CHAPTER 5

ANALYSIS OF A TWO-UNIT HOT STANDBY SYSTEM WITH OBSERVABLE FAILURES
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In the previous chapter, it has been assumed that the failure of a unit is not revealed until both the units get failed. However, there may be situations when systems are under constant observation and the failure of a unit is detected as soon as it fails.

Keeping this in view, we, in this chapter, analyse a two-unit hot standby system. It is assumed that failures are observable i.e. failure of a unit is detected as soon as it fails. On the failure of a unit, it goes under inspection which is carried out by an ordinary repairman to detect whether the unit is repairable or not. Two reliability models have been studied. In the first model, it is assumed that when the ordinary repairman declares that unit is not repairable, it is replaced with a new one. However, in the second model, on the declaration made by the ordinary repairman for a unit saying that it is not repairable, expert opinion is taken to confirm this and the unit is accordingly repaired or replaced. Other assumptions are same as taken in the previous chapter.

Following measures of the system effectiveness are obtained by making use of semi-Markov processes and regenerative point technique.

- Mean time to system failure (MTSF)
- Steady-state availability of the system
- Expected busy period per unit time (for repair only) by ordinary/expert repairman.
• Expected busy period per unit time (for inspection only) by ordinary/expert repairman.
• Expected busy period per unit time (for replacement only) by ordinary repairman.
• Expected number of visits per unit time by ordinary/expert repairman.
• Expected number of replacement per unit time.
• Expected profit incurred to the system

Study through graphs is also made.

NOTATIONS

\( \lambda \) : constant failure rate of operative unit
\( \alpha \) : constant failure rate of hot standby unit
\( p_1 \) : probability that unit is repairable
\( p_2 \) : probability that unit is irreparable

\( h_1(t), H_1(t) \): p.d.f. and c.d.f. of inspection time of ordinary repairman
\( h_2(t), H_2(t) \): p.d.f. and c.d.f. of replacement time of ordinary repairman
\( g(t), G(t) \): p.d.f. and c.d.f. of time to repair of ordinary repairman
\( h_3(t), H_3(t) \): p.d.f. and c.d.f. of time to inspection of expert repairman
\( g_3(t), G_3(t) \): p.d.f. and c.d.f. of time to repair of expert repairman

Symbols for the states of the system are

\( o \) : operative unit
\( hs \) : hot standby unit
\( Fui \) : failed unit under inspection of ordinary repairman
\( Fur \) : failed unit under repair of ordinary repairman
\( Rep \) : failed unit under replacement of ordinary repairman
\( Fuie \) : failed unit under inspection of expert repairman
\( Fre \) : failed unit under repair of expert repairman
wi failed unit waiting for repair
F_{UR} repair of failed unit is continuing by the ordinary
repairman from the previous state.
F_{UI}/F_{UIc} inspection of the failed unit is continuing by the ordinary/
expert repairman from the previous state
F_{Re} repair of the failed unit is continuing by the expert
repairman from the previous state.

MODEL 5.1

Here in this model the unit is replaced if the ordinary repairman
declares that unit is not repairable state transition diagram is shown as
in Fig. 5.1.

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The epochs of entry into states 0, 1, 2, 3, 7 and 8 are
regeneration points and thus 0, 1, 2, 3, 7 and 8 are regenerative states.
States 4, 5, 6, 7 and 8 are failed states. The transition probabilities are
given by:

\[ q_{01}(t) = (\lambda + \alpha) e^{-(\lambda + \alpha)t}; \]
\[ q_{13}(t) = p_2 h_1(t) e^{-\lambda_1}; \]
\[ q_{17}(t) = p_1 \left[\lambda e^{-\lambda_1} \mathbb{C} 1\right] h_1(t); \]
\[ q_{20}(t) = e^{-\lambda_1} g(t); \]
\[ q_{25}(t) = \lambda e^{-\lambda_1} \mathbb{G}(t); \]
\[ q_{30}(t) = e^{-\lambda_1} h_2(t); \]
\[ q_{36}(t) = \lambda e^{-\lambda_1} \mathbb{H}_2(t); \]

\[ q_{12}(t) = p_1 h_1(t) e^{\lambda_1}; \]
\[ q_{14}(t) = \lambda e^{-\lambda_1} \bar{h}_1(t); \]
\[ q_{18}(t) = p_2 \left[\lambda e^{-\lambda_1} \mathbb{C} 1\right] h_1(t); \]
\[ q_{21}(t) = \lambda e^{-\lambda_1} \mathbb{G}(t); \]
\[ q_{31}(t) = \left[\lambda e^{-\lambda_1} \mathbb{C} 1\right] h_2(t); \]
\[ q_{71}(t) = g(t); \]
\[ q_{81}(t) = h_2(t); \]

(5.1.1-5.1.14)
Fig. 5.1
The non-zero elements $p_{ij}$ are given as follows:

$$
\begin{align*}
p_{01} &= 1; & p_{12} &= p_{1}h_{1}(\lambda); & p_{13} &= p_{2}h_{1}(\lambda); & p_{14} &= 1-h_{1}(\lambda) \\
p_{17}^{(4)} &= p_{1}[1-h_{1}(\lambda)]; & p_{18}^{(4)} &= p_{2}(1-h_{1}(\lambda)); & p_{20} &= g^{*}(\lambda); \\
p_{21}^{(5)} &= 1-g^{*}(\lambda); & p_{25} &= 1-g^{*}(\lambda) \\
p_{30} &= h_{2}(\lambda); & p_{36} &= 1-h_{2}(\lambda); & p_{31}^{(6)} &= 1-h_{2}(t) \\
p_{71} &= 1, & p_{81} &= 1
\end{align*}
$$

By these transition probabilities, it can be verified that

$$
\begin{align*}
p_{01} &= 1, & p_{12} + p_{13} + p_{14} &= 1, & p_{12} + p_{13} + p_{18}^{(4)} + p_{17}^{(4)} &= 1 \\
p_{20} + p_{25} &= 1, & p_{20} + p_{21}^{(5)} &= 1, & p_{30} + p_{36} &= 1, & p_{30} + p_{31}^{(6)} &= 1 \\
p_{71} &= 1, & p_{81} &= 1
\end{align*}
$$

The mean sojourn times ($\mu_{i}$) in state ‘$i$’ are:

$$
\begin{align*}
\mu_{0} &= \frac{1}{\lambda + \alpha} , & \mu_{1} &= \frac{1}{\lambda} (1-h_{1}(\lambda)), \\
\mu_{3} &= \left[1-h_{2}(\lambda)\right] \frac{1}{\lambda} , & \mu_{2} &= \frac{1-g^{*}(\lambda)}{\lambda} \\
\mu_{7} &= \int_{0}^{\infty} G(t) \, dt = \int_{0}^{\infty} t \, g(t) \, dt \\
\mu_{8} &= \int_{0}^{\infty} t \, h_{2}(t) \, dt
\end{align*}
$$

The unconditional mean time taken by the system to transit for any state ‘$j$’ when it is counted from epoch of entrance into state ‘$i$’ is mathematically stated as:

$$
m_{ij} = \int_{0}^{\infty} t q_{ij}(t) \, dt = -q_{ij}^{*}(0)
$$

Thus,

$$
\begin{align*}
m_{01} &= \mu_{0} \\
m_{12} + m_{13} + m_{14} &= \mu_{1} \\
m_{12} + m_{13} + m_{17}^{(4)} + m_{18}^{(4)} &= \int_{0}^{\infty} h_{1}(t) \, dt = k_{1} \text{ (say)}
\end{align*}
$$
\[ m_{20} + m_{25} = \mu_2 \quad ; \quad m_{20} + m_{21}^{(5)} = \mu_7 \]
\[ m_{30} + m_{36} = \mu_3 \quad ; \quad m_{71} = \mu_7 \quad ; \quad m_{81} = \mu_8 \]

(5.1.45-5.1.52)

