CHAPTER 6

STUDY OF A TWO-UNIT HOT STANDBY SYSTEM WITH TWO TYPES OF INSPECTION AND CHANCES OF REPLACEMENT
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STUDY OF A TWO-UNIT HOT STANDBY SYSTEM WITH TWO TYPES OF INSPECTION AND CHANCES OF REPLACEMENT

Two-unit hot standby systems have been analysed in the last two chapters with one of the assumptions that failures are not self announcing. If the system is not under constant observation, the failure of a unit is detected only at the system failure. But if it is under the constant observation, failure of unit is revealed immediately.

However, there may be situations when the system is not put under the constant observation because a person engaged for this purpose may remain idle for a long time. So, there comes an idea of carrying out a random inspection for detecting the failure.

Thus, in the present chapter, we study a two-unit hot standby system with two types of inspection – an inspection for detecting the reparability of a failed unit and a random inspection for revealing the failure of a unit. On the failure of a unit (if revealed), an inspection is carried out by an ordinary repairman to detect the reparability of unit. Two models have been analysed. In first model, it is assumed that if ordinary repairman declares the unit as irreparable then it is replaced with new one; whereas in the second model, expert opinion is taken to confirm whether the unit is actually not repairable and is accordingly repaired by the expert himself or replaced by the ordinary repairman. Other assumptions are same as taken in the earlier chapters. System is analysed by making use of semi-Markov processes and regenerative
point technique. The following measures of system effectiveness have been obtained :

Mean Time to System Failure (MTSF)

- Mean time to system failure (MTSF)
- Steady-state availability of the system
- Expected busy period per unit time (for repair only) by ordinary/expert repairman
- Expected busy period per unit time (for inspection only) by ordinary/expert repairman
- Expected busy period per unit time (for replacement only) by ordinary repairman
- Expected number of visits per unit time by ordinary/expert repairman
- Expected number of replacements per unit time
- Expected number of times the inspection is carried out for revealing the failure
- Expected profit incurred to the system

**NOTATIONS**

- $\lambda$ Constant failure rate of operative unit
- $\alpha$ Constant failure rate of hot standby unit
- $p_1$ probability that unit is repairable
- $p_2$ probability that unit is not repairable
- $g(t), G(t)$ p.d.f. and c.d.f. of time to repair of ordinary repairman
- $g_e(t), G_e(t)$ p.d.f. and c.d.f. of time to repair of expert repairman
- $h_1(t), H_1(t)$ p.d.f. and c.d.f. of time to inspection of ordinary repairman
- $h_e(t), H_e(t)$ p.d.f. and c.d.f. of time to inspection of expert repairman
SYMBOLS FOR THE STATES OF THE SYSTEM ARE

- $o$: operative unit
- $hs$: hot standby unit
- $w$: failed unit waiting for repair
- $F_{ui}$: Failed unit under inspection by ordinary repairman
- $F_{uci}$: Failed unit under inspection by expert repairman
- $F_{ur}$: Failed unit under repair by ordinary repairman
- $F_{rc}$: Failed unit under repair by expert repairman
- $F_{rep}$: Failed unit under replacement
- $F_{U/i} F_{U/e}$: Inspection of the failed unit is continuing by the ordinary/expert from the previous state
- $F_{U/r} F_{R/e}$: Repair of the failed unit continuing by the ordinary/expert from the previous state
- $F_{REP}$: Replacement of the failed unit continuing by the ordinary repairman from the previous state

**MODEL 6.1**

In this model, it is assumed that the unit is replaced if ordinary repairman declares that the unit is irreparable. State transition is shown in Fig. 6.1.

**TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES**

The epochs of entry into states 0, 1, 2, 3, 4, 6 and 7 are regeneration points and thus 0, 1, 2, 3, 4, 6 and 7 are regenerative states. States 2, 5, 6, 7, 8 and 9 are failed states.

The transition probabilities are:

- $q_{00}(t) = e^{-(\lambda+\alpha)t} h(t)$
- $q_{01}(t) = [(\lambda+\alpha) e^{-(\lambda+\alpha)t} \odot e^{-\lambda t}] \overline{h}(t)$
- $q_{02}(t) = [(\lambda+\alpha)e^{-(\lambda+\alpha)t} \odot \lambda e^{-\lambda t}] \overline{H}(t)$
- $q_{13}(t) = p_2 h(t) e^{-\lambda t}$
Fig. 6.1
\[ q_{14}(t) = p_1 h_1(t) e^{-\lambda t} ; \quad q_{15}(t) = \lambda e^{-\lambda t} \bar{H}_1(t) \]
\[ q_{16}(5)(t) = p_1\{h_1(t) = p_1[1 - e^{-\lambda t}] h_1(t) \}
\[ q_{17}(5)(t) = p_2\{h_2(t) = p_2[1 - e^{-\lambda t}] h_1(t) \}
\[ q_{26}(t) = p_1 h_1(t) ; \quad q_{27}(t) = p_2 h_1(t) ; \quad q_{30}(t) = e^{-\lambda t} h_2(t) \]
\[ q_{38}(t) = \lambda e^{-\lambda t} \bar{H}_2(t) ; \quad q_{31}(8)(t) = [\lambda e^{-\lambda t} \bar{H}_2(t)] h_2(t) = (1 - e^{-\lambda t}) h_2(t) \]
\[ q_{40}(t) = e^{-\lambda t} g(t) ; \quad q_{49}(t) = \lambda e^{-\lambda t} \bar{G}(t) \]
\[ q_{46}(9)(t) = [\lambda e^{-\lambda t} \bar{G}(t)] h_2(t) = (1 - e^{-\lambda t}) g(t) \]
\[ q_{71}(t) = h_2(t) \quad (6.1.1-6.1.18) \]

The non-zero \( p_{ij} \) elements are given by

\[ p_{00} = h^*(\lambda + \alpha) \quad ; \quad p_{01} = \frac{\lambda + \alpha}{\alpha} [h^*(\lambda) - h^*(\lambda + \alpha)] \]
\[ p_{02} = 1 - \frac{\lambda + \alpha}{\alpha} h^*(\lambda) + \frac{\lambda}{\alpha} h^*(\lambda + \alpha) ; \quad p_{13} = p_2 h_1^*(\lambda) \]
\[ p_{14} = p_1 h_1^*(\lambda) \quad ; \quad p_{15} = 1 - h_1^*(\lambda) \quad ; \quad p_{16}(5) = p_1[1 - h_1^*(\lambda)] \]
\[ p_{17}(5) = p_2[1 - h_1^*(\lambda)] \quad ; \quad p_{26} = p_1 \quad ; \quad p_{27} = p_2 \]
\[ p_{30} = h_2^*(\lambda) \quad ; \quad p_{38} = 1 - h_2^*(\lambda) \quad ; \quad p_{31}(8) = 1 - h_2^*(\lambda) \]
\[ p_{40} = g^*(\lambda) \quad ; \quad p_{49} = 1 - g^*(\lambda) \quad ; \quad p_{41}(9) = 1 - g^*(\lambda) \]
\[ p_{61} = 1 \quad ; \quad p_{71} = 1 \quad (6.1.19-6.1.36) \]

By these probabilities, it can be verified that

\[ p_{00} + p_{01} + p_{02} = 1 \quad ; \quad p_{13} + p_{14} + p_{15} = 1 \]
\[ p_{13} + p_{14} + p_{16}(5) + p_{17}(5) = 1 \quad ; \quad p_{26} + p_{27} = 1 \quad ; \quad p_{30} + p_{38} = 1 \]
\[ p_{30} + p_{31}(8) = 1 \quad ; \quad p_{40} + p_{49} = 1 \quad ; \quad p_{40} + p_{41}(9) = 1 \]
\[ p_{61} = 1 \quad ; \quad p_{71} = 1 \quad (6.1.37-6.1.46) \]

The mean sojourn time (\( \mu_i \)) are

\[ \mu_0 = \frac{1}{\lambda + \alpha} \quad ; \quad \mu_1 = \frac{1 - h_1^*(\lambda)}{\lambda} \quad ; \quad \mu_2 = \int_0^\infty \bar{H}_1(t) \, dt \]
\[ \mu_3 = \frac{1}{\lambda} [(1 - h_2^*(\lambda)) \quad ; \quad \mu_4 = \frac{1}{\lambda} [(1 - g^*(\lambda)] \]
\[ \mu_6 = \int_0^\infty G(t) \, dt \quad ; \quad \mu_7 = \int_0^\infty H_2(t) \, dt \quad (6.1.47-6.1.53) \]

The unconditional mean time taken by the system to transit for any state \( j \) when it is counted from epoch of entrance into state \( i \) is mathematically stated as

\[ m_{ij} = \int_0^\infty t \, q_4(t) \, dt = -q_{ij}^*(0) \quad (6.1.54) \]

Thus,

\[
\begin{align*}
m_{00} + m_{01} + m_{02} &= \mu_0 \quad ; \quad m_{14} + m_{13} + m_{15} = \mu_1 \\
m_{26} + m_{27} &= \mu_2 \quad ; \quad m_{30} = m_{38} = \mu_3 \quad ; \quad m_{40} + m_{49} = \mu_4 \\
m_{61} &= \mu_6 \quad ; \quad m_{71} = \mu_7 \quad ; \quad m_{13} + m_{14} + m_{17}^{(5)} + m_{16}^{(5)} = \mu_2 \\
m_{30} + m_{31}^{(8)} &= \mu_7 \quad ; \quad m_{40} + m_{41}^{(9)} = \mu_6 \quad (6.1.55-6.1.64)
\end{align*}
\]

**MEAN TIME TO SYSTEM FAILURE**

By probabilistic arguments, we obtain the following recursive relations for \( \phi_i(t) \):

\[
\begin{align*}
\phi_0(t) &= Q_{00}(t) \, \phi_0(t) + Q_{01}(t) \, \phi_1(t) + Q_{02}(t) \\
\phi_1(t) &= Q_{14}(t) \, \phi_4(t) + Q_{13}(t) \, \phi_3(t) + Q_{15}(t) \\
\phi_3(t) &= Q_{30}(t) \, \phi_0(t) + Q_{38}(t) \\
\phi_4(t) &= Q_{40}(t) \, \phi_0(t) + Q_{49}(t) \quad (6.1.65-6.1.68)
\end{align*}
\]

