CHAPTER-VII

COST-BENEFIT ANALYSIS
OF A SYSTEM WHEREIN
OPERATION AND SOMETIMES
REPAIR OF MAIN UNIT
DEPENDS ON SUB-UNIT
CHAPTER - VII

COST-BENEFIT ANALYSIS OF A SYSTEM WHERE OPERATION AND SOMETIMES REPAIR OF MAIN UNIT DEPENDS ON SUB-UNIT

Introduction

In the previous chapter, it is assumed that only the working of main unit depends on the operation of sub-unit but there may be situations where the operation of sub-unit is also required for the repair of main unit. It is a practical situation e.g. if we consider computer as main unit and electricity as sub-unit then sometimes for the repair of computer if failed, electricity is required.

In present chapter, we introduce the above idea to study a system with main unit and sub-unit(s). The main unit works only if sub-unit is operative and for repair of the main unit the operation of sub-unit may be required.

In Model 1, the system consists of one main unit and one sub-unit and priority for repair is given to sub-unit. In Model 2, there is one main unit and two identical sub-units. Initially one sub-unit is in operating state and the other in cold standby state. Upon failure of operative sub-unit, cold standby sub-unit becomes operative immediately. On the failure of a unit, a repairman comes immediately to repair it. If both the sub-units are failed then main unit comes in state of rest and priority for repair is given to one of the sub-units. Other assumptions are as usual.

The models are analysed by making use of semi-Markov processes and regenerative point technique and the expressions for
various measures of system effectiveness such as mean time to system failure, steady state availability, total fraction of busy time of repairman per unit time and expected number of visits by repairman per unit time are determined. Profits for both models are calculated using the above measures. The profits for a particular case are also calculated where repair time distributions are exponential. Graphs pertaining to this particular case are also plotted and comparative study is made through the graphs.

**Notation**

- $\lambda$: constant failure rate of main unit
- $\lambda_1$: constant failure rate of sub-units
- $b_1$: constant rate of sub-units to become operative from rest
- $b_2$: constant rate of sub-units to come to rest from operating state
- $G(t), g(t)$: c.d.f. and p.d.f. of the repair time of main unit
- $G_i(t), g_i(t)$: c.d.f. and p.d.f. of the repair time of sub-units

Symbols for the states of the system:

- $S_i$: state number $i$, $i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$
- $O$: main unit in operating state
- $O_i$: sub-unit in operating state
- $C_{Si}$: sub-unit in cold standby state
- $Fr, Fr_1$: main unit, sub-unit are under repair respectively
- $Fr, Fr_1$: main unit, sub-unit are under repair respectively from the previous state i.e. repair is continuing from the previous state.
- $Fwr, Fwr_1$: main unit, sub-unit are waiting for repair respectively
- $Fwr_1$: sub-unit waiting for repair from previous state.
R.Ri main unit, sub-unit in state of rest.

**Model 1**

A transition diagram showing the various states of transition of the system is as shown in Fig 7.1. In this model, it is assumed that main unit works with the help of one sub-unit and when it is under repair, to become operative it needs the help of sub-unit.

**Transition Probabilities and Mean Sojourn Times**

The epochs of entry into states S₀, S₁, S₂, S₄, S₆ are regeneration points and hence these are regenerative states. States S₁, S₂, S₃, S₄, S₅, S₆ are down states. The transition probabilities are

\[ dQ_{01}(t) = \lambda e^{-(\lambda + \lambda_1)t} dt \]
\[ dQ_{02}(t) = \lambda_1 e^{-(\lambda + \lambda_1)t} dt \]
\[ dQ_{10}(t) = e^{-b_1 t} g(t) dt \]
\[ dQ_{10}^{(3)}(t) = (b_1 e^{-b_1 t} \odot e^{-(b_2 + \lambda_1)t}) g(t) dt \]
\[ dQ_{14}^{(3)}(t) = (b_1 e^{-b_1 t} \odot \lambda_1 e^{-(b_2 + \lambda_1)t}) g(t) dt \]
\[ dQ_{10}^{(3,5)}(t) = (b_1 e^{-b_1 t} \odot b_2 e^{-(b_2 + \lambda_1)t} \odot 1) g(t) dt \]
\[ dQ_{20}(t) = g_1(t) dt \]
\[ dQ_{46}(t) = g_1(t) dt \]
\[ dQ_{60}(t) = e^{-(b_2 + \lambda_1)t} g(t) dt \]
\[ dQ_{64}(t) = \lambda_1 e^{-(b_2 + \lambda_1)t} g(t) dt \]
\[ dQ_{60}^{(5)}(t) = (b_2 e^{-(b_2 + \lambda_1)t} \odot 1) g(t) dt \]  