**MEAN TIME TO SYSTEM FAILURE**

By probabilistic arguments, we obtain the following recursive relations for \( \phi_j(t) \):

\[
\begin{align*}
\phi_0(t) &= Q_{01}(t) \otimes \phi_1(t) \\
\phi_1(t) &= Q_{12}(t) \otimes \phi_2(t) + Q_{13}(t) \otimes \phi_3(t) + Q_{14}(t) \\
\phi_2(t) &= Q_{20}(t) \otimes \phi_0(t) + Q_{25}(t) \\
\phi_3(t) &= Q_{30}(t) \otimes \phi_0(t) + Q_{36}(t)
\end{align*}
\]

Taking (L.S.T.) of these relations and solving them for \( \phi_0^{**}(s) \), we obtain

\[
\phi_0^{**}(s) = \frac{N(s)}{D(s)}
\]

(5.1.57)

where

\[
N(s) = Q_{01}^{**}(s) [Q_{12}^{**}(s) Q_{25}^{**}(s) + Q_{14}^{**}(s) + Q_{13}^{**}(s) Q_{30}^{**}(s)]
\]

\[
D(s) = 1 - Q_{01}^{**}(s) [Q_{12}^{**}(s) Q_{20}^{**}(s) + Q_{13}^{**}(s) Q_{30}^{**}(s)]
\]

(5.1.58-5.1.59)

Now, the mean time to system failure (MTSF) when the system starts from the state ‘0’ is

\[
T_0 = \lim_{s \to 0} \frac{1 - \phi^{**}(s)}{s}
\]

(5.1.60)

Using L’Hospital rule and putting the value of \( \phi_0^{**}(s) \) from equation (5.1.57), we have

\[
T_0 = \frac{N}{D}
\]

where

\[
N = \mu_0 + \mu_1 + p_{12}\mu_2 + p_{13}\mu_3
\]

\[
D = 1 - (p_{12}p_{20} + p_{13}p_{30})
\]

(5.1.61-5.1.63)
AVAILABILITY ANALYSIS

Using the arguments of the theory of regenerative processes, the availability $A_i(t)$ is seen to satisfy the following recursive relations:

\[
A_0(t) = M_0(t) + q_{01}(t) \odot A_1(t)
\]

\[
A_1(t) = M_1(t) + q_{12}(t) \odot A_2(t) + q_{13}(t) \odot A_3(t) + q_{18}^{(4)}(t) \odot A_8(t)
\]

\[+ q_{17}^{(4)}(t) \odot A_7(t)\]

\[
A_2(t) = M_2(t) + q_{20}(t) \odot A_0(t) + q_{21}^{(5)}(t) \odot A_1(t)
\]

\[
A_3(t) = M_3(t) + q_{30}(t) \odot A_0(t) + q_{31}^{(6)}(t) \odot A_1(t)
\]

\[
A_7(t) = q_{71}(t) \odot A_1(t)
\]

\[
A_8(t) = q_{81}(t) \odot A_1(t)
\]

\[
(5.1.64-5.1.69)
\]

where

\[
M_0(t) = \exp(-\lambda + \alpha t), \quad M_1(t) = \exp(-\lambda t) \bar{H}_1(t),
\]

\[
M_2(t) = \exp(-\lambda t) \bar{G}(t), \quad M_3(t) = \exp(-\lambda t) \bar{H}_2(t)
\]

\[
(5.1.70-5.1.73)
\]

Taking L.T. of the above equations and solving them for $A_0^*(s)$ we get

\[
A_0^*(s) = \frac{N_1(s)}{D_1(s)}
\]

\[
(5.1.74)
\]

where

\[
N_1(s) = M_0^*(s) \left[1 - q_{12}^*(s) q_{21}^{(5)}(s) - q_{13}^*(s) q_{31}^{(8)}(s) - q_{17}^{(4)}(s) q_{71}^*(s) - q_{18}^{(4)}(s) q_{81}^*(s) + q_{01}^*(s) [M_1^*(s) + q_{12}^*(s) M_2^*(s) + q_{13}^*(s) M_3^*(s)]\right]
\]

\[
D_1(s) = 1 - q_{12}^*(s) q_{21}^{(5)}(s) - q_{13}^*(s) q_{31}^{(6)}(s) - q_{17}^{(4)}(s) q_{71}^*(s) - q_{18}^{(4)}(s) q_{81}^*(s) - q_{01}^*(s) [q_{12}^*(s) q_{20}^*(s) + q_{13}^*(s) q_{30}^*(s)]
\]

\[
(5.1.75-5.1.76)
\]

In steady-state, the availability of the system is given by

\[
A_0 = \lim_{s \to 0} \{s A_0^*(s)\} = \frac{N_1}{D_1}
\]

\[
(5.1.77)
\]

where
BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN
(Repair Time Only)

By probabilistic arguments, we have the following recursive relations for $B_i(t)$:

$B_0(t) = q_{00}(t) \circ B_1(t)$

$B_1(t) = q_{12}(t) \circ B_2(t) + q_{13}(t) \circ B_3(t) + q_{18}(t) \circ B_8(t)$

$+ q_{17}(t) \circ B_7(t)$

$B_2(t) = W_2(t) + q_{20}(t) \circ B_0(t) + q_{21}(t) \circ B_1(t)$

$B_3(t) = q_{30}(t) \circ B_0(t) + q_{31}(t) \circ B_1(t)$

$B_7(t) = W_7(t) + q_{71}(t) \circ B_1(t)$

$B_8(t) = q_{81}(t) \circ B_1(t)$

where

$W_2(t) = W_7(t) = \tilde{G}(t)$

Taking L.T. of the above equations and solving them for $B_0^*(s)$, we get

$B_0^*(s) = \frac{N_2(s)}{D_1(s)}$  \hspace{1cm} (5.1.87)

where

$N_2(s) = q_{01}(s)[q_{12}(s) W_2^*(s) + q_{17}(t) W_7^*(s)]$  \hspace{1cm} (5.1.88)

and $D_1(s)$ is already specified.

In steady-state, the total fraction of time for which the system is under repair of ordinary repairman is given by

$B_0 = \lim_{s \to 0} \{ s B_0^*(s) \} = \frac{N_2}{D_1}$  \hspace{1cm} (5.1.89)
where
\[ N_2 = \{p_{12} + p_{17}\mu_7\} \]  \hspace{1cm} (5.1.90)
and \( D_1 \) is already specified.

**BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN**

*(Inspection Time Only)*

By probabilistic arguments, we have the following recursive relations:

\[
BI_0(t) = q_{01}(t) \odot BI_1(t)
\]

\[
BI_1(t) = W_1(t) + q_{12}(t) \odot BI_2(t) + q_{13}(t) \odot BI_3(t) + q_{18}^{(4)}(t) \odot BI_8(t)
\]

\[ + q_{17}^{(4)}(t) \odot BI_7(t) \]

\[
BI_2(t) = q_{20}(t) \odot BI_0(t) + q_{21}^{(5)}(t) \odot BI_1(t)
\]

\[
BI_3(t) = q_{30}(t) \odot BI_0(t) + q_{31}^{(6)}(t) \odot BI_1(t)
\]

\[
BI_7(t) = q_{71}(t) \odot BI_1(t)
\]

\[
BI_8(t) = q_{81}(t) \odot BI_1(t)
\]

where
\[
W_1(t) = H_1(t) \]  \hspace{1cm} (5.1.90-5.1.97)

Taking L.T. of the above equations and solving them for \( BI_0^*(s) \), we get

\[
BI_0^*(s) = \frac{N_3(s)}{D_1(s)} \]  \hspace{1cm} (5.1.98)

where
\[
N_3(s) = q_{01}^*(s)W_1^*(s) \]  \hspace{1cm} (5.1.99)

and \( D_1(s) \) is already specified.

In steady-state, the total fraction of the time for which the system is under inspection of ordinary repairman is given by

\[
BI_0 = \lim_{s \to 0} \{s BI_0^*(s)\} = \frac{N_3}{D_1} \]  \hspace{1cm} (5.1.100)

where
\[ N_3 = \mu_1 \]  \hspace{1cm} (5.1.101)

And \( D_1 \) is already specified.

**BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN (Replacement Time Only)**

By probabilistic arguments, we have the following recursive relations:

\[
BR_0(t) = q_{01}(t) \odot BR_1(t)
\]

\[
BR_1(t) = q_{12}(t) \odot BR_2(t) + q_{13}(t) \odot BR_3(t) + q_{18}(t) \odot BR_8(t)
\]

\[
\quad + q_{17}(t) \odot BR_7(t)
\]

\[
BR_2(t) = q_{20}(t) \odot BR_0(t) + q_{21}(t) \odot BR_1(t)
\]

\[
BR_3(t) = W_3(t) + q_{30}(t) \odot BR_0(t) + q_{31}(t) \odot BR_1(t)
\]

\[
BR_7(t) = q_{71}(t) \odot BR_1(t)
\]

\[
BR_8(t) = W_8(t) + q_{81}(t) \odot BR_1(t)
\]

where

\[
W_3(t) = W_8(t) = H_2(t) \hspace{1cm} (5.1.102-5.1.108)
\]

Taking L.T. of the above equations and solving them for \( BR_0*(s) \), we get

\[
BR_0*(s) = N_4(s)/D_1(s) \hspace{1cm} (5.1.109)
\]

where

\[
N_4(s) = q_{01}*(s)[q_{13}*(s)W_3*(s) + q_{18}*(t)W_8*(s)]
\]

and \( D_1(s) \) is already specified.

In steady-state, the total fraction of the time for which the system is under replacement of ordinary repairman is given by

\[
BR_0 = \lim_{s \to 0} \{s \cdot BR_0*(s)\} = \frac{N_4}{D_1} \hspace{1cm} (5.1.111)
\]

where

\[
N_4 = (p_{13} + p_{18}^{(4)})\mu_8 \hspace{1cm} (5.1.112)
\]
and $D_1$ is already specified.

**EXPECTED NUMBER OF VISITS BY THE REPAIRMAN**

By probabilistic arguments, we have the following recursive relations:

\[ V(t) = Q_{i}(t) \odot V(t) + Q_{i+1}(t) \odot V(t+1) + \cdots \]

where $D$ is already specified.

Taking L.S.T. of the above equations and solving them for $V(t)$, we get

\[ V(t) = \frac{N_5(s)}{D_1(s)} \]

where

\[ N_5 = 1 - P_{12}P_{21} - P_{13}P_{31} - P_{17} - P_{18} \]

and $D_1(s)$ is specified earlier.

In steady-state, the expected number of visits of the repairman on the system is given by

\[ V_0 = \lim_{s \to 0} \{ N_5 \} = \frac{N_5}{D_1} \]

where

\[ N_5 = 1 - P_{12}P_{21} - P_{13}P_{31} - P_{17} - P_{18} \]

and $D_1$ is already specified.

**EXPECTED NUMBER OF REPLACEMENTS**

By probabilistic arguments, we have the following recursive relations:

\[ R(t) = Q_{i}(t) \odot R(t) \]

where $D$ is already specified.
\[ R_2(t) = Q_{20}(t) \mathcal{S} R_0(t) + Q_{21}(t) \mathcal{S} R_1(t) \]
\[ R_3(t) = Q_{30}(t) \mathcal{S} R_0(t) + Q_{31}(t) \mathcal{S} R_1(t) \]
\[ R_7(t) = Q_{71}(t) \mathcal{S} R_1(t) \]
\[ R_8(t) = Q_{81}(t) \mathcal{S} R_1(t) \]

Taking L.S.T. of the above equations and solving them for \( R_0^{**}(s) \), we get

\[ R_0^{**}(s) = \frac{N_6(s)}{D_1(s)} \] \hspace{1cm} (5.1.129)

where

\[ N_6(s) = Q_{01}^{**}(s)[Q_{13}^{**}(s) + Q_{18}^{(4)}(s)] \] \hspace{1cm} (5.1.130)

and \( D_1 \) is already specified.

In steady-state, the expected number of replacements of the failed units by the repairman is given by

\[ R_0 = \lim_{s \to 0} \{ s R_0^{**}(s) \} = \frac{N_6}{D_1} \] \hspace{1cm} (5.1.131)

where

\[ N_6 = p_{13} + p_{18}^{(4)} \] \hspace{1cm} (5.1.132)

and \( D_1 \) is already specified.

**PROFIT ANALYSIS**

The expected total profit incurred to the system in steady-state is given by

\[ P_{51} = C_0 A_0 - C_1 B_0 - C_2 B_1 - C_3 B R_0 - C_4 V_0 - C_5 R_0 \] \hspace{1cm} (5.1.133)

where

\( C_0 = \) revenue per unit up time of the system
\( C_1 = \) cost per unit time for which the repairman is busy under repair
\( C_2 = \) cost per unit time for which the repairman is busy under inspection
\[ + Q_{17}^{(4)}(t) \cdot R_7(t) \]
\[ R_2(t) = Q_{20}(t) \circ R_0(t) + Q_{21}^{(5)}(t) \circ R_1(t) \]
\[ R_3(t) = Q_{30}(t) \circ R_0(t) + Q_{31}^{(6)}(t) \circ R_1(t) \]
\[ R_7(t) = Q_{71}(t) \circ R_1(t) \]
\[ R_8(t) = Q_{81}(t) \circ R_1(t) \]
\[(5.1.123-5.1.128)\]

Taking L.S.T. of the above equations and solving them for \( R_{0**}(s) \), we get
\[ R_{0**}(s) = \frac{N_6(s)}{D_1(s)} \]
\[(5.1.129)\]

where
\[ N_6(s) = Q_{01**}(s)\{Q_{13**}(s) + Q_{18}^{(4)**}(s)\} \]
\[(5.1.130)\]

and \( D_1 \) is already specified.

In steady-state, the expected number of replacements of the failed units by the repairman is given by
\[ R_0 = \lim_{s \to 0} \{s \cdot R_{0**}(s)\} = \frac{N_6}{D_1} \]
\[(5.1.131)\]

where
\[ N_6 = p_{13} + p_{18}^{(4)} \]
\[(5.1.132)\]

and \( D_1 \) is already specified.

**PROFIT ANALYSIS**

The expected total profit incurred to the system in steady-state is given by
\[ P_{51} = C_0A_0 - C_1B_0 - C_2B_1 - C_3B_2R_0 - C_4V_0 - C_5R_0 \]
\[(5.1.133)\]

where

- \( C_0 \) = revenue per unit up time of the system
- \( C_1 \) = cost per unit time for which the repairman is busy under repair
- \( C_2 \) = cost per unit time for which the repairman is busy under inspection
\(C_3 = \text{cost per unit time for which the repairman is busy under replacement}\)

\(C_4 = \text{cost per visit of ordinary repairman}\)

\(C_5 = \text{cost per unit time for replacement}\)

**PARTICULAR CASE**

For graphical interpretation the following particular case is considered:

\[g(t) = \alpha_1 e^{-\alpha_1 t}; \quad h_1(t) = \gamma_1 e^{-\gamma_1 t}; \quad h_2(t) = \gamma e^{-\gamma t}\]

\[p_{01} = 1; \quad p_{12} = \frac{p_1 \gamma_1}{\gamma_1 + \lambda}; \quad p_{13} = \frac{p_2 \gamma_1}{\gamma_1 + \lambda}\]

\[p_{14} = \frac{\lambda}{\gamma_1 + \lambda}; \quad p_{17}^{(4)} = \frac{p_1 \lambda}{\gamma_1 + \lambda}; \quad p_{18}^{(4)} = \frac{p_2 \lambda}{\gamma_1 + \lambda}\]

\[p_{20} = \frac{\alpha_1}{\alpha_1 + \lambda}; \quad p_{21}^{(5)} = \frac{\lambda}{\alpha_1 + \lambda}; \quad p_{25} = \frac{\lambda}{\alpha_1 + \lambda}\]

\[p_{30} = \frac{\gamma}{\gamma + \lambda}; \quad p_{31}^{(6)} = \frac{\lambda}{\gamma + \lambda}; \quad p_{36} = \frac{\lambda}{\gamma + \lambda}\]

\[p_{71} = 1; \quad p_{81} = 1\]

\[\mu_0 = \frac{1}{\lambda + \alpha}; \quad \mu_1 = \frac{1}{\gamma_1 + \lambda}; \quad \mu_2 = \frac{1}{\alpha_1 + \lambda}\]

\[\mu_3 = \frac{1}{\gamma + \lambda}; \quad \mu_7 = \frac{1}{\alpha_1}; \quad \mu_8 = \frac{1}{\gamma}\]

\[K_1 = \frac{1}{\gamma_1}\]  \hspace{1cm} (5.1.134-5.1.157)

Using the above equations and the equations (5.1.61), (5.1.77), (5.1.89), (5.1.100), (5.1.111), (5.1.121), (5.1.131) and (5.1.133) we can have the expression for MTSF, availability and profit of the particular system.
MODEL 5.2

In this model expert opinion is taken if ordinary repairman declares the unit is irreparable. State transition diagram is shown as in Fig. 5.2.