Taking L.S.T. of these equations and solving them for \( \phi_0^{**}(s) \), we have

\[ \phi_0^{**}(s) = \frac{N(s)}{D(s)} \quad (6.1.69) \]

where

\[
\begin{align*}
N(s) &= Q_{01}^{**}(s) \{ Q_{14}^{**}(s) Q_{49}^{**}(s) + Q_{15}^{**}(s) + Q_{13}^{**}(s) Q_{38}^{**}(s) \} \\
&\quad + Q_{02}^{**}(s) \\
D(s) &= 1 - Q_{00}^{**}(s) - Q_{01}^{**}(s) \{ Q_{13}^{**}(s) Q_{30}^{**}(s) + Q_{14}^{**}(s) Q_{40}^{**}(s) \} \\
&\quad (6.1.70-6.1.71)
\end{align*}
\]
Now mean time to system (MTSF) when the system starts from the state '0' is given by

$$T_0 = \lim_{s \to 0} \frac{1 - \phi_0^* (s)}{s} = \frac{N}{D} \quad (6.1.72)$$

where

$$N = \mu_0 + p_{01} \mu_1 + p_{01} p_{13} \mu_3 + p_{01} p_{14} \mu_4 \quad (6.1.73)$$

$$D = [1 - p_{00} - p_{01} \{p_{13} p_{30} + p_{14} p_{40} \}] \quad (6.1.74)$$

**AVAILABILITY ANALYSIS**

The recursive relations for $A_i(t)$ are as follows

$$A_0(t) = M_0(t) + q_{00}(t) A_0(t) + q_{01}(t) A_1(t) + q_{02}(t) A_2(t)$$

$$A_1(t) = M_1(t) + q_{14}(t) A_4(t) + q_{13}(t) A_3(t) + q_{17}(t) A_7(t) + q_{16}(t) A_6(t)$$

$$A_2(t) = q_{26}(t) A_6(t) + q_{27}(t) A_7(t)$$

$$A_3(t) = M_3(t) + q_{30}(t) A_0(t) + q_{31}(t) A_1(t)$$

$$A_4(t) = M_4(t) + q_{40}(t) A_0(t) + q_{41}(t) A_1(t)$$

$$A_6(t) = q_{61}(t) A_1(t)$$

$$A_7(t) = q_{71}(t) A_1(t)$$

where

$$M_0(t) = e^{-\lambda t + \alpha t}, \quad M_1(t) = e^{-\lambda t \bar{H}_1(t)}, \quad M_3(t) = e^{-\lambda t \bar{H}_3(t)}$$

$$M_4(t) = e^{-\lambda t \bar{G}(t)} \quad (6.1.75-6.1.85)$$

Taking the L.T. of the above equations and solving them for $A_0^*(s)$, we have

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)} \quad (6.1.86)$$

where

$$N_1(s) = M_0^*(s) [1 - q_{13}^*(s) q_{13}^{(8)}(s) - q_{14}^*(s) q_{14}^{(9)}(s) - q_{61}^*(s) q_{16}^{(5)}(s) - q_{71}^*(s) q_{17}^{(5)}(s)] + [M_1^*(s)$$

$$+ M_3^*(s) q_{13}^*(s) + M_4^*(s) q_{14}^*(s)] [q_{01}^*(s)$$
In steady state, the availability of the system is given by

\[
A_0 = \lim_{s \to 0} \{ s A_0(s) \} = \frac{N_1}{D_1} \quad (6.1.89)
\]

where

\[
N_1 = (p_{13} p_{30} + p_{14} p_{40}) \mu_0 + (\mu_1 + p_{13} \mu_3 + p_{14} \mu_4) (p_{01} + p_{02})
\]

\[
D_1 = (p_{13} p_{30} + p_{14} p_{40}) \mu_0 + (1 - p_{00}) \mu_2 + p_{13} (p_{01} + p_{02}) \mu_7 + p_{14} (p_{01} + p_{02}) \mu_6 + [p_{16} (1 - p_{00}) + p_{02} p_{26} (p_{13} p_{30} + p_{14} p_{40})] \mu_6
\]

\[
+ [p_{17} (1 - p_{00}) + p_{02} p_{27} (p_{13} p_{30} + p_{14} p_{40})] \mu_7
\]

\[
(6.1.90-6.1.91)
\]

**BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN (Repair Time Only)**

The recursive relations for \( B_i(t) \) are as follows

\[
B_0(t) = Q_{00}(t) \otimes B_0(t) + Q_{01}(t) \otimes B_1(t) + Q_{02}(t) \otimes B_2(t)
\]

\[
B_1(t) = Q_{14}(t) \otimes B_4(t) + Q_{13}(t) \otimes B_3(t) + Q_{17}(t) \otimes B_7(t) + Q_{16}(t) \otimes B_6(t)
\]

\[
B_2(t) = Q_{26}(t) \otimes B_6(t) + Q_{27}(t) \otimes B_7(t)
\]

\[
B_3(t) = Q_{30}(t) \otimes B_0(t) + Q_{31}(t) \otimes B_1(t)
\]

\[
B_4(t) = W_4(t) + Q_{40}(t) \otimes B_0(t) + Q_{41}(t) \otimes B_1(t)
\]

\[
B_6(t) = W_6(t) + Q_{61}(t) \otimes B_1(t)
\]

\[
B_7(t) = Q_{71}(t) \otimes B_1(t)
\]

where

\[
W_4(t) = W_8(t) = \overline{G}(t)
\]

\[
(6.1.92-6.1.98)
\]
Taking L.T. of the above equations and solving them for \( B_0^*(s) \), we get

\[
B_0^*(s) = \frac{N_2(s)}{D_1(s)} \quad (6.1.100)
\]

where

\[
N_2(s) = W_4^*(s)[q_{01}^*(s) q_{14}^*(s) + q_{02}^*(s) q_{14}^*(s) (q_{26}^*(s) q_{61}^*(s) \\
+ q_{27}^*(s) q_{71}^*(s))] + W_6^*(s) [q_{02}^*(s) q_{26}^*(s) (I \\
- q_{13}^*(s) q_{31}^{(8)}(s) - q_{41}^{(8)}(s) q_{14}^*(s)) + q_{01}^*(s) q_{16}^{(5)}(s) \\
+ q_{02}^*(s) (q_{16}^{(5)}(s) q_{27}^*(s) - q_{17}^{(5)}(s) q_{26}^*(s))]
\]

(6.1.101)

and \( D_1(s) \) is already specified.

In steady state, the total fraction of time for which the system is busy under repair of ordinary repairman is given by

\[
B_0 = \frac{N_2}{D_1} \quad (6.1.102)
\]

where

\[
N_2 = [p_{14} (p_{01} + p_{02}) + p_{02} p_{26} (p_{13} p_{30} + p_{14} p_{40}) + p_{16}^{(5)}(p_{01} + p_{02})] \mu_6
\]

(6.1.103)

BUSTY PERIOD ANALYSIS OF ORDINARY REPAIRMAN
(Inspection Time Only)

By probabilistic arguments, we have the following recursive relations:

\[
\begin{align*}
B_{i0}(t) &= q_{00}(t) \odot B_{i0}(t) + q_{01}(t) \odot B_{i1}(t) + q_{02}(t) \odot B_{i2}(t) \\
B_{i1}(t) &= W_i(t) + q_{14}(t) \odot B_{i4}(t) + q_{13}(t) \odot B_{i3}(t) + q_{17}^{(5)}(t) \odot B_{i7}(t) \\
&\quad + q_{16}^{(5)}(t) \odot B_{i6}(t) \\
B_{i2}(t) &= W_2(t) + q_{26}(t) \odot B_{i6}(t) + q_{27}(t) \odot B_{i7}(t) \\
B_{i3}(t) &= q_{30}(t) \odot B_{i0}(t) + q_{31}^{(8)}(t) \odot B_{i1}(t) \\
B_{i4}(t) &= q_{40}(t) \odot B_{i0}(t) + q_{41}^{(9)}(t) \odot B_{i1}(t) \\
B_{i6}(t) &= q_{61}(t) \odot B_{i1}(t) \\
B_{i7}(t) &= q_{71}(t) \odot B_{i1}(t)
\end{align*}
\]

(6.1.104-6.1.110)
where
\[ W_1(t) = W_2(t) = \bar{H}_1(t) \]  \hspace{1cm} (6.1.111)

Taking L.T. of the above equations and solving them for \( BI_0^*(s) \), we get
\[ BI_0^*(s) = \frac{N_3(s)}{D_1(s)} \]  \hspace{1cm} (6.1.112)

where
\[
N_3(s) = [q_{111}^*(s) + q_{102}^*(s) (q_{26}^*(s) q_{61}^*(s) + q_{27}^*(s) q_{71}^*(s))] W_1^*(s) \\
+ q_{102}^*(s) W_2^*(s) [1 - q_{13}^*(s) q_{31}^{(8)*}(s) - q_{14}^*(s) q_{41}^{(9)*}(s)] \\
- q_{16}^{(5)*}(s) q_{61}^*(s) - q_{17}^{(5)*}(s) q_{71}^*(s)] \]  \hspace{1cm} (6.1.113)

and \( D_1(s) \) is already specified.

In steady state, the total fraction of time for which the system is under inspection of ordinary repairman is given by
\[ BI_0 = \frac{N_3}{D_1} \]  \hspace{1cm} (6.1.114)

where
\[
N_3 = \mu_2[(1 - p_{00}) + p_{02}(p_{13} p_{30} + p_{14} p_{40})] \]  \hspace{1cm} (6.1.115)

and \( D_1 \) is already specified.

**BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN**
(Replacement Time Only)

By probabilistic arguments, we have the following recursive relations:

\[ BR_0(t) = q_{00}(t) \odot BR_0(t) + q_{01}(t) \odot BR_1(t) + q_{02}(t) \odot BR_2(t) \]
\[ BR_1(t) = q_{14}(t) \odot BR_4(t) + q_{13}(t) \odot BR_3(t) + q_{17}^{(5)}(t) \odot BR_7(t) \]
\[ + q_{16}^{(5)}(t) \odot BR_6(t) \]
\[ BR_2(t) = q_{26}(t) \odot BR_6(t) + q_{27}(t) \odot BR_7(t) \]
\[ BR_3(t) = W_3(t) + q_{30}(t) \odot BR_0(t) + q_{31}^{(8)}(t) \odot BR_1(t) \]
\[ BR_4(t) = q_{40}(t) \odot BR_0(t) + q_{41}^{(9)}(t) \odot BR_1(t) \]
\[ BR_6(t) = q_{61}(t) \odot BR_1(t) \]
\[ BR_7(t) = W_7(t) + q_7(t) @ BR_1(t) \]  
(6.1.116-6.1.122)

where
\[ W_3(t) = W_7(t) = H_2(t) \]  
(6.1.123)

Taking L.T. of the above equations and solving them for \( BR_0*(s) \), we get

\[
BR_0*(s) = \frac{N_4(s)}{D_1(s)} \]  
(6.1.124)

where
\[
N_4(s) = W_3*(s)[q_{01}*(s) q_{13}*(s) + q_{02}*(s) q_{13}*(s) (q_{26}*(s) q_{61}*(s)) \\
+ q_{27}*(s) q_{71}*(s))] + W_7*(s) [q_{27}*(s) q_{02}*(s) \{1 \\
- q_{13}*(s) q_{31}^{(8)}(s) - q_{14}*(s) q_{41}^{(9)}(s) - q_{16}^{(5)}(s) q_{61}*(s)\} \\
+ q_{01}*(s) q_{17}^{(5)}(s) + q_{17}^{(5)}(s) q_{26}*(s) q_{61}*(s) q_{02}*(s)]
\]  
(6.1.125)

and \( D_1(s) \) is already specified.

In steady state, the total fraction of time for which the system is under replacement by ordinary repairman is given by

\[
BR_0 = \frac{N_4}{D_1} \]  
(6.1.126)

where
\[
N_4 = \{p_{13}(p_{01} + p_{02}) + \{p_{17}^{(5)}(p_{01} + p_{02}) + p_{02}p_{27}(p_{13} + p_{14} p_{40})\}\} \mu_7
\]  
and \( D_1 \) is already specified  \( (6.1.127) \)

**EXPECTED NUMBER OF VISITS BY ORDINARY REPAIRMAN**

By probabilistic arguments, we have the following recursive relations:
\[
V_0(t) = Q_{00}(t) @ [1+V_0(t)] + Q_{01}(t) @ [1+V_1(t)] + Q_{02}(t) @ [1+V_2(t)]
\]
\[
V_1(t) = Q_{14}(t) @ V_4(t) + Q_{13}(t) @ V_3(t) + Q_{17}^{(5)}(t) @ V_7(t) \\
+ Q_{16}^{(5)}(t) @ V_6(t)
\]
\[
V_2(t) = Q_{26}(t) @ V_6(t) + Q_{27}(t) @ V_7(t)
\]
By taking L.S.T. of the above equations and solving them for $V_0^{**}(s)$, we get

$$V_0^{**}(s) = \frac{N_5(s)}{D_1(s)} \quad (6.1.135)$$

where

$$N_5(s) = (Q_{00}^{*}(s) + Q_{01}^{*}(s) + Q_{02}^{*}(s) [1 - Q_{13}^{*}(s) Q_{31}^{(8)*}(s) - Q_{14}^{*}(s) Q_{41}^{(9)*}(s) - Q_{16}^{(5)*}(s) Q_{61}^{*}(s) - Q_{17}^{(5)*}(s) Q_{71}^{*}(s))]$$

and $D_1(s)$ is already specified.

