. Therefore, Model 6.1 is better or worse than Model 5.1 according as \(C_{10} < \) or > 128. Both the models are equally good if \(C_{10} = 128\).

(ii) For \(\alpha = 0.4\), \(\alpha = 0.1\), \(P_{6i} - P_{5i} > \) or = or < 0 according as \(C_{10} < \) or = or > 104. Therefore, Model 6.1 is better or worse than Model 5.1 according as \(C_{10} < \) or > 104. Both the models are equally good if \(C_{10} = 104\).

(iv) For \(\alpha = 0.7\), \(P_{6i} - P_{5i} > \) or = or < 0 according as \(C_{10} < \) or = or > 89. Therefore, Model 6.1 is better or worse than Model 5.1 according as \(C_{10} < \) or > 89. Both the models are equally good if \(C_{10} = 89\).

(v) For \(\alpha = 1\), \(P_{6i} - P_{5i} > \) or = or < 0 according as \(C_{10} < \) or = or > 78. Therefore, Model 6.1 is better or worse than Model 5.1 according as \(C_{10} < \) or > 78. Both the models are equally good if \(C_{10} = 78\).

**Comparison Between Model 5.2 and 6.2**

Fig. 7.13 shows the behaviour of the difference \((P_{62} - P_{52})\) with respect to cost per random inspection \((C_{10})\) for different values of failure rate \((\alpha)\); where \(P_{52}\) and \(P_{62}\) are the profits for Model 5.2 and Model 6.2 respectively. Following conclusions are drawn:

(i) For \(\alpha = 0.1\), \(P_{62} - P_{52} > \) or = or < 0 according as \(C_{10} < \) or = or > 102. So Model 6.2 is better or worse than Model 6.1 according as \(C_{10} < \) or > 102. Both the models are equally good if \(C_{10} = 102\).
(ii) For $\alpha = 0.4$, $P_{62} - P_{52} \geq 0$ according as $C_{10} < 125$. So Model 6.2 is better or worse than Model 6.1 according as $C_{10} < 125$. Both the models are equally good if $C_{10} = 125$.

(iii) For $\alpha = 0.7$, $P_{62} - P_{52} \geq 0$ according as $C_{10} < 129$. So Model 6.2 is better or worse than Model 6.1 according as $C_{10} < 129$. Both the models are equally good if $C_{10} = 129$.

(iv) For $\alpha = 1$, $P_{62} - P_{52} \geq 0$ according as $C_{10} < 130$. So Model 6.2 is better or worse than Model 6.1 according as $C_{10} < 130$. Both the models are equally good if $C_{10} = 130$.

(v) It can also be noticed that initially (approximately up to $C_{10} = 200$), $P_{62} - P_{52}$ is higher for higher values of $\alpha$, but for $C_{10} > 200$, this pattern reverses and becomes lower for higher values of failure rate ($\alpha$).
DIFFERENCE OF PROFIT (P_{22} - P_{21}) versus REPAIR RATE (a_2) for different values of probability (p_2).
DIFFERENCE OF PROFIT ($P_{32}-P_{31}$) VERSUS REPAIR RATE ($c_2$) FOR DIFFERENT VALUES OF PROBABILITY ($p_2$)

$\gamma = 5, \gamma_1 = 8, \gamma_2 = 10, \alpha = 2, \lambda = 0.5, \beta = 10, C_0 = 100, C_1 = 75, C_2 = 50, C_3 = 40, C_4 = 30, C_5 = 25, C_6 = 100, C_7 = 50, C_8 = 40, C_9 = 20$

Fig. 7.2
DIFFERENCE OF PROFIT ($P_{42} - P_{41}$) VERSUS REPAIR RATE ($a_2$) FOR DIFFERENT VALUES OF PROBABILITY ($p_2$)
DIFFERENCE OF PROFIT ($P^2-P_1^2$) VERSUS REPAIR RATE ($a_2$) FOR DIFFERENT VALUES OF PROBABILITY ($p_2$).
DIFFERENCE OF PROFIT ($P_{62} - P_{61}$) VERSUS REPAIR RATE ($a_2$) FOR DIFFERENT VALUES OF PROBABILITY ($p_2$)
Fig. 7.6

DIFFERENCE OF PROFIT \((P_3 - K) - (P_2 - 1)\) VERSUS COST \(K\) FOR DIFFERENT VALUES OF ACTIVATION RATE \((\gamma)\)

VALUES OF ACTIVATION RATE \((\gamma)\)

DIFFERENCE OF PROFIT \((P_3 - K) - (P_2 - 1)\) VERSUS COST \(K\) FOR DIFFERENT ACTIVATION RATES
Fig. 7.7

DIFFERENCE OF PROFIT ((P32-K)-P22) VERSUS COST (K) FOR DIFFERENT VALUES OF ACTIVATION RATE ($\alpha$)

Values of Activation Rate ($\alpha$)
DIFFERENCE OF PROFIT ($P_{41} - P_{31}$) VERSUS FAILURE RATE ($\alpha$) FOR DIFFERENT VALUES OF ACTIVATION RATE ($\beta$) AND COST ($C_9$)

- $\lambda = 0.5, \gamma = 5, \beta = 10, \alpha = 4, C_0 = 1000, C_1 = 50, C_2 = 40, C_3 = 30$
- $C_4 = 25, C_5 = 100, p_1 = 0.5, p_2 = 0.5$

Fig. 7.8
Failure Rate (a)

DIFFERENCE OF PROFIT ($P_3 - P_2$) VERSUS FAILURE RATE (a) FOR DIFFERENT VALUES OF ACTIVATION RATE ($P$) AND COST ($C$):

$C=25$, $C=100$, $C=75$, $C=50$, $C=30$, $P=0.5$, $P=0.25$, $y=5$, $y=8$, $y=10$, $a=2$, $a=3$, $C=30$, $C=40$, $C=50$, $C=60$, $C=450$

DIFFERENT VALUES OF ACTIVATION RATE ($P$) AND COST ($C$)
DIFFERENCE OF PROFIT ($P_{41} - P_{51}$) VERSUS COST ($C_4$) FOR DIFFERENT VALUES OF REVENUE PER UNIT UPTIME ($C_0$)

Fig. 7.10

$\lambda = 0.5, \gamma = 5, \gamma = 8, \alpha = 0.3, \alpha = 2, C_1 = 50, C_2 = 40, C_3 = 30$.

Values of revenue per unit uptime ($C_0$):
- $C_0 = 1000$
- $C_0 = 4000$
- $C_0 = 7000$
- $C_0 = 10000$
Fig. 7.11

Cost (C₄)

VALUES OF REVENUE PER UNIT UPTIME (C₀)

DIFFERENCE OF PROFIT (P₄² - P₅²) VERSUS COST (C₄) FOR DIFFERENT
Fig. 7.12

Values of failure rate ($\alpha$) difference of profit ($P_6 - P_{10}$) versus cost ($C_{10}$) for different
DIFFERENCE OF PROFIT ($P_6 - P_5$) VERSUS COST ($C_1$) FOR DIFFERENT VALUES OF FAILURE RATE ($a$)

VALUES OF FAILURE RATE ($a$)

Fig. 7.13

Cost ($C_1$)