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The epochs of entry into states 0, 1, 2, 3, 6, 7, 8, 9, 12 and 13 are regeneration points and thus there are regenerative states 4, 5, 8, 9, 10, 11, 12, 13 and 14 are failed states.

The transition probabilities are given by:

\[
q_{01}(t) = (\lambda + \alpha) e^{-\lambda t}, \quad q_{12}(t) = p_1 e^{-\lambda t} h_1(t) \\
q_{13}(t) = p_2 h_1 e^{-\lambda t}, \quad q_{14}(t) = \lambda e^{-\lambda t} \bar{H}_1(t) \\
q_{18}^{(4)}(t) = p_1 [\lambda e^{-\lambda t} \odot 1] h_1(t) = p_2 (1 - e^{-\lambda t}) h_1(t) \quad q_{19}^{(4)}(t) = p_2 [\lambda e^{-\lambda t} \odot 1] h_1(t) = p_2 (1 - e^{-\lambda t}) h_1(t) \\
q_{20}(t) = e^{-\lambda t} g(t), \quad q_{23}(t) = \lambda e^{-\lambda t} \bar{G}(t) \\
q_{21}^{(5)}(t) = [\lambda e^{-\lambda t} \odot 1] g(t) = [1 - e^{-\lambda t}] g(t) \quad q_{36}(t) = p_1 e^{-\lambda t} \bar{H}_c(t) \\
q_{37}(t) = p_2 e^{-\lambda t} h_c(t), \quad q_{3,14}(t) = \lambda e^{-\lambda t} \bar{H}_c(t) \\
q_{3,12}^{(14)}(t) = [\lambda e^{-\lambda t} \odot 1] p_1 h_c(t) = p_1 (1 - e^{-\lambda t}) h_c(t) \\
q_{3,13}^{(14)}(t) = [\lambda e^{-\lambda t} \odot 1] p_2 h_c(t) = p_2 (1 - e^{-\lambda t}) h_c(t) \\
q_{60}(t) = e^{-\lambda t} g_c(t), \quad q_{61} = [\lambda e^{-\lambda t} \odot 1] g_c(t), \quad q_{6,10} = \lambda e^{-\lambda t} \bar{G}_c(t) \\
q_{70}(t) = e^{-\lambda t} h_2(t), \quad q_{7,11}^{(11)}(t) = [\lambda e^{-\lambda t} \odot 1] \bar{H}_2(t) \\
q_{7,11}(t) = \lambda e^{-\lambda t} \bar{H}_2(t), \quad q_{81}(t) = g(t), \quad q_{9,12}(t) = p_1 h_c(t) \\
q_{9,13}(t) = p_2 h_c(t), \quad q_{12,1}(t) = g_c(t), \quad q_{13,1}(t) = h_2(t) \\
(5.2.1-5.2.25)

The non-zero elements \( p_{ij} \) are given by

\[
p_{01} = 1 \quad ; \quad p_{12} = p_1 h_1 \ast (\lambda) \quad ; \quad p_{13} = p_2 h_1 \ast (\lambda) \\
p_{14} = 1 - h_1 \ast (\lambda) \quad ; \quad p_{18}^{(4)} = p_1 (1 - h_1 \ast (\lambda))
\]
Up state
Failed state
Regeneration point

Fig. 5.2
\[ p_{19} = p_2 (1 - h_1^*(\lambda)) \quad ; \quad p_{20} = g^*(\lambda) \quad ; \quad p_{25} = 1 - g^*(\lambda) \]

\[ p_{21} = 1 - g^*(\lambda) \quad ; \quad p_{36} = p_1 h_c^*(\lambda) \quad ; \quad p_{37} = p_2 h_c^*(\lambda) \]

\[ p_{3,14} = 1 - h_c^*(\lambda) \quad ; \quad p_{3,12} = p_1 (1 - h_c^*(\lambda)) \]

\[ p_{3,13} = (1 - h_c^*(\lambda)) p_2 \quad ; \quad p_{60} = g_c^*(\lambda) \quad ; \quad p_{6,10} = 1 - g_c^*(\lambda) \]

\[ p_{6,1} = 1 - g_c^*(\lambda) \quad ; \quad p_{7,0} = h_2^*(\lambda) \quad ; \quad p_{7,1} = 1 - h_2^*(\lambda) \]

\[ p_{7,11} = 1 - h_2^*(\lambda) \quad ; \quad p_{8,1} = 1 \quad ; \quad p_{9,12} = p_1 \quad ; \quad p_{9,13} = p_2 \]

\[ p_{12,1} = 1 \quad ; \quad p_{13,1} = 1 \quad (5.2.26-5.2.50) \]

By these transition probabilities, it can be verified that

\[ p_{01} = 1 \quad ; \quad p_{12} + p_{13} + p_{14} = 1 \quad ; \quad p_{12} + p_{13} + p_{18} + p_{19} = 1 \]

\[ p_{20} + p_{25} = 1 \quad ; \quad p_{20} + p_{21} = 1 \quad ; \quad p_{36} + p_{37} + p_{3,14} = 1 \]

\[ p_{60} + p_{6,10} = 1 \quad ; \quad p_{6,10} + p_{6,1} = 1 \quad ; \quad p_{7,0} + p_{7,11} = 1 \quad ; \quad p_{7,0} + p_{7,1} = 1 \]

\[ p_{8,1} = 1 \quad ; \quad p_{9,12} + p_{9,13} = 1 \quad ; \quad p_{12,1} = 1 \quad (5.2.51-5.2.65) \]

The mean sojourn times \((\mu_i)\) are given as

\[ \mu_0 = \frac{1}{\lambda + \alpha} \quad ; \quad \mu_1 = \frac{1}{\lambda} (1 - h_1^*(\lambda)) \quad ; \quad \mu_2 = \frac{1}{\lambda} (1 - g^*(\lambda)) \]

\[ \mu_3 = \frac{1}{\lambda} (1 - h_c^*(\lambda)) \quad ; \quad \mu_6 = \frac{1}{\lambda} (1 - g_c^*(\lambda)) \quad ; \quad \mu_7 = \frac{1}{\lambda} (1 - h_2^*(\lambda)) \]

\[ \mu_8 = \int_0^\tau \bar{h}(t) \, dt \quad ; \quad \mu_9 = \int_0^\tau \bar{H}_c(t) \, dt \quad ; \quad \mu_{12} = \int_0^\tau \bar{G}_c(t) \, dt \]

\[ \mu_{13} = \int_0^\tau \bar{H}_2(t) \, dt \quad (5.2.66-5.2.75) \]