In steady state, the number of visits per unit time by ordinary repairman is given by

$$V_0 = \frac{N_5}{D_1} \quad (6.1.137)$$

where

$$N_5 = p_{13}p_{30} + p_{14}p_{40} \quad (6.1.138)$$

and $D_1$ is already specified

**EXPECTED NUMBER OF REPLACEMENTS**

By probabilistic arguments, we have the following recursive relations:

$$R_0(t) = Q_{00}(t) \circ R_0(t) + Q_{01}(t) \circ R_1(t) + Q_{02}(t) \circ R_2(t)$$

$$R_1(t) = Q_{13}(t) \circ [1 + R_3(t)] + Q_{14}(t) \circ R_4(t) + Q_{16}^{(5)}(t) \circ R_6(t)$$

$$+ Q_{17}^{(5)}(t) \circ [1 + R_7(t)]$$

$$R_2(t) = Q_{26}(t) \circ R_6(t) + Q_{27}(t) \circ [1 + R_7(t)]$$

$$R_3(t) = Q_{36}(t) \circ R_6(t) + Q_{31}^{(8)}(t) \circ R_1(t)$$

$$R_4(t) = Q_{40}(t) \circ R_0(t) + Q_{41}^{(9)}(t) \circ R_1(t)$$
\[ R_0(t) = Q_{61}(t) \otimes R_1(t) \]
\[ R_7(t) = Q_{71}(t) \otimes R_1(t) \quad (6.1.139-6.1.145) \]

By taking L.S.T. of the above equations and solving them for \( R_{ii}^{**}(s) \), we get

\[ R_{ii}^{**}(s) = \frac{N_6(s)}{D_i(s)} \quad (6.1.146) \]

where

\[ N_6(s) = \{ Q_{13}^{**}(s) + Q_{17}^{**}(s) \} \{ Q_{01}^{**}(s) + Q_{02}^{**}(s) (Q_{26}^{**}(s) Q_{61}^{**}(s) + Q_{27}^{**}(s) Q_{71}^{**}(s)) \} + Q_{02}^{**}(s) Q_{27}^{**}(s) \{ 1 - Q_{13}^{**}(s) Q_{31}^{**}(s) \} \]

\[ - Q_{14}^{**}(s) Q_{41}^{**}(s) - Q_{61}^{**}(s) Q_{16}^{**}(s) - Q_{71}^{**}(s) Q_{17}^{**}(s) \} \quad (6.1.147) \]

and \( D_i(s) \) is already specified.

In steady state, the number of replacements per unit time by ordinary repairman is given by

\[ R_0 = \frac{N_6}{D_1} \quad (6.1.148) \]

where

\[ N_6 = (p_{13} + p_{17}) + (p_{01} + p_{02}) + p_{02} p_{21} (p_{13} p_{30} + p_{14} p_{40}) \quad (6.1.149) \]

and \( D_1 \) is already specified

**EXPECTED NUMBER OF TIMES THE INSPECTION IS CARRIED OUT FOR REVEALING THE FAILURE**

By probabilistic arguments, we have the following recursive relations:

\[ T_{I0}(t) = Q_{00}(t) \otimes [1 + T_{I0}(t)] + Q_{01}(t) \otimes [1 + T_{I0}(t)] + Q_{02}(t) \otimes T_{I0}(t) \]
\[ T_{I1}(t) = Q_{13}(t) \otimes T_{I0}(t) + Q_{14}(t) \otimes T_{I1}(t) + Q_{17}(t) \otimes T_{I7}(t) \]
\[ + Q_{16}(t) \otimes T_{I6}(t) \]
\[ T_{I2}(t) = Q_{26}(t) \otimes T_{I0}(t) + Q_{27}(t) \otimes T_{I7}(t) \]
\[ T_{I3}(t) = Q_{30}(t) \otimes T_{I0}(t) + Q_{31}(t) \otimes T_{I1}(t) \]
\[ T_{14}(t) = Q_4(t) \circ T_{10}(t) + Q_{41}^{(9)}(t) \circ T_{11}(t) \]
\[ T_{16}(t) = Q_{61}(t) \circ T_{11}(t) \]
\[ T_{17}(t) = Q_{17}(t) \circ T_{11}(t) \quad (6.1.150-6.1.156) \]

By taking L.S.T. of the above equations and solving them for \( T_{10}^{**}(s) \), we get
\[
T_{10}^{**}(s) = \frac{N_7(s)}{D_1(s)} \quad (6.1.157)
\]
where
\[
N_7(s) = (Q_{00}^{*}(s) + Q_{01}^{*}(s)) [1 - Q_{13}^{*}(s) Q_{31}^{(8)*}(s) - Q_{14}^{*}(s) Q_{41}^{(9)*}(s) - Q_{16}^{(5)*}(s) Q_{61}^{*}(s) - Q_{17}^{(5)*}(s) Q_{71}^{*}(s)] \quad (6.1.158)
\]
In steady state, the number of times inspection is carried out for revealing the failure the failure is given by
\[
T_{10} = \frac{N_7}{D_1} \quad (6.1.159)
\]
where
\[
N_7 = (p_{00} + p_{01}) [p_{13}p_{30} + p_{14}p_{40}]
\]

**PROFIT ANALYSIS**

\[
P_{61} = C_0A_0 - C_1B_0 - C_2B_{10} - C_3B_{10} - C_4B_{R0} - C_4V_0 - C_5R_0 - C_{10}T_{10} \quad (6.1.161)
\]

where
\[
C_0 = \text{Revenue per unit up time of the system}
\]
\[
C_1 = \text{Cost per unit time for which ordinary repairman is busy, under repair}
\]
\[
C_2 = \text{Cost per unit time for which ordinary repairman is busy, under inspection}
\]
\[
C_3 = \text{Cost per unit time for which ordinary repairman is busy, under replacement.}
\]
\[
C_4 = \text{Cost per visit of ordinary repairman}
\]
C_5 = \text{Cost per unit time for replacement}

C_{10} = \text{cost per unit time for which inspection is carried out for revealing the failure.}

**PARTICULAR CASE**

For the geometrical interpretation, the following particular case is considered as:

\[ g(t) = \alpha_1 e^{-\eta t}, \quad h_1(t) = \gamma_1 e^{-\gamma t} \]

\[ h_2(t) = \gamma e^{-\gamma t} \]

We can easily obtain the following:

\[ p_{00} = \frac{\beta}{\lambda + \alpha + \beta}, \quad p_{01} = \frac{(\lambda + \alpha)\beta}{(\lambda + \beta)(\lambda + \alpha + \beta)} \]

\[ p_{02} = 1 - \frac{\beta}{\lambda + \alpha} \left( \frac{\lambda + \alpha}{\lambda + \beta} - \frac{\lambda}{\lambda + \alpha + \beta} \right) \]

\[ p_{13} = \frac{p_2 \gamma_1}{\lambda + \gamma_1}, \quad p_{14} = \frac{p_1 \gamma_1}{\lambda + \gamma_1} \]

\[ p_{15} = \frac{\lambda}{\lambda + \gamma_1}, \quad p_{16}^{(5)} = \frac{p_1 \lambda}{\lambda + \gamma_1} \]

\[ p_{17}^{(5)} = \frac{p_2 \lambda}{\lambda + \gamma_1}, \quad p_{26} = p_1 \]

\[ p_{27} = p_2, \quad p_{30} = \frac{\gamma}{\lambda + \gamma} \]

\[ p_{38} = \frac{\lambda}{\lambda + \gamma}, \quad p_{31}^{(8)} = \frac{\lambda}{\lambda + \gamma} \]

\[ p_{40} = \frac{\alpha_1}{\lambda + \alpha_1}, \quad p_{49} = \frac{\lambda}{\lambda + \alpha_1} \]

\[ p_{41}^{(9)} = \frac{\lambda}{\lambda + \alpha_1}, \quad p_{61} = 1 \quad p_{71} = 1 \]

\[ \mu_0 = \frac{1}{\lambda + \alpha}, \quad \mu_1 = \frac{1}{\lambda + \gamma_1} \]
\[
\mu_2 = \frac{1}{\gamma}, \quad \mu_3 = \frac{1}{\lambda + \gamma}, \quad \mu_4 = \frac{1}{\lambda + \alpha_i},
\]
\[
\mu_6 = \frac{1}{\alpha_i}, \quad \mu_7 = \frac{1}{\gamma}.
\]

With the help of the above equations and the equations (6.1.72), (6.1.89), (6.1.102), (6.1.114), (6.1.126), (6.1.137), (6.1.148), (6.1.159) and (6.1.161) we can have the expressions for MTSF, availability and profit for this particular case.

**MODEL 6.2**

Here, in this model expert opinion is taken if the ordinary repairman declares that unit is irreparable. State transition diagram is shown as in Fig. 62.

**TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES**

The epochs of entry into states 0, 1, 2, 3, 4, 6, 7, 9, 10, 12 and 13 are regeneration points and thus 0, 1, 2, 3, 4, 6, 7, 9, 10, 12 and 13 are regenerative states. States 2, 3, 6, 7, 8, 11, 12, 13, 14 and 15 are failed states. The transition probabilities are given by:

\[
q_{00}(t) = e^{-\lambda t} h(t) ; \quad q_{01}(t) = \left(1 + \alpha e^{-\lambda t}\right) h(t)
\]
\[
q_{02}(t) = \left(1 + \alpha e^{-\lambda t}\right) \lambda e^{-\lambda t} \bar{H}(t) ; \quad q_{13}(t) = p_1 h_1(t) e^{-\lambda t}
\]
\[
q_{14}(t) = p_2 h_1(t) e^{-\lambda t} ; \quad q_{15}(t) = \lambda e^{-\lambda t} \bar{H}(t);
\]
\[
q_{16}^{(5)}(t) = p_1 \left(\lambda e^{-\lambda t} \bar{G}(t) + 1\right) h_1(t) = p_1(1 - e^{-\lambda t}) h_1(t)
\]
\[
q_{17}^{(5)}(t) = p_2 \left(\lambda e^{-\lambda t} \bar{G}(t) + 1\right) h_1(t) = p_2(1 - e^{-\lambda t}) h_1(t)
\]
\[
q_{26}(t) = p_1 h_1(t) ; \quad q_{27}(t) = p_2 h_1(t) ; \quad q_{30}(t) = e^{-\lambda_t} g(t)
\]
\[
q_{38}(t) = \lambda e^{-\lambda t} \bar{G}(t) ; \quad q_{31}^{(8)}(t) = \left[\lambda e^{-\lambda t} \bar{G}(t) + 1\right] g(t) = [1 - e^{-\lambda t}] g(t)
\]
\[
q_{4,12}^{(11)}(t) = p_1 \left(\lambda e^{-\lambda t} \bar{G}(t) + 1\right) h_c(t) = p_1[1 - e^{-\lambda t}] h_c(t)
\]
\[
q_{40}(t) = p_1 e^{-\lambda t} h_c(t) ; \quad q_{4,13}^{(11)}(t) = p_2 \left(\lambda e^{-\lambda t} \bar{G}(t) + 1\right) h_c(t) = p_2[1 - e^{-\lambda t}] h_c(t)
\]
\[
q_{4,10}(t) = p_2 e^{-\lambda t} h_c(t) ; \quad q_{61}(t) = g(t)
\]
\[ q_{4.11}(t) = \lambda e^{-\lambda t} \quad \text{He}(t) \quad ; \quad q_{7.12}(t) = p_1h_e(t) \]
\[ q_{7.13}(t) = p_2h_e(t) \quad ; \quad q_{9.0}(t) = e^{-\lambda t}g_e(t) \quad ; \quad q_{9.14}(t) = \lambda e^{-\lambda t} \quad \text{Ge}(t) \]
\[ q_{9.1}^{(14)}(t) = [\lambda e^{-\lambda t} \mathcal{O} 1] \quad g_e(t) = (1 - e^{-\lambda t}) \quad g_e(t) \]
\[ q_{10.1}^{(15)}(t) = [\lambda e^{-\lambda t} \mathcal{O} 1] \quad h_2(t) = (1 - e^{-\lambda t}) \quad h_2(t) \]
\[ q_{10.0}(t) = e^{-\lambda t} \quad h_2(t) \quad ; \quad q_{10.15}(t) = \lambda e^{-\lambda t} \quad H_2(t) \quad ; \quad q_{12.1}(t) = g_e(t) \]
\[ q_{13.1}(t) = h_2(t) \quad (6.2.1-6.2.29) \]