(7.1.1)-(7.1.11)

The non-zero elements pᵢ are
Fig. 7.1: State transition diagram for Model 1.

\[ E = (S_0, S_1, S_2, S_4, S_6); \quad \bar{E} = (S_3, S_5) \]
\( p_{01} = \lambda/\(\lambda+\lambda_1\) \), \( p_{02} = \lambda_1/\(\lambda+\lambda_1\) \)

\( p_{10} = g^*(b_1) \), \( p_{10}^{(3)} = \(b_1/(b_1-b_1-\lambda_1)\) \) \[ g^*(b_2+\lambda_1) - g^*(b_1) \]

\( p_{14}^{(3)} = (b_1\lambda_1/(b_1-b_1-\lambda_1))[\(b_1-b_2-\lambda_1\)/(b_1(b_2+\lambda_1))+(g^*(b_1)/b_1) \)

\( -(g^*(b_2+\lambda_1)/(b_2+\lambda_1)) \]

\( p_{10}^{(3,5)} = b_1b_2[\{g^*(b_1)/(b_1(b_1-b_2-\lambda_1))\}-(g^*(b_2+\lambda_1)/((b_2+\lambda_1)/(b_1-b_2-\lambda_1))) \)

\( +(1/(b_1(b_2+\lambda_1))) \]

\( p_{20} = 1, \ p_{46} = 1, \ p_{60} = g^*(b_2+\lambda_1), \ p_{60}^{(5)} = (b_2/(b_2+\lambda_1))[1-g^*(b_2+\lambda_1)] \)

\( p_{64} = (\lambda_1/(\lambda_1+b_2)) \) \[1-g^*(b_2+\lambda_1)] \)

By these transition probabilities, it can be verified that

\( p_{01} + p_{02} = 1 \)

\( p_{10} + p_{10}^{(3)} + p_{10}^{(3,5)} + p_{14}^{(3)} = 1 \)

\( p_{20} = p_{46} = 1 \)

\( p_{60} + p_{64} + p_{60}^{(5)} = 1 \)

Also \( \mu_i \), the mean sojourn times in state \( S_j \) are

\( \mu_0 = 1/(\lambda+\lambda_1) \), \( \mu_1 = (1-g^*(b_1))/b_1 \)

\( \mu_2 = (1-g^*(\lambda_1))/\lambda_1 \), \( \mu_4 = (1-g^*(\lambda_1))/\lambda_1 \)

\( \mu_6 = (1-g^*(b_2+\lambda_1))/(b_2+\lambda_1) \)

The unconditional mean time taken by the system to transit for any state \( S_j \in E \) when it (time) is counted from epoch of entrance into state \( S_i \in E \) is mathematically stated as:

\[ m_{ij} = \int_0^\infty Q_j(t) \, dt = -q_{ij}^*(0) \] (7.1.30)

Thus

\[ m_{01} + m_{02} = \mu_0 \]

\[ m_{10} + m_{10}^{(3)} + m_{14}^{(3)} + m_{10}^{(3,5)} = (\lambda_1/(b_1-b_2-\lambda_1))[\(b_1\mu_6/(b_2+\lambda_1)-\mu_1\] + \(b_2/(b_2+\lambda_1))K \]

\[ = e_1 \text{(say)} \]
\[ K = \int_0^\infty G(t) dt \]

\[ m_{20} = \int_0^\infty G_1(t) dt = K_1 \text{ (say)} = m_{46} \]

\[ m_{60} + m_{60}(5) + m_{64} = \frac{X_i}{(A_i + b_2)} + (b_2/(\lambda_1 + b_2))K_1 = \epsilon_6 \text{ (say)} \quad (7.1.31)-(7.1.34) \]

**Mean Time to System Failure**

To determine the mean time to system failure (MTSF) of the system, we regard the failed states of the system absorbing. By probabilistic arguments, we obtain the following recursive relation for \( \phi_0(t) \):

\[ \phi_0(t) = Q_{01}(t) + Q_{02}(t) \quad \text{(7.1.35)} \]

Taking Laplace-Stieltjes Transforms (L.S.T) of this relation and solving for \( \phi_0^{**}(s) \), we obtain

\[ \phi_0^{**}(s) = \frac{N(s)}{D(s)} \quad (7.1.36) \]

where

\[ N(s) = [Q_{01}^{**}(t) + Q_{02}^{**}(t)] \quad (7.1.37) \]

\[ D(s) = 1 \quad (7.1.38) \]

Now the mean time to system failure (MTSF) when the system starts from the state \( S_0 \) is

\[ T_0 = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s} \quad (7.1.39) \]