The unconditional mean time taken by the system to transit for any state \(j\) when it is counted from the epoch of entrance into state \(i\) is mathematically stated as

\[ m_{ij} = \int_0^\tau t q_{ij}(t) \, dt = - q_{ij}^*(0) \quad (5.2.76) \]
Thus,

\[ m_{11} = \mu_0 \]

\[ m_{12} + m_{13} + m_{14} = \mu_1 \quad ; \quad m_{20} + m_{25} = \mu_2 \]

\[ m_{36} + m_{37} + m_{3.14} = \mu_3 \quad ; \quad m_{60} + m_{6.10} = \mu_6 \]

\[ m_{70} + m_{7.11} = \mu_7 \]

\[ m_{12} + m_{13} + m_{14}^{(4)}_{18} + m_{14}^{(4)}_{19} = \int_0^\infty H_1(t) = k_1 \text{ (say)} \]

\[ m_{20} + m_{21}^{(5)} = \mu_8 \quad ; \quad m_{36} + m_{37} + m_{3.12}^{(14)} + m_{3.13}^{(14)} = \mu_9 \]

\[ m_{60} + m_{6.10}^{(10)} = \mu_{12} \quad ; \quad m_{70} + m_{7.11}^{(11)} = \mu_{13} \]

\[ m_{81} = \mu_8 \quad ; \quad m_{9.12} + m_{9.13} = \mu_9 \]

\[ m_{12,1} = \mu_{12} \quad ; \quad m_{13.1} = \mu_{13} \quad (5.2.77-5.2.91) \]

**MEAN TIME TO SYSTEM FAILURE**

\( \phi_i(t) \) is seen to satisfy the following recursive relations:

\[ \phi_0(t) = Q_{01}(t) \bigg[ \phi_1(t) \bigg] \]

\[ \phi_1(t) = Q_{12}(t) \bigg[ \phi_2(t) + Q_{13}(t) \bigg] \bigg[ \phi_3(t) + Q_{14}(t) \bigg] \]

\[ \phi_2(t) = Q_{20}(t) \bigg[ \phi_0(t) + Q_{25}(t) \bigg] \]

\[ \phi_3(t) = Q_{36}(t) \bigg[ \phi_0(t) + Q_{37}(t) \bigg] \bigg[ \phi_7(t) + Q_{3.14}(t) \bigg] \]

\[ \phi_4(t) = Q_{60}(t) \bigg[ \phi_0(t) + Q_{6.10}(t) \bigg] \]

\[ \phi_7(t) = Q_{70}(t) \bigg[ \phi_0(t) + Q_{7.11}(t) \bigg] \quad (5.2.92-5.2.97) \]

Taking L.S.T. of the above relations and solving them for \( \phi_0^{**}(s) \), we get

\[ \phi_0^{**}(s) = N(s)/D(s) \quad (5.2.98) \]

where

\[ N(s) = Q_{01}^{**}(s)\{Q_{12}^{**}(s)Q_{25}^{**}(s) + Q_{14}^{**}(s) \]

\[ + Q_{13}^{**}(s)\{Q_{36}^{**}(s)Q_{6.10}^{**}(s) + Q_{37}^{**}(s)Q_{7.11}^{**}(s)\}\} \]

\[ D(s) = 1 - Q_{01}^{**}(s)\{Q_{12}^{**}(s)Q_{20}^{**}(s) + Q_{13}^{**}(s)\{Q_{36}^{**}(s)Q_{60}^{**}(s) \]

\[ + Q_{37}^{**}(s)Q_{70}^{**}(s)\}\} \quad (5.2.99-5.2.100) \]
Now, the mean time to system failure (MTSF) when the system starts from the state ‘0’ is

\[ T_0 = \lim_{s \to 0} \frac{1 - \phi^{**}(s)}{s} = \frac{N}{D} \quad (5.2.101) \]

where

\[ N = \mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6 + \mu_7 \]

\[ D = 1 - \left[ p_{12}p_{20} + p_{13}(p_{36}p_{60} + p_{37}p_{70}) \right] \quad (5.2.102-5.2.103) \]

**AVAILABILITY ANALYSIS**

The recursive relations for \( A_i(t) \) are as follows:

\[
\begin{align*}
A_0(t) &= M_0(t) + q_{01}(t) \circ A_1(t) \\
A_1(t) &= M_1(t) + q_{12}(t) \circ A_2(t) + q_{13}(t) \circ A_3(t) + q_{18}(4)(t) \circ A_8(t) \\
& \quad + q_{19}(4)(t) \circ A_9(t) \\
A_2(t) &= M_2(t) + q_{20}(t) \circ A_0(t) + q_{21}(5)(t) \circ A_1(t) \\
A_3(t) &= M_3(t) + q_{36}(t) \circ A_6(t) + q_{37}(t) \circ A_7(t) + q_{3,12}(14)(t) \circ A_{12}(t) \\
& \quad + q_{3,13}(14)(t) \circ A_{13}(t) \\
A_6(t) &= M_6(t) + q_{60}(t) \circ A_0(t) + q_{6,1}(10)(t) \circ A_1(t) \\
A_7(t) &= M_7(t) + q_{70}(t) \circ A_0(t) + q_{7,1}(11)(t) \circ A_1(t) \\
A_8(t) &= q_{81}(t) \circ A_1(t) \\
A_9(t) &= q_{9,12}(t) \circ A_{12}(t) + q_{9,13}(t) \circ A_{13}(t) \\
A_{12}(t) &= q_{12,1}(t) \circ A_1(t) \\
A_{13}(t) &= q_{13,1}(t) \circ A_1(t)
\end{align*}
\]

where

\[
\begin{align*}
M_0(t) &= e^{-\lambda t} + n(t) \\
M_1(t) &= e^{-\lambda t} \quad \text{\{since \} } M_1(t) = e^{-\lambda t} \bar{H}_1(t) \\
M_2(t) &= e^{\lambda t} \quad \text{\{since \}} H_2(t) \\
M_3(t) &= e^{\lambda t} \bar{H}_3(t) \\
M_6(t) &= e^{\lambda t} \bar{H}_6(t) \\
M_7(t) &= e^{\lambda t} \bar{H}_7(t)
\end{align*}
\]

(5.2.104-5.2.119)

Taking L. T. of the above equations and solving them for \( A_0^*(s) \), we get
where

\[ N_1(s) = M_{01}(s) \left[ 1 - q_{12}(s) q_{21}(s) q_{18}(s) q_{81}(s) - q_{19}(s) \right] \]

\[ A_0(s) = \frac{N_1(s)}{D_1(s)} \quad (5.2.120) \]

In steady-state, the availability of the system is given by

\[ A_0 = \frac{N_1}{D_1} \quad (5.2.123) \]

BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN
(Repair Time Only)

By probabilistic arguments, we have the following recursive relations for \( B_j(t) : \)

\[ B_0(t) = q_{01}(t) \otimes B_1(t) \]
\[ B_1(t) = q_{12}(t) \otimes B_2(t) + q_{13}(t) \otimes B_3(t) + q_{18}^{(4)}(t) \otimes B_8(t) + q_{19}^{(4)}(t) \otimes B_9(t) \]
\[ B_2(t) = W_2(t) + q_{20}(t) \otimes B_9(t) + q_{21}^{(5)}(t) \otimes B_1(t) \]
\[ B_3(t) = q_{36}(t) \otimes B_6(t) + q_{37}(t) \otimes B_7(t) + q_{3.12}^{(14)}(t) \otimes B_{12}(t) + q_{3.13}^{(14)}(t) \otimes B_{13}(t) \]
\[ B_6(t) = q_{60}(t) \otimes B_0(t) + q_{61}^{(10)}(t) \otimes B_1(t) \]
\[ B_7(t) = q_{70}(t) \otimes B_0(t) + q_{71}^{(11)}(t) \otimes B_1(t) \]
\[ B_8(t) = W_8(t) + q_{81}(t) \otimes B_1(t) \]
\[ B_9(t) = q_{9.12}(t) \otimes B_{12}(t) + q_{9.13}(t) \otimes B_{13}(t) \]
\[ B_{12}(t) = q_{12.1}(t) \otimes B_1(t) \]
\[ B_{13}(t) = q_{13.1}(t) \otimes B_1(t) \]

where
\[ W_2(t) = W_8(t) = \tilde{G}(t) \quad (5.2.126-5.2.136) \]

Taking L.T. of the above equations and solving them for \( B_0^*(s) \), we get
\[ B_0^*(s) = \frac{N_2(s)}{D_1(s)} \quad (5.2.137) \]

where
\[ N_2(s) = q_{01}^*(s)[q_{12}^*(s) W_2^*(s) + q_{18}^{(4)*}(s) W_8^*(s)] \quad (5.2.138) \]

and \( D_1(s) \) is already specified.