The non-zero elements \( p_{ij} \) are given as follows:

\[ p_{00} = h^*(\lambda + \alpha) \quad ; \quad p_{01} = \frac{\lambda + \alpha}{\alpha} \quad [h^*(\lambda) - h^*(\lambda + \alpha)] \]
\[ p_{02} = 1 - \frac{\lambda + \alpha}{\alpha} \quad h^*(\lambda) + \frac{\lambda}{\alpha} h^*(\lambda + \alpha) \quad ; \quad p_{13} = p_1h_1^*(\lambda) \]
\[ p_{14} = p_2h_1^*(\lambda) \quad ; \quad p_{15} = 1 - h_1^*(\lambda) \quad ; \quad p_{16}^{(5)} = p_1[1 - h_1^*(\lambda)] \]
\[ p_{17}^{(5)} = p_2[1 - h_1^*(\lambda)] \quad ; \quad p_{26} = p_1 \quad ; \quad p_{27} = p_2 \]
\[ p_{30} = g^*(\lambda) \quad ; \quad p_{38} = 1 - g^*(\lambda) \quad ; \quad p_{31}^{(8)} = 1 - g^*(\lambda) \]
\[ p_{49} = p_1h_c^*(\lambda) \quad ; \quad p_{4j10} = p_2h_c^*(\lambda) \quad ; \quad p_{4j11} = 1 - h_c^*(\lambda) \]
\[ p_{4j12}^{(11)} = p_1[1 - h_c^*(\lambda)] \quad ; \quad p_{4j13}^{(11)} = p_2[1 - h_c^*(\lambda)] \]
\[ p_{61} = 1 \quad ; \quad p_{7j12} = p_1 \quad ; \quad p_{7j13} = p_2 \quad ; \quad p_{9j1} = g_c^*(\lambda) \]
\[ p_{9j4} = 1 - g_c^*(\lambda) \quad ; \quad p_{9j1}^{(14)} = 1 - g_c^*(\lambda) \quad ; \quad p_{11j0} = h_2^*(\lambda) \]
\[ p_{10.15} = 1 - h_2^*(\lambda) \quad ; \quad p_{11j1}^{(15)} = 1 - h_2^*(\lambda) \quad ; \quad p_{12j1} = 1 \]
\[ p_{13.1} = 1 \quad (6.2.30-6.2.58) \]

By these transition probabilities, it can be verified that

\[ p_{00} + p_{01} + p_{02} = 1 \quad ; \quad p_{13} + p_{14} + p_{15} = 1 \quad ; \quad p_{30} + p_{38} = 1 \]
\[ p_{49} + p_{4j10} + p_{4j11} = 1 \quad ; \quad p_{9j0} + p_{9j14} = 1 \]
\[ p_{10j0} + p_{10j15} = 1 \quad ; \quad p_{13} + p_{14} + p_{16}^{(5)} + p_{17}^{(5)} = 1 \]
\[ p_{26} + p_{27} = 1 \quad ; \quad p_{30} + p_{31}^{(8)} = 1 \]
\[ p_{49} + p_{4j10} + p_{4j12}^{(11)} + p_{4j13}^{(11)} = 1 \quad ; \quad p_{61} = 1 \]
\[ p_{7j12} + p_{7j13} = 1 \quad ; \quad p_{9j0} + p_{9j1}^{(14)} = 1 \quad ; \quad p_{10j0} + p_{10j1}^{(15)} = 1 \]
\[ p_{12j1} = 1 \quad ; \quad p_{13j1} = 1 \quad (6.2.59-6.2.74) \]
FIG. 6.2
\[
\mu_0 = \frac{1}{\lambda + \alpha}, \quad \mu_1 = \frac{1}{\lambda} [1 - h_1^*(\lambda)], \quad \mu_2 = \int_0^x \overline{H}_1(t) \, dt
\]
\[
\mu_3 = \frac{1}{\lambda} [1 - g^*(\lambda)], \quad \mu_4 = \frac{1}{\lambda} [1 - h_2^*(\lambda)], \quad \mu_6 = \int_0^x \overline{G}(t) \, dt
\]
\[
\mu_7 = \int_0^x \overline{H}_3(t) \, dt, \quad \mu_9 = \frac{1}{\lambda} [1 - g_c^*(\lambda)], \quad \mu_{10} = \frac{1}{\lambda} [1 - h_2^*(\lambda)]
\]
\[
\mu_{12} = \int_0^x \overline{G}_c(t) \, dt ; \quad \mu_{13} = \int_0^x \overline{H}_2(t) \, dt \quad (6.2.75-6.2.85)
\]

The unconditional mean time taken by the system to transit for any state \( j \) when it is counted from the epoch of entrance into state 1 is mathematically stated as:

\[
M_{ij} = \int_0^x q_{ij}(t) \, dt = -q_{ij}^*'(0) \quad (6.2.86)
\]

Thus,

\[
m_{00} + m_{01} + m_{02} = \mu_0, \quad m_{13} + m_{14} + m_{15} = \mu_1
\]
\[
m_{30} + m_{38} = \mu_3, \quad m_{30} + m_{31}^{(8)} = \mu_6, \quad m_{13} + m_{14} + m_{16}^{(5)} + m_{17}^{(5)} = \mu_2
\]
\[
m_{26} + m_{27} = \mu_2, \quad m_{49} + m_{4,10} + m_{4,11} = \mu_4
\]
\[
m_{49} + m_{4,10} + m_{4,12}^{(11)} + m_{4,13}^{(11)} = \mu_7, \quad m_{61} = \mu_6
\]
\[
m_{7,12} + m_{7,13} = \mu_7 ; \quad m_{90} + m_{9,14} = \mu_9, \quad m_{90} + m_{9,1}^{(14)} = \mu_{12}
\]
\[
m_{10,0} + m_{10,15} = \mu_{10}, \quad m_{10,0} + m_{10,1}^{(15)} = \mu_{13}, \quad m_{12,1} = \mu_{12}
\]
\[
m_{13,1} = \mu_{13} \quad (6.2.87-6.2.102)
\]

**MEAN TIME TO SYSTEM FAILURE**

By probabilistic arguments, we obtain the following recursive relations for \( \phi_i(t) \):

\[
\phi_0(t) = Q_{00}(t) \circ \phi_0(t) + Q_{01}(t) \circ \phi_1(t) + Q_{02}(t)
\]
\[
\phi_1(t) = Q_{13}(t) \circ \phi_3(t) + Q_{14}(t) \circ \phi_4(t) + Q_{15}(t)
\]
\[
\phi_3(t) = Q_{30}(t) \circ \phi_0(t) + Q_{38}(t)
\]
\[
\phi_4(t) = Q_{40}(t) \circ \phi_0(t) + Q_{4,10}(t) \circ \phi_{10}(t) + Q_{4,11}(t)
\]
\[ \phi_0(t) = Q_{9,0}(t) \odot \phi_0(t) + Q_{9,14}(t) \]
\[ \phi_{10}(t) = Q_{10,0}(t) \odot \phi_0(t) + Q_{10,15}(t) \]  
\[ \phi_{10}(t) = Q_{10,0}(t) \odot \phi_0(t) + Q_{10,15}(t) \]  
(6.2.103-6.2.108)

Taking the L.S.T. of these equations and solving them for \( \phi_0**(s) \), we have
\[ \phi_0**(s) = \frac{N(s)}{D(s)} \]  
(6.2.109)

where
\[ N(s) = Q_{02}**(s) + Q_{01}**(s) [Q_{15}**(s) + Q_{3k}**(s)Q_{13}**(s) \]
\[ + Q_{14}**(s) \{Q_{49}**(s) Q_{9,14}**(s) + Q_{4,11}**(s) \]
\[ + Q_{4,10}**(s) Q_{10,0}**(s) \} \]
\[ D(s) = 1 - Q_{00}**(s) - Q_{01}**(s) [Q_{13}**(s) Q_{30}**(s) \]
\[ + Q_{14}**(s) \{Q_{49}**(s) Q_{9,0}**(s) + Q_{4,10}**(s) Q_{10,0}**(s) \} \]  
(6.2.110-6.2.111)

Now, the mean time to system failure (MTSF) when the system starts from the state ‘0’ is
\[ T_0 = \lim_{s \to 0} \frac{1 - \phi_0**(s)}{s} = \frac{N}{D} \]  
(6.2.112)

where
\[ N = \mu_0 + p_{01}\mu_1 + p_{01}p_{13}\mu_3 + p_{14}(\mu_4 + p_{49}\mu_9 + p_{4,10}\mu_{10}) \]
\[ D = 1 - p_{00} - p_{01}[p_{13}p_{30} + p_{14}(p_{49}p_{9,0} + p_{4,10}p_{10,0}) \]  
(6.2.113-6.2.114)

**AVAILABILITY ANALYSIS**

The recursive relations for \( A_i(t) \) are as follows:
\[ A_0(t) = M_0(t) + q_{00}(t) \odot A_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) \]
\[ A_1(t) = M_1(t) + q_{13}(t) \odot A_3(t) + q_{14}(t) \odot A_4(t) + q_{16}^{(5)}(t) \odot A_6(t) \]
\[ + q_{17}^{(5)}(t) \odot A_7(t) \]
\[ A_2(t) = q_{27}(t) \odot A_7(t) + q_{26}(t) \odot A_6(t) \]
\[ A_3(t) = M_3(t) + q_{30}(t) \odot A_0(t) + q_{31}^{(k)}(t) \odot A_1(t) \]
\[ A_4(t) = M_4(t) + q_{49}(t) \odot A_9(t) + q_{4,10}(t) \odot A_{10}(t) + q_{4,12}^{(11)}(t) A_{12}(t) \]
\[
A_6(t) = q_{61}(t) \odot A_1(t)
\]
\[
A_7(t) = q_{7,12}(t) \odot A_{12}(t) + q_{7,13}(t) \odot A_{13}(t)
\]
\[
A_9(t) = q_{9,0}(t) \odot A_0(t) + q_{9,1}(t) \odot A_1(t)
\]
\[
A_{10}(t) = q_{10,0}(t) \odot A_0(t) + q_{10,1}(t) \odot A_1(t)
\]
\[
A_{12}(t) = q_{12,1}(t) \odot A_1(t)
\]
\[
A_{13}(t) = q_{13,1}(t) \odot A_1(t)
\]

where

\[
M_0(t) = e^{-\lambda t + \alpha t} ; \quad M_1(t) = e^{-\lambda t} H_1(t)
\]
\[
M_3(t) = e^{\lambda t} G(t) ; \quad M_4(t) = e^{\lambda t} H_4(t)
\]