Using L’Hospital’s Rule and putting the value of \( \phi_0^{**}(s_0) \) from equation (7.1.36) we have

\[ T_0 = \frac{N}{D} \quad (7.1.40) \]

where

\[ N = \mu_0 \quad (7.1.41) \]

\[ D = 1 \quad (7.1.42) \]
Availability Analysis

Using the arguments of the theory of regenerative processes, the availability \( A_i(t) \) is seen to satisfy the following recursive relations :

\[
\begin{align*}
A_0(t) &= M_0(t) + q_01(t) \odot A_1(t) + q_02(t) \odot A_2(t) \\
A_1(t) &= q_{10}(t) \odot A_0(t) + q_{10}^{(3,5)}(t) \odot A_0(t) + q_{10}^{(3)}(t) \odot A_0(t) + q_{14}^{(3)}(t) \odot A_4(t) \\
A_2(t) &= q_{20}(t) \odot A_0(t) \\
A_4(t) &= q_{40}(t) \odot A_0(t) \\
A_5(t) &= q_{60}(t) \odot A_0(t) + q_{60}^{(5)}(t) \odot A_0(t) + q_{64}(t) \odot A_4(t)
\end{align*}
\]

(7.1.43)-(7.1.47)

where

\[
M_0(t) = e^{-(\lambda + \lambda_1)t}
\]

(7.1.48)

Taking Laplace transform of the equation (7.1.48) and letting \( s \to 0 \), we get

\[
M_0^*(0) = \mu_0
\]

(7.1.49)

Taking the Laplace transform of the above equations in (7.1.43)-(7.1.47) and solving them for \( A_0^*(s) \), we get

\[
A_0^*(s) = \frac{N_1(s)}{D_1(s)}
\]

(7.1.50)

where

\[
N_1(s) = M_0^*(s)(1-q_{46}^*(s)q_{64}^*(s))
\]

(7.1.51)

\[
D_1(s) = [1 - q_{01}^*(s)(q_{10}^*(s)+q_{10}^{(3)}(s)+q_{10}^{(3,5)}(s))- q_{02}^*(s)q_{20}^*(s)]
\]

(7.1.52)

In steady state availability of the system is given by

\[
A_0 = \lim_{s \to 0} (sA_0^*(s)) = \frac{N_1}{D_1}
\]

(7.1.53)

where

\[
N_1 = \mu_0(1-p_{64})
\]

(7.1.54)

\[
D_1 = (1-p_{64})[\mu_0 + \mu_1 \epsilon + p_{02} K_1] + p_{01} p_{14}^{(3)} [\epsilon_6 + K_1]
\]

(7.1.55)
Busy Period Analysis of Repairman

By probabilistic arguments we have the following recursive relations:

\[ B_0(t) = q_{01}(t) B_1(t) + q_{02}(t) B_2(t) \]

\[ B_1(t) = W_1(t) + q_{10}(t) B_0(t) + q_{11}^{(3)}(t) B_0(t) + q_{10}^{(3,5)}(t) B_0(t) \]

\[ B_2(t) = W_2(t) + q_{20}(t) B_0(t) \]

\[ B_4(t) = W_4(t) + q_{40}(t) B_0(t) \]

\[ B_6(t) = W_6(t) + q_{60}(t) B_0(t) + q_{64}(t) B_4(t) + q_{60}^{(5)}(t) B_0(t) \]

(7.1.56)-(7.1.60)

where

\[ W_1(t) = \frac{b_1 b_2}{b_1(b_1-b_2-\lambda_1)} e^{-b_1 t} G(t) - \frac{b_1 b_2}{(b_2+\lambda_1)(b_1-b_2-\lambda_1)} e^{-(b_2+\lambda_1)t} G(t) \]

\[ + \frac{b_1 b_2}{b_1(b_1-b_2-\lambda_1)} G(t) + \frac{b_1}{(b_1-b_2-\lambda_1)} \left[ e^{-(b_2+\lambda_1)t} - e^{-b_1 t} \right] G(t) \]

\[ W_2(t) = G_1(t), \quad W_4(t) = G_1(t) \]

\[ W_6(t) = e^{-(b_2+\lambda_1)t} G(t) + (b_2 e^{-(b_2+\lambda_1)t} \odot 1) G(t) \]

(7.1.61)-(7.1.63)

Taking L.T. of above equations in (7.1.61)-(7.1.63) and letting \( s \to 0 \) we have,

\[ W_1^*(0) = \frac{\lambda_1}{b_1-b_2-\lambda_1} \left[ \frac{b_1}{b_2+\lambda_1} \mu_6 - b_2 \right] + \frac{b_2}{b_2+\lambda_1} K \]

\[ = \epsilon_1 \text{ (already specified)} \]

\[ W_2^*(0) = K_1 = W_4^*(0) \text{ (already specified)} \]

\[ W_6^*(0) = (\lambda_1/(b_2+\lambda_1)) \mu_6 + (b_2/(b_2+\lambda_1)) K = \epsilon_6 \text{ (say)} \]