In steady-state, the total fraction of time for which the system is under repair of ordinary repairman is given by
\[ B_0 = \frac{N_2}{D_1} \quad (5.2.139) \]

where
\[ N_2 = [p_{12} + p_{18}^{(4)}] \mu_8 \quad (5.2.140) \]
BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN

By probabilistic arguments, we have the following recursive relations:

\[ B_{I0}(t) = q_{01}(t) \odot B_{I1}(t) \]
\[ B_{I1}(t) = W_1(t) + q_{12}(t) \odot B_{I2}(t) + q_{13}(t) B_{I3}(t) + q_{18}(t) \odot B_{I8}(t) + q_{19}(t) \odot B_{I9}(t) \]
\[ B_{I2}(t) = q_{20}(t) \odot B_{I0}(t) + q_{21}(t) \odot B_{I1}(t) \]
\[ B_{I3}(t) = q_{36}(t) \odot B_{I6}(t) + q_{37}(t) \odot B_{I7}(t) + q_{3,12}(t) \odot B_{I12}(t) + q_{3,13}(t) \odot B_{I13}(t) \]
\[ B_{I6}(t) = q_{60}(t) \odot B_{I0}(t) + q_{61}(t) \odot B_{I1}(t) \]
\[ B_{I7}(t) = q_{70}(t) \odot B_{I0}(t) + q_{7,11}(t) \odot B_{I1}(t) \]
\[ B_{I8}(t) = q_{81}(t) \odot B_{I1}(t) \]
\[ B_{I9}(t) = q_{9,12}(t) \odot B_{I12}(t) + q_{9,13}(t) \odot B_{I13}(t) \]
\[ B_{I12}(t) = q_{12,1}(t) \odot B_{I1}(t) \]
\[ B_{I13}(t) = q_{13,1}(t) \odot B_{I1}(t) \]

where

\[ W_1(t) = \int_0^t \bar{H}_1(t) \, dt = k_1 \quad (5.2.141-5.2.151) \]

Taking L.T. of the above equations and solving them for \( B_{I0}*(s) \), we get

\[ B_{I0}*(s) = \frac{N_3(s)}{D_1(s)} \quad (5.2.152) \]

where

\[ N_3(s) = q_{01}*(s) W_1*(s) \quad (5.2.153) \]

and \( D_1(s) \) is already specified.

In steady-state, the total fraction of the time for which the system is under inspection of ordinary repairman is given by

\[ B_{I0} = \frac{N_3}{D_1} \quad (5.2.154) \]
where \( N_3 = k_1 \)  

**BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN**  
(Replacement Time Only)

By probabilistic arguments, we have the following recursive relations for \( BR_0(t) \):

\[
BR_0(t) = q_{01}(t) \circ BR_1(t)
\]

\[
BR_1(t) = q_{12}(t) \circ BR_2(t) + q_{13}(t) \circ BR_3(t) + q_{18}^{(4)}(t) \circ BR_8(t) + q_{19}^{(4)}(t) \circ BR_9(t)
\]

\[
BR_2(t) = q_{20}(t) \circ BR_0(t) + q_{21}^{(5)}(t) \circ BR_1(t)
\]

\[
BR_3(t) = q_{36}(t) \circ BR_6(t) + q_{37}(t) \circ BR_7(t) + q_{3,12}^{(14)}(t) \circ BR_{12}(t) + q_{3,13}^{(14)}(t) \circ BR_{13}(t)
\]

\[
BR_6(t) = q_{60}(t) \circ BR_0(t) + q_{61}^{(10)}(t) \circ BR_1(t)
\]

\[
BR_7(t) = W_7(t) + q_{70}(t) \circ BR_0(t) + q_{71}^{(11)}(t) \circ BR_1(t)
\]

\[
BR_8(t) = q_{81}(t) \circ BR_1(t)
\]

\[
BR_9(t) = q_{9,12}(t) \circ BR_{12}(t) + q_{9,13}(t) \circ BR_{13}(t)
\]

\[
BR_{12}(t) = q_{12,1}(t) \circ BR_1(t)
\]

\[
BR_{13}(t) = W_{13}(t) + q_{13,1}(t) \circ BR_1(t)
\]

where

\[
W_7(t) = W_{13}(t) = \bar{H}_2(t)
\]

Taking L.T. of the above equations and solving them for \( BR_0^*(s) \), we get

\[
BR_0^*(s) = N_4(s)/D_1(s)
\]

where

\[
N_4(s) = W_7^*(s) q_{13}^*(s) q_{37}^*(s) + W_{13}^*(s) \left[ q_{13}^*(s) q_{3,13}^{(14)}(s) + q_{19}^{(4)}(s) q_{9,13}^*(s) \right]
\]

In steady-state, the total fraction of the time for which the system is under replacement of ordinary repairman is given by

\[
BR_0 = \frac{N_4}{D_1}
\]
where

\[ N_4 = [p_{13}(p_{37} + p_{3.13}^{(14)}) + p_{19}^{(4)} p_{9.13}] \mu_{13} \]  

(5.2.170)

EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

The following recursive relations for \( V_i(t) \) are obtained:

\[
V_0(t) = Q_0(t) + [1 + V_1(t)]
\]

\[
V_1(t) = Q_{12}(t) + V_2(t) + Q_{13}(t) + V_3(t) + Q_{18}^{(4)}(t) \sum V_8(t)
\]

\[
+ Q_{19}^{(4)}(t) \sum V_9(t)
\]

\[
V_2(t) = Q_{20}(t) + Q_{21}(t) + V_1(t)
\]

\[
V_3(t) = Q_{36}(t) + V_6(t) + Q_{37}(t) + V_7(t) + Q_{33}^{(14)}(t) \sum V_{12}(t)
\]

\[
+ Q_{33}^{(14)}(t) \sum V_{13}(t)
\]

\[
V_6(t) = Q_{60}(t) + Q_{61}(t) + V_0(t) + Q_{61}^{(10)}(t) [1 + V_1(t)]
\]

\[
V_7(t) = Q_{70}(t) + Q_{71}(t) + V_7(t) + Q_{71}(t) \sum V_7(t)
\]

\[
V_8(t) = Q_{81}(t) + V_8(t)
\]

\[
V_9(t) = Q_{9,12}(t) + Q_{9,13}(t) + Q_{9,13}(t) \sum V_{13}(t)
\]

\[
V_{12}(t) = Q_{12,1}(t) + [1 + V_1(t)]
\]

\[
V_{13}(t) = Q_{13,1}(t) \sum V_1(t)
\]

(5.2.171-5.2.180)

Taking L.S.T. of the above equations and solving them for \( V_0^{**}(s) \), we get

\[
V_0^{**}(s) = \frac{N_5(s)}{D_1(s)}
\]

(5.2.181)

where

\[
N_5(s) = Q_{01}^{*}(s) [1 - Q_{12}^{*}(s) Q_{21}^{(5)}(s) - Q_{18}^{(4)}(s) Q_{81}^{*}(s)
\]

\[
- Q_{13,1}^{*}(s) (Q_{19}^{(4)}(s) Q_{9,13}^{*}(s) + Q_{13}^{*}(s) Q_{3,13}^{(14)}(s))]
\]

(5.2.182)

In steady-state, the number of visits per unit time by the ordinary repairman is given by

\[
V_0 = \frac{N_5}{D_1}
\]