Taking L.T. of the above equations and solving them for \( A_0^*(s) \), we get

\[
A_0^*(s) = \frac{N_1(s)}{D_1(s)}
\]

where

\[
N_1(s) = q_{01}^*(s) [M_1^*(s) + q_{13}^*(s) M_3^*(s) + M_4^*(s) q_{14}^*(s)]
\]
\[
+ M_0^*(s) [1 - q_{13}^*(s) q_{31}^{(8)}(s) - q_{16}^{(5)}(s) q_{61}^*(s)]
\]
\[
- q_{17}^{(5)} [q_{7,12}^*(s) q_{12,1}^*(s) + q_{7,13}^*(s) + q_{13,1}^*(s)]
\]
\[
- q_{14}^*(s) [q_{49}^*(s) q_{9,1}^{(14)}(s) + q_{4,10}^*(s) q_{10,1}^{(15)}(s)]
\]
\[
+ q_{4,12}^{(11)}(s) q_{12,1}^*(s) + q_{14,13}^*(s) q_{13,1}^*(s)]
\]
\[
D_1(s) = (1 - q_{00}^*(s)) [1 - q_{61}^*(s) q_{16}^{(5)}(s) - q_{13}^*(s) q_{31}^{(8)}(s)]
\]
\[
- q_{17}^{(5)}(s) (q_{7,13}^*(s) q_{13,1}^*(s) + q_{7,12}^*(s) q_{12,1}^*(s))
\]
\[
- q_{14}^*(s) [q_{49}^*(s) q_{9,1}^{(14)}(s) + q_{4,10}^*(s) q_{10,1}^{(15)}(s)]
\]
\[
+ q_{4,12}^{(11)}(s) q_{12,1}^*(s) + q_{14,13}^*(s) q_{13,1}^*(s)]
\]
\[
- [q_{13}^*(s) q_{30}^*(s) + q_{14}^*(s) q_{49}^*(s)]
\]
\[
+ q_{4,10}^*(s) q_{10,0}^*(s)] [q_{01}^*(s) + q_{02}^*(s) [q_{26}^*(s) q_{61}^*(s)]
\]
\[
+ q_{27}^*(s) (q_{7,12}^*(s) q_{12,1}^*(s) + q_{7,13}^*(s) q_{13,1}^*(s)]
\]

(6.2.115-6.2.125)
In steady state, the availability of the system is given by
\[ A_0 = \frac{N_1}{D_1} \quad (6.2.133) \]

where
\[ N_1 = [p_{13}p_{30} + p_{14}(p_{49}p_{9,0} + p_{4,10}p_{10,0})] \mu_0 + (p_{01} + p_{02}) \]
\[ [\mu_1 + p_{13}\mu_3 + p_{14}\mu_4] \]
\[ D_1 = [p_{13}p_{30} + p_{14}(p_{49}p_{9,0} + p_{4,10}p_{10,0})] \mu_2 + (1 - p_{00})\mu_2 \]
\[ + p_{02}[p_{13}p_{30} + p_{14}(p_{49}p_{9,0} + p_{4,10}p_{10,0})] \mu_2 + (1 - p_{00})p_{13}\mu_6 \]
\[ + [p_{16}(1 - p_{00}) + p_{02}p_{26}(p_{13}p_{30} + p_{14}(p_{49}p_{9,0} + p_{4,10}p_{10,0}))] \mu_6 \]
\[ + [(1 - p_{00})(p_{14} + p_{17}(5)) + p_{02}p_{27}(p_{13}p_{30} + p_{14}(p_{49}p_{9,0} + p_{4,10}p_{10,0}))] \mu_7 \]
\[ + [p_{14}(p_{49}(1 - p_{00}) + p_{4,12}(11)) + p_{27}p_{7,12}p_{13}p_{30} + p_{14}(p_{49}p_{9,0} + p_{4,10}p_{10,0})] \mu_7 \]
\[ + p_{27}p_{7,13}(p_{13}p_{30} + p_{14}(p_{49}p_{9,0} + p_{4,10}p_{10,0}))] \mu_{13} \]
\[ (6.2.134-6.2.135) \]

**BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN**
(Repair Time Only)

By probabilistic arguments, we have the following recursive relations:

\[ B_0(t) = q_{00}(t) \odot B_0(t) + q_{01}(t) \odot B_1(t) + q_{02}(t) \odot B_2(t) \]
\[ B_1(t) = q_{13}(t) \odot B_3(t) + q_{14}(t) \odot B_4(t) + q_{16}(5)(t) \odot B_6(t) + q_{17}(5)(t) \odot B_7(t) \]
\[ B_2(t) = q_{27}(t) \odot B_7(t) + q_{26}(t) \odot B_6(t) \]
\[ B_3(t) = W_3(t) + q_{30}(t) \odot B_0(t) + q_{31}(8)(t) \odot B_1(t) \]
\[ B_4(t) = q_{49}(t) \odot B_9(t) + q_{4,10}(t) \odot B_{10}(t) + q_{4,12}(11)(t) B_{12}(t) + q_{4,13}(11)(t) \odot B_{13}(t) \]
\[ B_6(t) = W_6(t) + q_{61}(t) \odot B_1(t) \]
\[ B_7(t) = q_{7,12}(t) \odot B_{12}(t) + q_{7,13}(t) \odot B_{13}(t) \]
\[ B_9(t) = q_{9,0}(t) \odot B_0(t) + q_{9,1}(14)(t) \odot B_1(t) \]
\[ B_{10}(t) = q_{10,0}(t) \odot B_0(t) + q_{10,1}(15)(t) \odot B_1(t) \]
\( B_{12}(t) = q_{12,1}(t) \odot B_1(t) \)
\( B_{13}(t) = q_{13,1}(t) \odot B_1(t) \)

(6.2.136-6.2.147)

where

\( W_3(t) = W_6(t) = \bar{G}(t) \)

Taking L.T. of the above equations and solving them for \( B_0^*(s) \), we get

\[
B_0^*(s) = \frac{N_2(s)}{D_1(s)}
\]

where

\[
N_2(s) = W_3^*(s) q_{13}^*(s) \{ q_{01}^*(s) + q_{02}^*(s) \{ q_{26}^*(s) q_{61}^*(s) + q_{27}^*(s) (q_{7,12}^*(s) + q_{7,13}^*(s) q_{13,1}^*(s)) \} + W_6^*(s) \{ q_{20}^*(s) q_{20}^*(s) \} + q_{26}^*(s) q_{27}^*(s) (q_{7,12}^*(s) qi_{2,1}^*(s) + q_{7,13}^*(s) qi_{3,1}^*(s)) \}
\]

and \( D_1(s) \) is already specified. (6.2.149)

In steady state, the total fraction of time for which the system is under repair of ordinary repairman

\[
B_0 = \frac{N_2}{D_1}
\]

(6.2.150)

where

\[
N_2 = [P_{13}(P_{01} + P_{02}) + P_{16}^{(5)}(P_{01} + P_{02}) + P_{02}\mu_6 \{ P_{13}P_{30} + P_{14}(P_{49}P_{9,0} + P_{4,10}P_{10,0}) \}]
\]

(6.2.151)

BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN

(Inspection Time for detecting the type of failure)

The recursive relations for \( B_{I_i}(t) \) are as follows.

\( B_{I_0}(t) = q_{00}(t) \odot B_{I_0}(t) + q_{01}(t) \odot B_{I_1}(t) + q_{02}(t) \odot B_{I_2}(t) \)
\[ \begin{align*}
& BI_1(t) = W_1(t) + q_{13}(t) \odot BI_3(t) + q_{14}(t) \odot BI_4(t) + q_{16}(5)(t) \odot BI_6(t) \\
& \quad + q_{17}(5)(t) \odot BI_7(t) \\
& BI_2(t) = W_2(t) + q_{27}(t) \odot BI_7(t) + q_{26}(t) \odot BI_6(t) \\
& BI_3(t) = q_{30}(t) \odot BI_0(t) + q_{31}(8)(t) \odot BI_1(t) \\
& BI_4(t) = q_{40}(t) \odot BI_9(t) + q_{4,10}(t) \odot BI_{10}(t) + q_{4,12}(11)(t) \ BI_{12}(t) \\
& \quad + q_{4,13}(11)(t) \odot BI_{13}(t) \\
& BI_6(t) = q_{61}(t) \odot BI_1(t) \\
& BI_7(t) = q_{7,12}(t) \odot BI_{12}(t) + q_{7,43}(t) \odot BI_{13}(t) \\
& BI_9(t) = q_{9,10}(t) \odot BI_0(t) + q_{9,1}(14)(t) \odot BI_1(t) \\
& BI_{10}(t) = q_{10,10}(t) \odot BI_0(t) + q_{10,1}(15)(t) \odot BI_1(t) \\
& BI_{12}(t) = q_{12,1}(t) \odot BI_1(t) \\
& BI_{13}(t) = q_{13,1}(t) \odot BI_1(t) \quad (6.2.152-6.2.163)
\end{align*} \]

where

\[ W_1(t) = W_2(t) = H_1(t) \]

Taking L.T. of the above equations and solving them for \( BI_0^*(s) \), we get

\[ BI_0^*(s) = \frac{N_3(s)}{D_1(s)} \quad (6.2.164) \]

where

\[ N_3(s) = W_1^*(s) [q_{01}^*(s) + q_{02}^*(s) \{q_{26}^*(s)q_{61}^*(s) \\
+ q_{27}^*(s) (q_{7,12}^*(s)q_{12,1}^*(s) + q_{7,13}^*(s)q_{13,1}^*(s))\}] \\
+ W_2^*(s) q_{02}^*(s) [1 - q_{13}^*(s)q_{31}(8)^*(s) - q_{16}(5)^*(s)q_{61}^*(s) \\
- q_{17}(5)^*(s) (q_{7,12}^*(s)q_{12,1}^*(s) + q_{7,13}^*(s)q_{13,1}^*(s)) \\
- q_{14}^*(s) \{q_{40}^*(s)q_{9,1}(14)^*(s) + q_{4,10}^*(s) q_{10,1}(15)^*(s) \\
+ q_{4,12}(11)^*(s)q_{12,1}^*(s) + q_{4,13}(11)^*(s)q_{13,1}^*(s)\}] \]

and \( D_1(s) \) is already specified. \quad (6.2.165)
In steady state, the total fraction of time for which the system is under inspection (for detecting the type of failure) of ordinary repairman is given by

$$BI_0 = \frac{N_3}{D_1}$$  \hspace{1cm} (6.2.166)

where

$$N_3 = [(p_{01} + p_{02}) + p_{02}(p_{13}p_{30} + p_{14}(p_{49}p_{9.0} + p_{4.10}p_{10.0})))] \mu_2$$  \hspace{1cm} (6.2.167)

**BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN**

(Replacement Time Only)

The recursive relations for $BR_i(t)$ are as follows.