(7.1.64)-(7.1.66)

Now taking L.T. of equations in (7.1.56)-(7.1.60) and solving them for \( B_0^*(s) \), we get
\[ B_0^*(s) = \frac{N_2(s)}{D_1(s)} \quad (7.1.67) \]

where

\[ N_2(s) = q_{01}^* (s) W_1^*(s) (1 - q_{46}^* (s) q_{64}^* (s)) + W_2^* (s) q_{02}^* (s) (1 - q_{46}^* (s) q_{64}^* (s)) + W_4^* (s) q_{01}^* (s) q_{14}^{(3)*} (s) + W_6^* (s) q_{01}^* (s) q_{14}^{(3)*} (s) q_{46}^* (s) \quad (7.1.68) \]

\[ D_1(s) \text{ is already specified in equation (7.1.52).} \]

In steady-state, the total fraction of the time for which the system is under repair is given by

\[ B_0 = \lim_{s \to 0} (s B_0^*(s)) = \frac{N_2}{D_1} \quad (7.1.69) \]

where

\[ N_2 = (1 - p_{64}) (p_{01} + p_{02} K_1) + p_{01} p_{14}^{(3)} (K_1 + e_6) \quad (7.1.70) \]

and \( D_1 \) is already specified in equation (7.1.55)

**Expected Number of Visits by Repairman**

By probabilistic arguments we have the following recursive relations:

\[ V_0(t) = Q_{01}(t) \circ [1 + V_1(t)] + Q_{02}(t) \circ [1 + V_2(t)] \]

\[ V_1(t) = Q_{10}(t) \circ V_0(t) + Q_{10}^{(3)}(t) \circ V_0(t) + Q_{10}^{(3,5)}(t) \circ V_0(t) + Q_{14}^{(3)}(t) \circ V_4(t) \]

\[ V_2(t) = Q_{20}(t) \circ V_0(t) \]

\[ V_4(t) = Q_{46}(t) \circ V_0(t) \]

\[ V_6(t) = Q_{60}(t) \circ V_0(t) + Q_{64}(t) \circ V_4(t) + Q_{60}^{(5)}(t) \circ V_0(t) \quad (7.1.71) - (7.1.75) \]

Taking L.S.T. of equations in (7.1.71) - (7.1.75) and solving for \( V_0^{**}(s) \), we get

\[ V_0^{**}(s) = \frac{N_4(s)}{D_1(s)} \quad (7.1.76) \]

where

\[ N_4(s) = (q_{01}^* (s) + q_{02}^* (s)) (1 - q_{46}^* (s) q_{64}^* (s)) \quad (7.1.77) \]

and \( D_1(s) \) is already specified in equation (7.1.52).

In steady-state, the number of visits then per unit time is given by
\[ V_0 = \lim_{t \to \infty} \frac{V_0(t)}{t} = \lim_{s \to 0} s V_0(s) \]
\[ = \frac{N_4}{D_1} \]  \hspace{1cm} (7.1.78)

\[ N_4 = (1-p_{64}) \]  \hspace{1cm} (7.1.79)

and \( D_1 \) is already specified in equation (7.1.55)

**Cost-Benefit Analysis**

The expected total profit incurred to the system in steady-state is given by

\[ P = C_0 A_0 - C_1 B_0 - C_2 V_0 \]  \hspace{1cm} (7.1.80)

where

- \( C_0 \) = revenue per unit up time of the system
- \( C_1 \) = cost per unit time for which the repairman is busy.
- \( C_2 \) = cost per visit of repairman.

**Particular Case**

For graphical representation, the following particular case is considered. Let us assume that \( g(t) = e^{-\alpha t} \), \( g_1(t) = e^{-\beta t} \) and the remaining distributions are the same as in the general case. Therefore, we have