(5.2.183)

where
The following recursive relations for $R_i(t)$ are obtained:

\[
R_0(t) = Q_{01}(t) \circ R_1(t)
\]
\[
R_1(t) = Q_{12}(t) \circ R_2(t) + Q_{13}(t) \circ R_3(t) + Q_{18}(t) \circ R_8(t)
\]
\[
+ Q_{19}(t) \circ R_9(t)
\]
\[
R_2(t) = Q_{20}(t) \circ R_0(t) + Q_{21}(t) \circ R_1(t)
\]
\[
R_3(t) = Q_{36}(t) \circ R_6(t) + Q_{37}(t) \circ [1+R_7(t)] + Q_{3,12}(t) \circ R_{12}(t)
\]
\[
+ Q_{3,13}(t) \circ R_{13}(t)
\]
\[
R_6(t) = Q_{60}(t) \circ R_0(t) + Q_{61}(t) \circ R_1(t)
\]
\[
R_7(t) = Q_{70}(t) \circ R_0(t) + Q_{71}(t) \circ R_1(t)
\]
\[
R_8(t) = Q_{81}(t) \circ R_1(t)
\]
\[
R_9(t) = Q_{9,12}(t) \circ R_{12}(t) + Q_{9,13}(t) \circ [1+R_{13}(t)]
\]
\[
R_{12}(t) = Q_{12,1}(t) \circ R_1(t)
\]
\[
R_{13}(t) = Q_{13,1}(t) \circ R_1(t)
\]

(5.2.185-5.2.194)

Taking L.S.T. of the above equations and solving them for $R_0^{**}(s)$, we get

\[
R_0^{**}(s) = \frac{N_6(s)}{D_1(s)}
\]

(5.2.195)

where

\[
N_6(s) = Q_{01}^{**}(s) [Q_{13}*(s) Q_{37}*(s) + Q_{19}^{(4)}(s) Q_{9,13}*(s)]
\]

(5.2.196)

In steady-state, the expected number of expected replacements is given by

\[
R_0 = \frac{N_6}{D_1}
\]

(5.2.197)

where

\[
N_6 = (p_{13}p_{37} + p_{19}^{(4)}p_{9,13})
\]

(5.2.198)
BUSY PERIOD ANALYSIS OF EXPERT REPAIRMAN
(Repair Time only)

By probabilistic arguments, we have the following recursive relations for $B_i^c(t)$:

$$B_0^c(t) = Q_{0i}(t) \otimes B_1^c(t)$$

$$B_1^c(t) = Q_{12}(t) \otimes B_2^c(t) + Q_{13}(t) B_3^c(t) + Q_{18}^{(4)}(t) \otimes B_8^c(t) + Q_{19}^{(4)}(t) \otimes B_9^c(t)$$

$$B_2^c(t) = Q_{20}(t) \otimes B_0^c(t) + Q_{21}^{(5)}(t) \otimes B_1^c(t)$$

$$B_3^c(t) = Q_{30}(t) B_0^c(t) + Q_{31}(t) \otimes B_7^c(t) + Q_{3_{12}}^{(14)}(t) \otimes B_{12}^c(t) + Q_{3_{13}}^{(14)}(t) \otimes B_{13}^c(t)$$

$$B_4^c(t) = W_6(t) + Q_{60}(t) \otimes B_0^c(t) + Q_{61}^{(10)}(t) \otimes B_1^c(t)$$

$$B_5^c(t) = Q_{70}(t) \otimes B_0^c(t) + Q_{71}^{(11)}(t) \otimes B_1^c(t)$$

$$B_6^c(t) = Q_{80}(t) \otimes B_0^c(t)$$

$$B_7^c(t) = Q_{912}(t) \otimes B_{12}^c(t) + Q_{9,13}(t) \otimes B_{13}^c(t)$$

$$B_{12}^c(t) = W_12(t) + Q_{12,1}(t) \otimes B_1^c(t)$$

$$B_{13}^c(t) = Q_{13,1}(t) \otimes B_1^c(t)$$

where

$$W_i(t) = G_i(t) \quad \text{(5.2.199-5.2.209)}$$

Taking L.T. of the above equations and solving them for $B_0^*(s)$, we get

$$B_i^c(s) = \frac{N_i(s)}{D_i(s)} \quad \text{(5.2.210)}$$

where

$$N_i(s) = q_{0i}^*(s) \left[ W_6^*(s) q_{13}^*(s) q_{36}^*(s) + W_{12}^*(s) \{ q_{13}^*(s) + q_{3_{12}}^{(14)}(s) + q_{19}^{(14)}(s) q_{9,12}^*(s) \} \right] \quad \text{(5.2.211)}$$

and $D_i(s)$ is already specified.

In steady-state, the total fraction of time for which the system is under repair of expert repairman is given by
where
\[ N_7 = [p_{13} p_{36} + p_{13} p_{3.12}^{(14)} + p_{19} p_{9,12}^{(4)}] \mu_{12} \] 

BUSY PERIOD ANALYSIS OF EXPERT REPAIRMAN
(Inspection Time Only)

The following recursive relations are obtained for \( B_1^c(t) \):

- \( B_1^c(t) = q_{12}(t) \odot B_1^c(t) + q_{13}(t) B_3^c(t) + q_{18}^{(4)}(t) \odot B_8^c(t) \)
- \( B_2^c(t) = q_{20}(t) \odot B_0^c(t) + q_{21}^{(5)}(t) \odot B_1^c(t) \)
- \( B_3^c(t) = W_3(t) + q_{36}(t) B_6^c(t) + q_{37}(t) \odot B_7^c(t) + q_{3.12}^{(14)}(t) \odot B_{12}^c(t) \)
- \( B_4^c(t) = q_{22}(t) \odot B_0^c(t) + q_{23}(t) \odot B_1^c(t) \)
- \( B_5^c(t) = W_5(t) + q_{3.13}^{(14)}(t) \odot B_{13}^c(t) \)
- \( B_6^c(t) = q_{60}(t) \odot B_0^c(t) + q_{61}^{(10)}(t) \odot B_1^c(t) \)
- \( B_7^c(t) = q_{70}(t) \odot B_0^c(t) + q_{7.1}^{(11)}(t) \odot B_1^c(t) \)
- \( B_8^c(t) = q_{81}(t) \odot B_1^c(t) \)
- \( B_9^c(t) = W_9(t) + q_{9,12}(t) \odot B_{12}^c(t) + q_{9,13}(t) \odot B_{13}^c(t) \)
- \( B_{12}^c(t) = q_{12,1}(t) \odot B_1^c(t) \)
- \( B_{13}^c(t) = q_{13,1}(t) \odot B_1^c(t) \)

where
\[ W_3 = W_9 = H_0(t) \] 

Taking L.T. of the above equations and solving them for \( B_1^c(s) \), we get

\[ B_1^c(s) = \frac{N_8(s)}{D_1(s)} \] 

where
\[ N_8(s) = q_{01}^*(s) [q_{13}^*(s) W_3^*(s) + q_{19}^{(4)}(s) W_9^*(s)] \]
and $D_1(s)$ is already specified.

In steady-state, the total fraction of the time for which the system is under inspection of expert repairman is given by

$$BI_s^0 = \frac{N_8}{D_1}$$ (5.2.227)

where $N_8 = (p_{13} + p_{19}^{(4)}) \mu_9$ (5.2.228)

**EXPECTED NUMBER OF VISITS BY THE EXPERT REPAIRMAN**

$$V_e^0(t) = Q_{01}(t) \otimes V_1^e(t)$$

$$V_1^e(t) = Q_{12}(t) \otimes V_2^e(t) + Q_{13}(t)\left[1 + V_3^e(t)\right] + Q_{18}^{(4)}(t) \otimes V_8^e(t)$$

$$+ Q_{19}^{(4)}(t) \otimes \left[1 + V_9^e(t)\right]$$

$$V_2^e(t) = Q_{20}(t) \otimes V_0^e(t) + Q_{21}^{(5)}(t) \otimes V_1^e(t)$$

$$V_3^e(t) = Q_{36}(t) \otimes V_6^e(t) + Q_{37}(t) \otimes V_7^e(t) + Q_{3,12}^{(14)}(t) \otimes V_{12}^e(t)$$

$$+ Q_{3,13}^{(14)}(t) \otimes V_{13}^e(t)$$

$$V_6^e(t) = Q_{60}(t) \otimes V_0^e(t) + Q_{61}^{(10)}(t) \otimes V_1^e(t)$$

$$V_7^e(t) = Q_{70}(t) \otimes V_0^e(t) + Q_{7,1}^{(11)}(t) \otimes V_1^e(t)$$

$$V_8^e(t) = Q_{81}(t) \otimes V_1^e(t)$$

$$V_9^e(t) = Q_{9,12}(t) \otimes V_{12}^e(t) + Q_{9,13}(t) \otimes V_{13}^e(t)$$

$$V_{12}^e(t) = Q_{12,1}(t) \otimes V_1^e(t)$$

$$V_{13}^e(t) = Q_{13,1}(t) \otimes V_1^e(t)$$ (5.2.229-5.2.238)