- $BR_0(t) = q_{00}(t) \circ BR_0(t) + q_{01}(t) \circ BR_1(t) + q_{02}(t) \circ BR_2(t)$
- $BR_1(t) = q_{13}(t) \circ BR_3(t) + q_{14}(t) \circ BR_4(t) + q_{16(5)}(t) \circ BR_6(t) + q_{17(5)}(t) \circ BR_7(t)$
- $BR_2(t) = q_{27}(t) \circ BR_7(t) + q_{26}(t) \circ BR_6(t)$
- $BR_3(t) = q_{30}(t) \circ BR_0(t) + q_{31}(t) \circ BR_1(t)$
- $BR_4(t) = q_{49}(t) \circ BR_9(t) + q_{410}(t) \circ BR_{10}(t) + q_{412(11)}(t) \circ BR_{12}(t) + q_{413(11)}(t) \circ BR_{13}(t)$
- $BR_6(t) = q_{61}(t) \circ BR_1(t)$
- $BR_7(t) = q_{712}(t) \circ BR_{12}(t) + q_{713}(t) \circ BR_{13}(t)$
- $BR_9(t) = q_{90}(t) \circ BR_0(t) + q_{91(14)}(t) \circ BR_1(t)$
- $BR_{10}(t) = W_{10}(t) + q_{10.0}(t) \circ BR_0(t) + q_{10.1(15)}(t) \circ BR_1(t)$
- $BR_{12}(t) = q_{12.1}(t) \circ BR_1(t)$
- $BR_{13}(t) = W_{13}(t) + q_{13.1}(t) \circ BR_1(t)$

where

$$W_{10}(t) = W_{13}(t) = \tilde{H}_2(t)$$  \hspace{1cm} (6.2.168-6.2.179)

Taking L.T. of the above equations and solving them for $BR_i*(s)$, we get
\[ \text{BR}_0(s) = \frac{N_4(s)}{D_1(s)} \]  

(6.2.180)

where

\[ N_4(s) = W_{10}(s) q_{4.10}(s) q_{14}(s) [q_{01}(s) + q_{02}(s) q_{26}(s) q_{61}(s) + q_{02}(s) q_{27}(s)(q_{12.1}(s) q_{7.12}(s) + q_{13.1}(s) q_{7.13}(s))]
+ W_{13}(s) [(q_{01}(s) + q_{02}(s) q_{26}(s) q_{61}(s))(q_{14}(s) q_{4.13}(s))
+ q_{7.13}(s) q_{17}(s) + q_{02}(s) q_{27}(s) q_{7.13}(s) \{1 - q_{13}(s) q_{31}(s) - q_{61}(s) q_{16}(s) - q_{14}(s) \{q_{49}(s) q_{9.14}(s) + q_{4.10}(s) q_{10.1}(s) + q_{4.12}(s) q_{12.1}(s)\},
+ q_{02}(s) q_{27}(s) q_{14}(s) q_{12.1}(s) q_{7.12}(s) q_{4.13}(s)\}
\]

and \( D_1(s) \) is already specified.  

(6.2.181)

In steady state, the total fraction of time for which the system is busy, under replacement by ordinary repairman is given by

\[ \text{BR}_0 = \frac{N_4}{D_1} \]  

(6.2.182)

where

\[ N_4 = \mu_{13}[(1 - p_{00})(p_{14} p_{4.10} + p_{14} p_{4.13} + p_{17}(s) p_{7.13}) - p_{02} p_{27} p_{7.13} p_{13} + p_{14} p_{12} p_{31}(s) [1 - p_{40} p_{90} - p_{4.10} p_{10.0}]] \]  

(6.2.183)

**EXPECTED NUMBER OF VISITS BY ORDINARY REPAIRMAN**

\[ V_0(t) = Q_{00}(t) \mathbb{S} [1 + V_0(t)] + Q_{01}(t) \mathbb{S} [1+V_1(t)] + Q_{02}(t) \mathbb{S} [1+V_2(t)] \]
\[ V_1(t) = Q_{13}(t) \mathbb{S} V_3(t) + Q_{14}(t) \mathbb{S} V_4(t) + Q_{16}(t) \mathbb{S} V_6(t) \]
\[ + Q_{17}(t) \mathbb{S} V_7(t) \]
\[ V_2(t) = Q_{27}(t) \mathbb{S} V_7(t) + Q_{26}(t) \mathbb{S} V_6(t) \]
\[ V_3(t) = Q_{30}(t) \mathbb{S} V_0(t) + Q_{31}(s) \mathbb{S} V_1(t) \]
\[ V_4(t) = Q_{40}(t) \mathbb{S} V_0(t) + Q_{4.10}(t) \mathbb{S} [1+V_{10}(t)] + Q_{4.12}(11)(t) \mathbb{S} V_{12}(t) \]
\[ + Q_{4.13}(11)(t) \mathbb{S} [1+V_{13}(t)] \]
\[ V_6(t) = Q_{61}(t) \mathbb{S} V_1(t) \]
\[ V_7(t) = Q_{7.12}(t) + Q_{7.13}(t) [1 + V_{13}(t)] \]
\[ V_9(t) = Q_{9.0}(t) + Q_{9.1}(t) [1 + V_1(t)] \]
\[ V_{10}(t) = Q_{10.0}(t) + Q_{10.1}(t) V_1(t) \]
\[ V_{12}(t) = Q_{12.1}(t) V_1(t) \]
\[ V_{13}(t) = Q_{13.1}(t) V_1(t) \]

where

Taking L.S.T. of the above equations and solving them for \( V_0**(s) \), we get

\[ V_0**(s) = \frac{N_5(s)}{D_1(s)} \]

where

\[ N_5(s) = [Q_{00}^*(s) + Q_{01}^*(s) + Q_{02}^*(s)] [1 - Q_{13}^*(s) Q_{31}^*(s)] - Q_{16}^*(s) Q_{61}^*(s) - Q_{14}^*(s) \{Q_{49}^*(s) Q_{9.1}^*(s)\}
+ Q_{4.10}^*(s) Q_{10.1}^*(s) + Q_{4.12}^*(s) Q_{12.1}^*(s)
+ Q_{4.13}^*(s) Q_{13.1}^*(s) - Q_{17}^*(s) \{Q_{7.12}^*(s) + Q_{7.13}^*(s) \}
+ Q_{7.13}^*(s) Q_{13.1}^*(s)\]

\[ [Q_{14}^*(s) (Q_{4.10}^*(s) + Q_{4.13}^*(s) + Q_{4.12}^*(s) Q_{9.1}^*(s))] + Q_{7.13}^*(s)\]
\[ [Q_{17}^*(s) (Q_{01}^*(s) + Q_{02}^*(s) Q_{26}^*(s) Q_{61}^*(s))] + Q_{02}^*(s) Q_{27}^*(s)\]
\[ \{1 - Q_{13}^*(s) Q_{31}^*(s) - Q_{61}^*(s) Q_{16}^*(s) - Q_{14}^*(s) (Q_{49}^*(s) Q_{9.1}^*(s) + Q_{4.10}^*(s) Q_{10.1}^*(s) + Q_{4.12}^*(s) Q_{12.1}^*(s)\}
+ Q_{4.13}^*(s) Q_{13.1}^*(s)\}\]

and \( D_1(s) \) is already specified.

In steady state, the number of visits per unit time by the ordinary repairman is given by

\[ V_0 = \frac{N_5}{D_1} \]
\[ N_5 = (p_{01} + p_{02})[p_{7,13}p_{17}^{(5)} + p_{14}(p_{4,10} + p_{4,13}^{(11)} + p_{49}p_{9,1}^{(14)})] + [p_{13}p_{30} \\
+ p_{14}(p_{49}p_{90} + p_{4,10}p_{10,0})] [1 + p_{02}p_{27}p_{7,13}] \] (6.2.198)

and \( D_i \) is already specified

**EXPECTED NUMBER OF REPLACEMENTS**

\[ R_0(t) = Q_{00}(t) \otimes R_0(t) + Q_{01}(t) \otimes R_1(t) + Q_{02}(t) \otimes R_2(t) \]
\[ R_1(t) = Q_{13}(t) \otimes R_3(t) + Q_{14}(t) \otimes R_4(t) + Q_{16}^{(5)}(t) \otimes R_6(t) \]
\[ + Q_{17}^{(5)}(t) \otimes R_7(t) \]
\[ R_2(t) = Q_{27}(t) \otimes R_7(t) + Q_{26}(t) \otimes R_6(t) \]
\[ R_3(t) = Q_{30}(t) \otimes R_0(t) + Q_{31}^{(8)}(t) \otimes R_1(t) \]
\[ R_4(t) = Q_{49}(t) \otimes R_9(t) + Q_{4,10}(t) \otimes [1 + R_{10}(t)] + Q_{4,12}^{(11)}(t) \otimes R_{12}(t) \]
\[ + Q_{4,13}^{(11)}(t) \otimes [1 + R_{13}(t)] \]
\[ R_6(t) = Q_{61}(t) \otimes R_1(t) \]
\[ R_7(t) = Q_{7,12}(t) \otimes R_{12}(t) + Q_{7,13}(t) \otimes R_{13}(t) \]
\[ R_9(t) = Q_{9,0}(t) \otimes R_0(t) + Q_{9,1}^{(14)}(t) \otimes R_1(t) \]
\[ R_{10}(t) = Q_{10,0}(t) \otimes R_0(t) + Q_{15}^{(15)}_{10,1}(t) \otimes R_1(t) \]
\[ R_{12}(t) = Q_{12,1}(t) \otimes R_1(t) \]
\[ R_{13}(t) = Q_{13,1}(t) \otimes R_1(t) \] (6.2.199-6.2.209)

Taking L.S.T. of the above equations and solving them for \( R_0^{**}(s) \), we get

\[ R_0^{**}(s) = \frac{N_6(s)}{D_1(s)} \] (6.2.210)

where

\[ N_6(s) = Q_{14}^{*}(s)(Q_{4,10}^{*}(s) + Q_{4,12}^{(11)*}(s)) [Q_{02}^{*}(s) \{Q_{26}^{*}(s) Q_{61}^{*}(s) \]
\[ + Q_{27}^{*}(s) (Q_{7,12}^{*}(s) Q_{12,1}^{*}(s) + Q_{7,13}^{*}(s) Q_{13,1}^{*}(s)) \} + Q_{01}^{*}(s)] \] (6.2.211)

In steady state, the total fraction of time for which system is under replacement is given by
where

\[ N_6 = p_{14}(p_{4,10} + p_{4,12}) (p_{01} + p_{02}) \]  

**BUSY PERIOD ANALYSIS OF EXPERT REPAIRMAN**  
(Repair Time Only)

By probabilistic arguments, we have the following recursive relations:

\[ B_0^\epsilon(t) = q_{00}(t) \otimes B_0^\epsilon(t) + q_{01}(t) \otimes B_1^\epsilon(t) + q_{02}(t) \otimes B_2^\epsilon(t) \]

\[ B_1^\epsilon(t) = q_{13}(t) \otimes B_3^\epsilon(t) + q_{14}(t) \otimes B_4^\epsilon(t) + q_{16}(5)(t) \otimes B_6^\epsilon(t) + q_{17}(5)(t) \otimes B_7^\epsilon(t) \]

\[ B_2^\epsilon(t) = q_{27}(t) \otimes B_7^\epsilon(t) + q_{28}(t) \otimes B_9^\epsilon(t) \]

\[ B_3^\epsilon(t) = q_{30}(t) \otimes B_9^\epsilon(t) + q_{31}(8)(t) \otimes B_1^\epsilon(t) \]

\[ B_4^\epsilon(t) = q_{49}(t) \otimes B_4^\epsilon(t) + q_{40}(t) \otimes B_{10}^\epsilon(t) + q_{41}(11)(t) \otimes B_{12}^\epsilon(t) + q_{42}(11)(t) \otimes B_{13}^\epsilon(t) \]

\[ B_5^\epsilon(t) = q_{51}(t) \otimes B_1^\epsilon(t) \]

\[ B_6^\epsilon(t) = q_{61}(t) \otimes B_2^\epsilon(t) \]

\[ B_7^\epsilon(t) = q_{7,12}(t) \otimes B_{12}^\epsilon(t) + q_{7,13}(t) \otimes B_{13}^\epsilon(t) \]

\[ B_8^\epsilon(t) = W_9(t) + q_{9,0}(t) \otimes B_0^\epsilon(t) + q_{9,1}(14)(t) \otimes B_1^\epsilon(t) \]

\[ B_{10}^\epsilon(t) = q_{10,0}(t) \otimes B_9^\epsilon(t) + q_{10,1}(15)(t) \otimes B_1^\epsilon(t) \]

\[ B_{12}^\epsilon(t) = W_{12}(t) + q_{12,1}(t) \otimes B_1^\epsilon(t) \]

\[ B_{13}^\epsilon(t) = q_{13,1}(t) \otimes B_1^\epsilon(t) \]

where

\[ W_9(t) = W_{12}(t) = \bar{G}(t) \]