\[ p_{01} = \lambda / (\lambda + \lambda_1), \quad p_{02} = \lambda_1 / (\lambda + \lambda_1), \quad p_{10} = \alpha / (b_1 + \alpha) \]
\[ p_{10}^{(3)} = (\alpha b_1) / ((b_2 + \lambda_1 + \alpha)(b_1 + \alpha)), \quad p_{14}^{(3)} = (b_1 \lambda_1) / ((b_2 + \lambda_1 + \alpha)(b_1 + \alpha)) \]
\[ p_{10}^{(3,5)} = (b_1 b_2) / ((b_1 + \alpha)(b_2 + \lambda_1 + \alpha)), \quad p_{20} = 1, \quad p_{46} = 1 \]
\[ p_{60} = \alpha / (b_2 + \lambda_1 + \alpha), \quad p_{64} = \lambda_1 / (b_2 + \lambda_1 + \alpha), \quad p_{60}^{(5)} = b_2 / (b_2 + \lambda_1 + \alpha) \]
\[ \mu_0 = 1 / (\lambda + \lambda_1), \quad \mu_1 = 1 / (b_1 + \alpha), \quad \mu_2 = 1 / (\lambda_1 + \alpha), \quad \mu_4 = 1 / (\lambda_1 + \alpha) \]
\[ \mu_6 = 1 / (b_2 + \lambda_1 + \alpha), \quad K = 1 / \alpha, \quad K_1 = 1 / \beta \]  \hspace{1cm} (7.1.90) - (7.1.107)

Using above equations (7.1.90) - (7.1.107) and equations (7.1.40), (7.1.53), (7.1.69) and (7.1.78) we have expressions for MTSF and profit for this particular case.
Variation in Failure Rate
Fig. 7.4

Failure Rate (A) vs Profit (P)

Variation in Failure Rate (A) vs Profit (P)
Graphical Interpretation

Figs. 7.2 and 7.4 show the relationship between MTSF w.r.t. failure rate ($\lambda_1$) and profit ($P_1$) w.r.t. failure rate ($\lambda_1$). It is clear from these figures that as the failure rate increases MTSF as well as the profit decreases. Here variation is taken in failure rate ($\lambda$) which is $\lambda = 0.01$, $\lambda = 0.02$ and $\lambda = 0.03$ for MTSF and $\lambda = 1$, $\lambda = 1.05$ and $\lambda = 1.1$ for profit and as it increases there is decrease in MTSF as well as in the profit.

Fig. 7.3 shows the behaviour of profit ($P_1$) w.r.t. rate ($b_1$) (where $b_1$ is the constant rate of sub-units to become operative from rest). It is obvious from the figure that as the rate ($b_1$) increases profit ($P_1$) decreases. Also as variation is taken in the rate ($b_2$) (where $b_2$ is constant rate of sub-units to come to rest from operating state) which is $b_2 = 0.1$, $b_2 = 0.5$ and $b_2 = 1$ in this case and as it increases profit decreases.

Model 2

A state transition diagram showing the various states of transition of the system is as shown in the Fig. 7.5. In this model, there is one main unit and two identical sub-units. For system to be operative main unit and one sub-unit should be in operating state.

Transition Probabilities and Mean Sojourn Times

The epochs of entry into states $S_0$, $S_1$, $S_2$, $S_4$ and $S_{10}$ are regeneration points and hence these states are regenerative states. States $S_1$, $S_3$, $S_4$, $S_5$, $S_6$, $S_7$, $S_8$, $S_9$, $S_{10}$ and $S_{11}$ are down states. The transition probabilities are:

\[ dQ_{01}(t) = \lambda e^{- (\lambda + \lambda_1) t} \, dt \]

\[ dQ_{02}(t) = \lambda_1 e^{- (\lambda + \lambda_1) t} \, dt \]
Fig. 7.5. State transition diagram for Model 2.

\[ E = (S_0, S_1, S_2, S_4, S_6, S_7, S_8, S_9, S_{10}) \]

Up state • Failed state • Regeneration point

\[ E = (S_0, S_{11}, S_2, S_4, S_6, S_7, S_8, S_{10}) \]
\[ dQ^{(3)}_1(t) = (b_1 e^{-b_1 t} \otimes (\lambda_1 e^{-(\lambda_1 + b_2)t})) g(t) dt \]
\[ dQ^{(3,7)}_1(t) = (b_1 e^{-b_1 t} \otimes (\lambda_1 e^{-(\lambda_1 + b_2)t} \otimes 1)) g(t) dt \]
\[ dQ^{(3,8)}_1(t) = (b_1 e^{-b_1 t} \otimes (\lambda_1 e^{-(\lambda_1 + b_2)t} \otimes \lambda_1 e^{-(\lambda_1 + b_2)t} \otimes 1)) g(t) dt \]
\[ dQ^{(3,6,9)}_1(t) = (b_1 e^{-b_1 t} \otimes (\lambda_1 e^{-(\lambda_1 + b_2)t} \otimes 1)) g(t) dt \]
\[ dQ^{(5)}_2(t) = (\lambda_1 e^{-(\lambda_1 + \lambda_1)t} \otimes 1) g(t) dt \]
\[ dQ^{(8)}_2(t) = (b_1 e^{-b_1 t} \otimes e^{-(\lambda_1 + b_2)t}) g(t) dt \]
\[ dQ^{(8,9)}_2(t) = (b_1 e^{-b_1 t} \otimes b_2 e^{-(\lambda_1 + b_2)t} \otimes 1) g(t) dt \]
\[ dQ^{(8)}_4(t) = (b_1 e^{-b_1 t} \otimes \lambda_1 e^{-(\lambda_1 + b_2)t}) g(t) dt \]
\[ dQ^{(10,11)}_4(t) = g(t) dt \]
\[ dQ^{(11)}_4(t) = e^{-(\lambda_1 + b_2)t} g(t) dt \]
\[ dQ^{(9)}_4(t) = (b_2 e^{-(\lambda_1 + b_2)t} \otimes 1) g(t) dt \]
\[ dQ^{(11,10)}_4(t) = \lambda_1 e^{-(\lambda_1 + b_2)t} G(t) dt \]