Taking L.S.T. of the above equations and solving them for $V_e^0*(s)$, we get

$$V_e^0*(s) = N_9(s)/D_1(s)$$ (5.2.239)

where

$$N_9(s) = Q_{01}*(s) \left[Q_{13}*(s) + Q_{19}^{(4)}*(s)\right]$$ (5.2.240)

In steady-state, the total number of visits of the expert on the system is given by

$$V_e^0 = \frac{N_9}{D_1}$$ (5.2.241)
where
\[ N_0 = p_{13} + p_{19}^{(4)} \] (5.2.242)

**PROFIT ANALYSIS**

The expected total profit incurred to the system is steady-state is given by
\[
P_{52} = C_0A_0 - C_1B_0 - C_2B_10 - C_3B_10 - C_4V_0 - C_5R_0 - C_6B_0^c - C_7B_1^c - C_8V_0^c
\] (5.2.243)

where
\[ C_0, C_1, C_2, C_3, C_4, C_5 \text{ are same as already defined in model 1.} \]
\[ C_6 = \text{Cost per unit time where expert repairman busy under repair} \]
\[ C_7 = \text{Cost per unit time for which expert repairman is busy under inspection} \]
\[ C_8 = \text{Cost per visit of expert repairman} \]

**PARTICULAR CASES**

For graphical interpretation, the following particular case is considered
\[ g(t) = \alpha_1 e^{-\alpha_1 t} ; \quad g_a(t) = \alpha_2 e^{-\alpha_2 t} ; \quad h_1(t) = \gamma_1 e^{-\gamma_1 t} \]
\[ h_3(t) = \gamma_3 e^{\gamma_3 t} ; \quad h_2(t) = \gamma e^{\gamma t} \]

We can easily obtain the following:
\[
p_{01} = 1 ; \quad p_{12} = \frac{p_1\gamma_1}{\lambda + \gamma_1} ; \quad p_{13} = \frac{p_2\gamma_1}{\lambda + \gamma_1} \\
p_{14} = \frac{\lambda}{\lambda + \gamma_1} ; \quad p_{18}^{(4)} = \frac{p_1\lambda}{\lambda + \gamma_1} ; \quad p_{19}^{(4)} = \frac{p_2\lambda}{\lambda + \gamma_1} \\
p_{20} = \frac{\alpha_1}{\lambda + \alpha_1} ; \quad p_{25} = \frac{\lambda}{\lambda + \alpha_1} ; \quad p_{21}^{(5)} = \frac{\lambda}{\lambda + \alpha_1} \\
p_{36} = \frac{p_1\gamma_2}{\lambda + \gamma_2} ; \quad p_{37} = \frac{p_2\gamma_2}{\lambda + \gamma_2} ; \quad p_{3,14} = \frac{\lambda}{\lambda + \gamma_2}
Using the above equations and equations (5.2.101), (5.2.123), (5.2.139), (5.2.154), (5.2.169), (5.2.183), (5.2.197), (5.2.212), (5.2.227), (5.2.241) and (5.2.243). We can have expressions for MTSF, availability and profit for this particular case.

**GRAPHICAL INTERPRETATION OF MODEL 5.1**

For the graphical interpretation, the mentioned particular case is considered. The behaviour of MTSF and the availability ($A_0$) with respect to failure rate ($\lambda$) for different values of repair rate ($\alpha_1$) is shown as in Fig. 5.3 and 5.4 respectively. It is clear from the graphs the MTSF and availability decrease with increase in the values of failure rate. However, their values become higher for higher values of repair rate ($\alpha_1$).
Fig. 5.5 shows the behaviour of profit ($P_{51}$) with respect to cost per replacement ($C_5$) for different values of probability ($p_2$) that the unit is not repairable. Following conclusions can be drawn:

(i) If $p_2 = 0.1$, this profit will become negative for a very high value of cost ($C_5$) and hence it can be concluded that if there are less chances of replacement, then the system is more profitable.

(ii) If $p_2 = 0.5$ then $P_{51} > = or < 0$ according as $C_5 < or = or > 2300$ i.e. the system is profitable if $C_5 \leq 2300$

(iii) If $p_2 = 0.9$ then $P_{51} > = or < 0$ according as $C_5 < or = or > 1256$ i.e. the system is profitable if $C_5 \leq 1256$.

GRAPHICAL INTERPRETATION OF MODEL 5.2

For the graphical interpretation, the mentioned particular case is considered. The behaviour of MTSF and the availability ($A_n$) with respect to failure rate ($\lambda$) for different values of repair rate ($\alpha_1$) is shown as in Fig. 5.6 and 5.7 respectively. It is clear from the graphs the MTSF and availability decrease with increase in the values of failure rate. However, their values become higher for higher values of repair rate ($\alpha_1$).

Fig. 5.8 shows the behaviour of profit ($P_{52}$) with respect to cost per replacement ($C_5$) for different values of probability ($p_2$). The following conclusions can be drawn:

(i) If $p_2 = 0.1$ then profit will become negative for a very high value of cost ($C_5$) and hence it can be concluded that if there are less chances of replacements then the system is more profitable.

(ii) If $p_2 = 0.5$ then $P_{52} > = or < 0$ according as $C_5 < or = or > 4900$

(iii) If $p_2 = 0.9$ then $P_{52} > = or < 0$ according as $C_5 < or = or > 1499$ i.e. the system is profitable if $C_5 \leq 1499$. 

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MTSF VERSUS FAILURE RATE ($\lambda$) FOR DIFFERENT VALUES OF REPAIR RATE ($\alpha_1$)

$\alpha=0.5$, $p_1=0.5$, $p_2=0.5$, $\gamma_1=8$, $\gamma_2=5$

Failure Rate ($\lambda$)

Fig. 5.3
AVAILABILITY VERSUS FAILURE RATE (\(a_r\)) FOR DIFFERENT VALUES OF REPAIR RATE (\(a_p\)).

**Figure 5.4**

- Line for \(a_r = 3\)
- Line for \(a_r = 2\)
- Line for \(a_r = 1\)
PROFIT VERSUS COST ($C_5$) FOR DIFFERENT VALUES OF PROBABILITY ($p_2$)

$C_2 = 40, C_3 = 30, C_4 = 25$

$p_2 = 0.5, p_2 = 0.1$

$\delta = 0.5, \lambda = 0.5, \gamma = 8.5, C_3 = 1000, C_4 = 50$
Figure 5.6

Availability versus failure rate (\( \lambda \)) for different values of repair rate (\( \mu \)).
AVAILABILITY VERSUS FAILURE RATE (λ) FOR DIFFERENT VALUES OF REPAIR RATE (α)

Fig. 5.7

Failure Rate (λ)
0.2 0.4 0.6 0.8 1 1.3 1.6 1.9 2.2 2.5 2.8 3.1 3.4 3.7 4 4.3 4.6 4.9

A_o

p_1=0.5, p_2=0.2, γ_1=5, γ_2=10, α=0.3, α_1=3

α_1=2

α_1=3
Fig. 5.8

PROFIT VERSUS COST (C5) FOR DIFFERENT VALUES OF PROBABILITY (p2)

Cost (C5)
Profit

PROBABILITY (p2)

PROFIT VERSUS COST (C5) FOR DIFFERENT VALUES OF

C1=50, C2=40, C3=30, C4=25, C5=25, C6=15, C7=50, C8=40

P2=0.9, P2=0.5, P2=0.1, P2=0.3