Taking L.T. of the above equations and solving them for \( B_0^\epsilon^*(s) \), we get

\[ B_0^\epsilon^*(s) = \frac{N_7(s)}{D_1(s)} \]

where

\[ N_7(s) = W_9^*(s) q_{14}^*(s) q_{49}^*(s) [q_{01}^*(s) + q_{02}^*(s) \{q_{26}^*(s) q_{61}^*(s) \]

\[ + q_{27}^*(s) (q_{7,12}^*(s) q_{12,1}^*(s) + q_{7,13}^*(s) q_{13,1}^*(s)) \{] \]
In steady state, the total fraction of time for which the system is busy under repair of expert repairman is given by

$$B_0^c = \frac{N_2}{D_1}$$  \hspace{1cm} \text{(6.2.228)}$$

where

$$N_2 = p_{144}p_{49}(p_{01} + p_{02}) + (p_{144}p_{49}(p_{01} + p_{02}) + p_{02}p_{27}p_{712}) \cdot \{p_{01} + p_{02}\} \cdot l_{12}$$  \hspace{1cm} \text{(6.2.229)}$$

and $D_1$ is already specified

**BUSY PERIOD ANALYSIS OF EXPERT REPAIRMAN**

*(Inspection Time Only)*

By probabilistic arguments, we have the following recursive relations:

$$B_{10^c}(t) = q_{00}(t) \circ B_{10^c}(t) + q_{01}(t) \circ B_{11^c}(t) + q_{02}(t) \circ B_{12^c}(t)$$

$$B_{11^c}(t) = q_{14}(t) \circ B_{11^c}(t) + q_{14}(t) \circ B_{12^c}(t) + q_{14}(t) \circ B_{13^c}(t)$$

$$B_{12^c}(t) = q_{26}(t) \circ B_{12^c}(t) + q_{26}(t) \circ B_{13^c}(t)$$

$$B_{13^c}(t) = q_{30}(t) \circ B_{13^c}(t) + q_{30}(t) \circ B_{14^c}(t)$$

$$B_{14^c}(t) = W_4(t) + q_{40}(t) \circ B_{14^c}(t) + q_{410}(t) \circ B_{15^c}(t)$$

$$B_{15^c}(t) = q_{61}(t) \circ B_{15^c}(t)$$

$$B_{16^c}(t) = W_7(t) + q_{712}(t) \circ B_{16^c}(t) + q_{713}(t) \circ B_{17^c}(t)$$

$$B_{17^c}(t) = q_{90}(t) \circ B_{17^c}(t) + q_{914}(t) \circ B_{18^c}(t)$$

$$B_{18^c}(t) = q_{190}(t) \circ B_{18^c}(t) + q_{191}(t) \circ B_{19^c}(t)$$

$$B_{19^c}(t) = q_{00}(t) \circ B_{19^c}(t)$$

$$B_{20^c}(t) = q_{00}(t) \circ B_{20^c}(t) + q_{00}(t) \circ B_{21^c}(t)$$
\[
B_{12}^e(t) = q_{12,i}(t) \odot B_{1}^e(t)
\]
\[
B_{13}^e(t) = q_{13,j}(t) \odot B_{1}^e(t)
\]
(6.2.230-6.2.240)

where
\[
W_4(t) = W_7(t) = H_c(t)
\]
(6.2.241)

Taking L.T. of the above equations and solving them for \(B_{10}^e(s)\), we get
\[
B_{10}^e(s) = \frac{N_8(s)}{D_1(s)}
\]
(6.2.242)

where
\[
N_8(s) = W_4^*(s) q_{14}^*(s) [q_{01}^*(s) + q_{02}^*(s) \{q_{26}^*(s) q_{61}^*(s) + q_{27}^*(s)\}
\]
\[
(q_{7,12}^*(s) q_{12,j}^*(s) + q_{7,13}^*(s) q_{13,i}^*(s))\} + W_7^*(s)
\]
\[
[q_{01}^*(s) q_{17}^{(5)}(s) + q_{02}^*(s) \{q_{26}^*(s) q_{61}^*(s) q_{17}^{(5)}(s)
\]
\]
\[
+ q_{27}^*(s) \{1 - q_{13}^*(s) q_{31}^{(8)}(s) - q_{16}^{(5)}(s) q_{61}^*(s)
\]
\]
\[
- q_{14}^*(s) (q_{49}^*(s) q_{9,12}^{(14)}(s) + q_{4,10}^*(s) q_{10,12}^{(15)}(s) + q_{4,12}^{(11)}(s)
\]
\]
\[
+ q_{4,13}^{(11)}(s) q_{13,j}^*(s))\} + W_7^*(s)
\]
(6.2.243)

where \(D_1(s)\) is already specified

In steady state, the total fraction of time for which the system is busy under inspection of expert repairman is given by
\[
B_{10} = \frac{N_8}{D_1}
\]
(6.2.244)

where
\[
N_8 = [(p_{01} + p_{02}) (p_{14} + p_{17}^{(5)}) + p_{02}p_{27}\{p_{13} + p_{14}p_{9,0} + p_{4,10}p_{10,0}\}] \mu_7
\]
(6.2.245)

EXPECTED NUMBER OF VISITS BY EXPERT REPAIRMAN

The recursive relations for \(V_j^e(t)\) are obtained
\[
V_0^e(t) = Q_{00}(t) V_0^e(t) + Q_{01}(t) V_1^e(t) + Q_{02}(t) V_2^e(t)
\]
\[
V_1^e(t) = Q_{13}(t) V_3^e(t) + Q_{16}^{(5)}(t) V_6^e(t) + Q_{14}(t) [1 + V_4^e(t)]
\]
\[
+ Q_{17}^{(5)}(t) [1 + V_7^e(t)]
\]
\[ V_2'(t) = Q_27(t) [1 + V_7 c(t)] + Q_26(t) V_6 c(t) \]

\[ V_3 c(t) = Q_30(t) V_6 c(t) + Q_31 c(t) V_1 c(t) \]

\[ V_4 c(t) = Q_49(t) V_9 c(t) + Q_{4,10}(t) V_{10} c(t) + Q_{4,12}(t) V_{12} c(t) \]

\[ + Q_{4,13}(t) V_{13} c(t) \]

\[ V_5 c(t) = Q_61(t) V_1 c(t) \]

\[ V_7 c(t) = Q_{7,12}(t) V_{12} c(t) + Q_{7,13}(t) V_{13} c(t) \]

\[ V_9 c(t) = Q_{9,0}(t) V_0 c(t) + Q_{9,1}(t) V_1 c(t) \]

\[ V_{10} c(t) = Q_{10,0}(t) V_{10} c(t) + Q_{10,1}(t) V_1 c(t) \]

\[ V_{12} c(t) = Q_{12,1}(t) V_1 c(t) \]

\[ V_{13} c(t) = Q_{13,1}(t) V_1 c(t) \]

(6.2.246-6.2.256)

Taking L.S.T. of the above equations and solving them for \( V_0 c**(s) \), we get

\[ V_0 c*(s) = \frac{N_9(s)}{D_1(s)} \]  

(6.2.257)

where

\[ N_9(s) = Q_{02}(s) Q_{27}(s) \left[ Q_{14}(s) \right. \left. \{ Q_{7,12}(s) Q_{12,1}(s) + Q_{7,13}(s) Q_{13,1}(s) \right. \]

\[ + Q_{49}(s) Q_{9,1}(s) + Q_{4,10}(s) Q_{10,3}(s) - Q_{4,12}(s) \]

\[ \left. \left. (Q_{13}(s) - Q_{12,1}(s)) \right\} + 1 - Q_{13}(s) Q_{31}(s) - Q_{61}(s) Q_{16}(s) \right) \]

(6.2.258)

where \( D_1(s) \) is already specified.

In steady state, the total number of visits per unit time by the expert repairman is given by

\[ V_0 c = \frac{N_9}{D_1} \]  

(6.2.259)

where

\[ N_9 = p_{14} p_{27} \left\{ \frac{1}{1 + p_{49} p_{9,1}} + p_{4,10} p_{10,1} + p_{4,12}(1 - p_{13}) \right\} \]

\[ + 1 - p_{13} p_{31} - p_{16}(1 - p_{13}) \]

(6.2.260)
EXPECTED NUMBER OF TIMES THE INSPECTION IS CARRIED OUT FOR REVEALING THE FAILURE

BY probabilistic arguments, we have the following recursive relations

\[ T_{I_0}(t) = Q_{00}(t) \mathbb{S} [1 + T_{I_0}(t)] + Q_{01}(t) \mathbb{S} [1 + T_{I_1}(t)] + Q_{02}(t) \mathbb{S} T_{I_2}(t) \]

\[ T_{I_1}(t) = Q_{13}(t) \mathbb{S} T_{I_3}(t) + Q_{14}(t) \mathbb{S} T_{I_4}(t) + Q_{16}(t) \mathbb{S} T_{I_6}(t) + Q_{17}(t) \mathbb{S} T_{I_7}(t) \]

\[ T_{I_2}(t) = Q_{27}(t) \mathbb{S} T_{I_7}(t) + Q_{26}(t) \mathbb{S} T_{I_6}(t) \]

\[ T_{I_3}(t) = Q_{30}(t) \mathbb{S} T_{I_0}(t) + Q_{31}(t) \mathbb{S} T_{I_1}(t) \]

\[ T_{I_4}(t) = Q_{40}(t) \mathbb{S} T_{I_9}(t) + Q_{41}(t) \mathbb{S} T_{I_{10}}(t) + Q_{412}(t) \mathbb{S} T_{I_{12}}(t) \]

\[ T_{I_5}(t) = Q_{50}(t) \mathbb{S} T_{I_0}(t) + Q_{51}(t) \mathbb{S} T_{I_1}(t) \]

\[ T_{I_6}(t) = Q_{60}(t) \mathbb{S} T_{I_0}(t) + Q_{61}(t) \mathbb{S} T_{I_1}(t) \]

\[ T_{I_7}(t) = Q_{70}(t) \mathbb{S} T_{I_0}(t) + Q_{71}(t) \mathbb{S} T_{I_1}(t) \]

\[ T_{I_8}(t) = Q_{80}(t) \mathbb{S} T_{I_0}(t) + Q_{81}(t) \mathbb{S} T_{I_1}(t) \]

\[ T_{I_9}(t) = Q_{90}(t) \mathbb{S} T_{I_0}(t) + Q_{91}(t) \mathbb{S} T_{I_1}(t) \]

\[ T_{I_{10}}(t) = Q_{100}(t) \mathbb{S} T_{I_0}(t) + Q_{101}(t) \mathbb{S} T_{I_1}(t) \]

\[ T_{I_{11}}(t) = Q_{110}(t) \mathbb{S} T_{I_0}(t) + Q_{111}(t) \mathbb{S} T_{I_1}(t) \]

\[ T_{I_{12}}(t) = Q_{120}(t) \mathbb{S} T_{I_0}(t) + Q_{121}(t) \mathbb{S} T_{I_1}(t) \]

\[ T_{I_{13}}(t) = Q_{130}(t) \mathbb{S} T_{I_0}(t) + Q_{131}(t) \mathbb{S} T_{I_1}(t) \]

Taking L.S.T. of the above equations and solving them for \( T_{I_0}(s) \), we get

\[ T_{I_0}(s) = \frac{N_{10}(s)}{D_1(s)} \quad \text{(6.2.272)} \]

where

\[ N_{10}(s) = (Q_{00}(s) + Q_{01}(s) [1 - Q_{13}(s)Q_{31}(s) - Q_{16}(s)] - Q_{17}(s)Q_{712}(s)Q_{121}(s) + Q_{17}(s)Q_{713}(s)Q_{131}(s) - Q_{14}(s)Q_{49}(s)Q_{91}(s) + Q_{14}(s)Q_{410}(s)Q_{101}(s) + Q_{412}(s)Q_{121}(s) + Q_{412}(s)Q_{131}(s)] \quad \text{(6.2.273)} \]

and \( D_1(s) \) is already specified.
In steady state, the total number of times inspection is carried out for revealing the failure.

\[ T_{I_0} = \frac{N_{10}}{D_1} \]  

(6.2.274)

where

\[ N_{10} = (1 - p_{02}) \{ p_{13} p_{30} + p_{14} (p_{49} p_{9.0} + p_{4.10} p_{10.0}) \} \]  

(6.2.275)

**PROFIT ANALYSIS**

\[ P_{62} = C_0 A_0 - C_1 B_0 - C_2 B I_0 - C_6 B R_0 - C_4 V_0 - C_5 R_0 - C_6 B^c_0 - C_7 B I^c_0 \]

\[ - C_8 V^c_0 - C_{10} T_{I_0} \]  

(6.2.276)

where \( C_0, C_1, C_2, C_3, C_5 \) are already defined in model 1.