The non-zero elements $p_{ij}$ are:
\[ p_{01} = \lambda/(\lambda+\lambda_1), \quad p_{02} = \lambda_1/(\lambda+\lambda_1) \]

\[ p_{10} = g^*(b_1), \quad p_{10}^{(3)} = (b_1/(b_2+\lambda_1-b_1))(g^*(b_1)-g^*(b_2+\lambda_1)) \]

\[ p_{10}^{(3,7)} = b_1b_2 \left[ \frac{g^*(b_2^2+\lambda_1)}{(b_2+\lambda_1)(b_2+\lambda_1-b_1)} - \frac{g^*(b_1)}{b_1(b_2+\lambda_1-b_1)} + \frac{1}{b_1(b_2+\lambda_1)} \right] \]

\[ p_{12}^{(3,6)} = b_1\lambda_1 \left[ \frac{g^*(b_2^2+\lambda_1)}{(b_2+\lambda_1-b_1)} - \frac{g^*(b_2+\lambda_1)}{(b_2+\lambda_1-b_1)^2} + \frac{g^*(b_1)}{(b_2+\lambda_1-b_1)^2} \right] \]

\[ p_{12}^{(3,6,9)} = b_1\lambda_1 b_2 \left[ \frac{g^*(b_2^2+\lambda_1)b_2}{(b_2+\lambda_1-b_1)^2(b_2+\lambda_1)^2} - \frac{g^*(b_2^2+\lambda_1)}{(b_2+\lambda_1)(b_2+\lambda_1-b_1)} \right. \\
\left. - \frac{g^*(b_1)}{b_1(b_2+\lambda_1-b_1)^2} + \frac{1}{(b_2+\lambda_1)^2b_1} \right] \]

\[ p_{20} = g_1^*(\lambda+\lambda_1), \quad p_{24} = (\lambda/(\lambda_1+\lambda_1))[1-g_1^*(\lambda+\lambda_1)] \]

\[ p_{25} = p_{22}^{(5)} = (\lambda_1/(\lambda+\lambda_1)) \left[ 1-g_1^*(\lambda+\lambda_1) \right], \quad p_{42} = g^*(b_1) \]

\[ p_{42}^{(8)} = \frac{b_1}{b_2+\lambda_1-b_1} [g^*(b_1)-g^*(b_2+\lambda_1)]g(t) \]

\[ p_{42}^{(8,9)} = \frac{g^*(b_2+\lambda_1)}{(b_2+\lambda_1-b_1)(b_2+\lambda_1)} - \frac{g^*(b_1)}{b_1(b_2+\lambda_1-b_1)} + \frac{1}{b_1(b_2+\lambda_1)} \]
By these transition probabilities, it can be verified that

\[ P_{01} + P_{02} = 1 \]
\[ P_{10} + P_{10}^{(3)} + P_{10}^{(3,7)} + P_{12}^{(3,6)} + P_{12}^{(3,6,9)} + P_{11,10}^{(3,6)} = 1 \]
\[ P_{20} + P_{24} + P_{25} = 1 = P_{20} + P_{24} + P_{22}^{(5)} \]
\[ P_{42} + P_{42}^{(8,9)} + P_{4,10}^{(8)} + P_{42}^{(8)} = 1, P_{10,11} = 1 \]
\[ P_{11,12} + P_{11,2}^{(9)} + P_{11,10} = 1 \]

(7.2.40)-(7.2.45)

Also \( \mu_i \), the mean sojourn time in state \( S_i \) are

\[ \mu_0 = \frac{1}{\lambda + \lambda_1}, \quad \mu_1 = \frac{1-g^*(b_1)}{b_1} \]
\[ \mu_2 = \frac{1-g^*(\lambda + \lambda_1)}{(\lambda + \lambda_1)}, \quad \mu_4 = \frac{1-g^*(b_1)}{b_1} \]
\[ \mu_{10} = K_i \text{(already specified)} \]
\[ \mu_{11} = \frac{1-g^*(b_2 + \lambda_1)}{(b_2 + \lambda_1)} \]