\( C_6 = \) Cost per unit time for which expert repairman.

\( C_7 = \) Cost per unit time for which expert repairman is busy under inspection.

\( C_8 = \) Cost per visit of the expert repairman

\( C_{10} = \) Cost per unit time for which inspection is carried out for revealing the failure

**PARTICULAR CASE**

For graphical interpretation the following particular case is assumed

\[ g(t) = \alpha_1 e^{-\alpha_1 t} \quad ; \quad h_1(t) = \gamma_1 e^{-\gamma_1 t} \quad ; \quad h_2(t) = \gamma_2 e^{-\gamma_2 t} \]

\[ g_2(t) = \alpha_2 e^{-\alpha_2 t} \quad ; \quad h_2(t) = \gamma e^{-\gamma t} \quad ; \quad h(t) = \beta e^{-\beta t} \]

We can easily obtain the following:

\[ p_{00} = \frac{\beta}{\lambda + \alpha + \beta} \quad ; \quad p_{01} = \frac{(\lambda + \alpha)\beta}{(\lambda + \beta)(\lambda + \alpha + \beta)} \]

\[ p_{02} = 1 - \frac{\beta}{\alpha} \left( \frac{\lambda + \alpha}{\lambda + \beta} - \frac{\lambda}{\lambda + \alpha + \beta} \right) \quad ; \quad p_{13} = \frac{p_{11} \gamma_1}{\lambda + \gamma_1} \]

\[ p_{14} = \frac{p_2 \gamma_1}{\lambda + \gamma_1} \quad ; \quad p_{15} = \frac{\lambda}{\lambda + \gamma_1} \quad ; \quad p_{16} = \frac{p_1 \lambda}{\lambda + \gamma_1} \]
\( p_{17}^{(5)} = \frac{p_3 \lambda}{\lambda + \gamma_1} \); \( p_{26} = p_1 \); \( p_{27} = p_2 \)

\( p_{30} = \frac{\alpha_1}{\lambda + \alpha_1} \); \( p_{38} = \frac{\lambda}{\lambda + \alpha_1} \); \( p_{31}^{(8)} = \frac{\lambda}{\lambda + \alpha_1} \)

\( p_{49} = \frac{p_1 \gamma_2}{\lambda + \gamma_2} \); \( p_{4,10} = \frac{p_2 \gamma_2}{\lambda + \gamma_2} \); \( p_{4,11} = \frac{\lambda}{\lambda + \gamma_2} \)

\( p_{4,12}^{(11)} = \frac{p_1 \lambda}{\lambda + \gamma_2} \); \( p_{4,13}^{(11)} = \frac{p_2 \lambda}{\lambda + \gamma_2} \); \( p_{61} = 1 \)

\( p_{7,12} = p_1 \); \( p_{7,13} = p_2 \); \( p_{90} = \frac{\alpha_2}{\lambda + \alpha_2} \)

\( p_{9,14} = \frac{\lambda}{\lambda + \alpha_2} \); \( p_{9,1}^{(14)} = \frac{\lambda}{\lambda + \alpha_2} \); \( p_{10,0} = \frac{\gamma}{\lambda + \gamma} \)

\( p_{10,15} = \frac{\lambda}{\lambda + \gamma} \); \( p_{10,1}^{(15)} = \frac{\lambda}{\lambda + \gamma} \); \( p_{12,1} = 1 \)

\( p_{13,1} = 1 \)

\( \mu_0 = \frac{1}{\lambda + \alpha} \); \( \mu_1 = \frac{1}{\lambda + \gamma_1} \); \( \mu_2 = \frac{1}{\gamma_1} \)

\( \mu_3 = \frac{1}{\lambda + \alpha_1} \); \( \mu_4 = \frac{1}{\lambda + \gamma_2} \); \( \mu_6 = \frac{1}{\alpha_1} \)

\( \mu_7 = \frac{1}{\gamma_2} \); \( \mu_9 = \frac{1}{\lambda + \alpha_2} \); \( \mu_{10} = \frac{1}{\lambda + \gamma} \)

\( \mu_{12} = \frac{1}{\alpha_2} \); \( \mu_{13} = \frac{1}{\gamma} \)

With the help of the above equations and the equations (6.2.112), (6.2.133), (6.2.150), (6.2.166), (6.2.182), (6.2.197), (6.2.212), (6.2.228), (6.2.244), (6.2.259), (6.2.274) and (6.2.276) we can have the expression for MTSF, availability and profit for this particular case.
For the graphical interpretation, the mentioned particular case is considered. The behaviour of MTSF and availability ($A_0$) with respect to failure rate ($\lambda$) for different values of repair rate ($\alpha_1$) is shown as in Fig. 6.3 and 6.4 respectively. It is clear from the graphs that the MTSF and availability decrease with increase in the values of failure rate. However, their values become higher for higher values of repair rate ($\alpha_1$).

Fig. 6.5 the behaviour of profit ($P_6$) with respect to cost replacement ($C_5$) for different values of probability ($p_2$) that the unit is not repairable. Following conclusions can be drawn:

(i) If $p_2 = 0.1$ then profit will become negative for a very high value of cost ($C_5$) and hence it can be concluded that if there are less chances of replacement, then the system is more profitable.

(ii) If $p_2 = 0.5$ then $P_6 > 0$ or $< 0$ according as $C_5 < \text{ or } = \text{ or } > 15990$ i.e. the system is profitable if $C_5 \leq 15990$.

(iii) If $p_2 = 0.9$ then $P_6 > 0$ or $< 0$ according as $C_5 < \text{ or } = \text{ or } > 9090$ i.e. the system is profitable if $C_5 \leq 9090$.

Fig. 6.6 shows the behaviour profit ($P_6$) with respect to cost per random inspection ($C_{10}$) for different values of inspection rate ($\beta$). Following conclusions are drawn:

(i) If $\beta = 1$ then $P_6 > 0$ or $< 0$ according as $C_{10} < \text{ or } = \text{ or } > 1660$ i.e. the system is profitable if $C_{10} \leq 1660$.

(ii) If $\beta = 6$ then $P_6 > 0$ or $< 0$ according as $C_{10} < \text{ or } = \text{ or } > 1283$ i.e. the system is profitable if $C_{10} \leq 1283$.

(iii) If $\beta = 11$ then $P_6 > 0$ or $< 0$ according as $C_{10} < \text{ or } = \text{ or } > 1252$ i.e. the system is profitable if $C_{10} \leq 1252$. 
GRAPHICAL INTERPRETATION FOR MODEL 6.2

For the graphical interpretation, the mentioned particular case is considered. The behaviour of MTSF and availability \( (A_0) \) with respect to failure rate \( (\lambda) \) for different values of repair rate \( (\alpha_1) \) is shown as in Fig. 6.7 and 6.8 respectively. It is clear from the graphs that the MTSF and availability decrease with increase in the values of failure rate. However, their values become higher for higher values of repair rate \( (\alpha_1) \).

Fig. 6.9 shows the behaviour of profit \( (P_{62}) \) with respect to cost per replacement \( (C_5) \) for different values of probability \( (p_2) \). Following conclusions are drawn:

(i) If \( p_2 = 0.1 \) then profit will become negative for a very high value of cost \( (C_5) \) and hence it can be concluded that if there are less chances of replacement, then the system is more profitable.

(ii) If \( p_2 = 0.5 \) then \( P_{62} > \) or = or \( < 0 \) according as \( C_5 < \) or = or \( > 16695 \) i.e. the system is profitable if \( C_5 \leq 16695 \).

(iii) If \( p_2 = 0.9 \) then \( P_{62} > \) or = or \( < 0 \) according as \( C_5 < \) or = or \( > 12099.5 \) i.e. the system is profitable if \( C_5 \leq 12099.5 \).

Fig. 6.10 shows the behaviour of profit \( (P_{62}) \) with respect to cost per random inspection \( (C_{10}) \) for different values of inspection rate \( (\beta) \). Following conclusion are drawn:

(i) If \( \beta = 1 \) then \( P_{62} > \) or = or \( < 0 \) according as \( C_{10} < \) or = or \( > 1674 \) i.e. the system is profitable if \( C_{10} \leq 1674 \)

(ii) If \( \beta = 6 \) then \( P_{62} > \) or = or \( < 0 \) according as \( C_{10} < \) or = or \( > 1290 \) i.e. the system is profitable if \( C_{10} \leq 1290 \)

(iii) If \( \beta = 11 \), then \( P_{62} < \) or = or \( > 0 \) according as \( C_{10} < \) or = or \( > 1260 \) i.e. the system is profitable if \( C_{10} \leq 1260 \).
MTSF versus Failure Rate ($\lambda$) for different values of repair rate ($\alpha$).
Figure 6.4

Availability versus failure rate (λ) for different values of repair rate (α).
PROFIT VERSUS COST \((C_5)\) FOR DIFFERENT VALUES OF PROBABILITY \((p_2)\)

\[
\lambda = 0.5, \; \alpha_1 = 2, \; \gamma = 5, \; \gamma_1 = 8, \; \beta = 10, \; C_0 = 1000, \; C_1 = 500, \; C_2 = 40, \; C_3 = 30, \; C_4 = 25, \; C_5 = 500
\]

Fig 6.5
Fig. 6.6

PROFIT VERSUS COST (C_n) FOR DIFFERENT VALUES OF RATE OF RANDOM INSPECTION (β)

λ = 0.5, α = 2, y = 5, y' = 8, C_0 = 1000, C_1 = 50, C_2 = 40.
C_3 = 30, C_4 = 25, C_5 = 100, p_1 = 0.5, p_2 = 0.5,

Cost (C_n)
Failure Rate ($X$) versus MTSF for different values of repair rate ($\alpha$).
AVAILABILITY VERSUS FAILURE RATE ($\lambda$) FOR DIFFERENT VALUES OF
REPAIR RATE ($\alpha$)

\[ \text{Failure Rate} \]

Fig. 6.8 (a)

\[ \text{Repair Rate ($\alpha$)} \]
PROFIT VERSUS COST (C₅) FOR DIFFERENT VALUES OF PROBABILITY (p₂)

Fig. 6.9

Profit

Cost (C₅)

λ = 1, α = 0.5, q₁ = 2, y₂ = 5. y₁ = 8, C₀ = 1000. C₅ = 50
C₂ = 40, C₃ = 30, C₄ = 25, C₅ = 75, C₆ = 50, C₇ = 40, C₈ = 500
Figure 6.10

Profit versus cost (C10) for different values of rate of random inspection (P).

Graph showing the relationship between profit and cost for different values of P, with specific values and constants noted on the graph.

Cost (C10)

Profit

Rate of Random Inspection (P)

Values used in the graph:
- C2 = 40, C3 = 30, C4 = 25, C5 = 100, P1 = 10, P5 = 0.5
- P1 = 0.5, n = 2, C1 = 2.5, l = 8, C6 = 1000, C7 = 50, C8 = 40.