(7.2.46)-(7.2.51)

The unconditional mean time taken by the system to transit for any state \( S_j \in E \), when it (time) is counted from the epoch of entrance into state \( S_i \in E \) is mathematically stated as:

\[ m_j = \int_0^\infty Q_{ij}(t) \, dt = -q_{ij}^*(0) \]

(7.2.52)

Thus,
\[m_{01} + m_{02} = \mu_0\]
\[m_{10}^{(3,7)} + m_{10} + m_{10}^{(3)} + m_{12}^{(3,6)} + m_{12}^{(3,6,9)} + m_{1,10}^{(3,5)}\]
\[\frac{b_1\lambda_1^2}{b_1(b_2+\lambda_1-b_1)^2} \mu_1 - \frac{b_1\lambda_1^2(3b_2+3\lambda_1-2b_1)}{(b_2+\lambda_1)^2(b_2+\lambda_1-b_1)^2} \mu_{11}\]
\[+ \frac{b_1\lambda_1^2}{(b_2+\lambda_1)^2(b_2+\lambda_1-b_1)^2} \int_0^{\infty} t e^{-(b_2+\lambda_1)t} g(t) \, dt\]
\[+ \frac{b_2(b_1+2\lambda_1)}{(b_2+\lambda_1)^2} K = \epsilon_1 \text{ (say)}\]
\[\int G(t) \, dt = K.\]
\[m_{20} + m_{24} + m_{25} = \mu_2,\]
\[m_{20} + m_{24} + m_{22}^{(5)} = \frac{\lambda_1}{\lambda+\lambda_1} K_1 + \frac{\lambda}{\lambda+\lambda_1} \mu_2 = \epsilon_2 \text{ (say)}\]
\[m_{42} + m_{42}^{(8)} + m_{42}^{(8,9)} + m_{4,10}^{(8)}\]
\[= \frac{b_2}{b_2+\lambda_1} K + \frac{\lambda_1}{(b_2+\lambda_1-b_1)(b_2+\lambda_1)} [(b_2+\lambda_1)\mu_1 - b_1\mu_{11}] = \epsilon_4 \text{ (say)}\]
\[m_{10,11} = K_1\]
\[m_{11,2} + m_{11,2}^{(9)} + m_{11,10} = \frac{\lambda_1}{b_2+\lambda_1} \mu_{11} + \frac{b_2}{b_2+\lambda_1} K = \epsilon_{11} \text{ (say)}\]

(7.2.53)-(7.2.69)

**Mean Time to System Failure**

To determine the mean time to system failure (MTSF) of the system, we regard the failed states of the system absorbing. By probabilistic arguments, we obtain the following relations for \(\phi_i(t)\):
\[\phi_0(t) = Q_{02}(t)\Phi_0(t) + Q_{01}(t)\]
\[\phi_2(t) = Q_{20}(t)\Phi_0(t) + Q_{24}(t) + Q_{25}(t)\]

(7.2.70)-(7.2.71)
Taking Laplace-Stieltjes Transforms (L.S.T.) of these relations and solving for $\phi_0^{**}(s)$, we obtain

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)} \quad (7.2.72)$$

where

$$N(s) = Q_{01}^{**}(s) + Q_{02}^{**}(s) (Q_{24}^{**}(s) + Q_{25}^{**}(s)) \quad (7.2.73)$$

$$D(s) = 1 - Q_{02}^{**}(s) Q_{20}^{**}(s) \quad (7.2.74)$$

Now the mean time to system failure (MTSF) when the system starts from the state $S_0$ is

$$T_0 = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s} \quad (7.2.75)$$

Using L'Hospital Rule and putting the value of $\phi_0^{**}(s)$ from equation (7.2.72), we have

$$T_0 = \frac{N}{D} \quad (7.2.76)$$

where $N = \mu_0 + \mu_2 p_{02}$
$$D = 1 - p_{02} p_{20} \quad (7.2.77)$$

**Availability Analysis**

Using the argument of the theory of regenerative processes, the availability $A_i(t)$ is seen to verify the following recursive relations:

$$A_0(t) = M_0(t) + q_{01}(t) \circ A_1(t) + q_{02}(t) \circ A_2(t)$$

$$A_1(t) = q_{10}(t) \circ A_0(t) + q_{10}^{(3)}(t) \circ A_0(t) + q_{10}^{(3,7)}(t) \circ A_0(t)$$
$$+ q_{12}^{(3,6)}(t) \circ A_2(t)$$
$$+ q_{12}^{(3,6,9)}(t) \circ A_2(t) + q_{1,10}^{(3,6)}(t) \circ A_{10}(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) \circ A_0(t) + q_{24}(t) \circ A_4(t) + q_{22}^{(5)}(t) \circ A_2(t)$$

$$A_4(t) = q_{42}(t) \circ A_2(t) + q_{42}^{(8)}(t) \circ A_2(t) + q_{42}^{(8,9)}(t) \circ A_2(t)$$
$$+ q_{4,10}^{(8)}(t) \circ A_{10}(t)$$

$$A_{10}(t) = q_{10,11}(t) \circ A_{11}(t)$$
\[ A_{11}(t) = q_{11,2}(t) \odot A_2(t) + q_{11,2}^{(9)}(t) \odot A_2(t) + q_{11,10}(t) \odot A_{10}(t) \]  

(7.2.79)-(7.2.84)

where

\[ M_0(t) = e^{-(\lambda \tau_2 + \lambda_1) t} \]

\[ M_2(t) = e^{-(\lambda \tau_2 + \lambda_1) t} G_1(t) \]  

(7.2.85)-(7.2.86)

Taking the Laplace transform of equations in (7.2.85)-(7.2.86) and letting \( s \to 0 \), we get

\[ M_0^*(0) = \mu_0, \quad M_2^*(0) = \mu_2 \]  

(7.2.87)-(7.2.88)

Taking the Laplace transform of the above equations in (7.2.79)-(7.2.84) and solving them for \( A_0^*(s) \), we get

\[ A_0^*(s) = \frac{N_1(s)}{D_1(s)} \]  

(7.2.89)

where

\[ N_1(s) = M_0^*(s) [(1-q_{22}^{(5)}(s))(1-q_{10,11}^*(s)q_{11,10}^*(s)) \]

\[ - q_{24}^*(s)((q_{42}^*(s)+q_{42}^{(8)}(s)+q_{42}^{(8,9)}(s))(1-q_{10,11}^*(s)q_{11,10}^*(s)) \]

\[ + q_{4,10}^{(8)}(s)(q_{10,11}^*(s)(q_{11,2}^*(s) + q_{11,2}^{(9)}(s))) \]

\[ + M_2^*(s) [(q_{10}^*(s))(q_{12}^{(3,6,9)}(s)+q_{12}^{(3,6)}(s)) \]

\[ (1-q_{10,11}^*(s)q_{11,10}^*(s)) - q_{11,10}^{(3,6)}(s) \]

\[ (q_{10,11}^*(s)(q_{11,2}^*(s)+q_{11,2}^{(9)}(s))) \]

\[ +q_{0,2}^*(s)(1-q_{10,11}^*(s)q_{11,10}^*(s)) \]  

(7.2.90)

and

\[ D_1(s) = q_{24}^*(s)(-(q_{42}^*(s)+q_{42}^{(8)}(s)+q_{42}^{(8,9)}(s))) \]

\[ [1+q_{01}^*(s)(-q_{10}^*(s)-q_{10}^{(3)}(s)-q_{10}^{(3,7)}(s))] \]

\[ + [1-q_{22}^{(5)}(s) - q_{01}^*(s)((1-q_{22}^{(5)}(s))(q_{10}^*(s)+q_{10}^{(3)}(s) \]

\[ + q_{10}^{(3,7)}(s)) + q_{20}^*(s)(q_{12}^{(3,6,9)}(s)+q_{12}^{(3,6)}(s))] + q_{0,2}^*(s)q_{20}^*(s) \]

\[ + q_{10,11}^*(s)q_{01}^*(s)q_{24}^*(s)(-q_{10}^*(s)-q_{10}^{(3)}(s)-q_{10}^{(3,7)}(s)) \]

\[ [q_{11,10}^*(s)(q_{42}^*(s)+q_{42}^{(8)}(s)+q_{42}^{(8,9)}(s))-q_{4,10}^{(8)}(s) \]
In steady state availability of the system is given by:

\[ A_0 = \lim_{s \to 0} (s A_0^\ast(s)) = N_1/D_1 \]  (7.2.92)

where

\[ N_1 = (1-p_{11,10})[p_{20} \mu_0 + \mu_2 \{p_{01}(p_{12}^{(3,6)} + p_{12}^{(3,6,9)} + p_{110}^{(3,6)}) + p_{02}\}] \]  (7.2.93)

and

\[ D_1 = p_{20}(1-p_{11,10}) (\mu_0 + p_{01} \epsilon_1) \]
\[ + (1-p_{11,10})[1+p_{01}(p_{12}^{(3,6)} + p_{12}^{(3,6,9)} + p_{110}^{(3,6)})] [\epsilon_2+ \epsilon_4 \ p_{24}] \]
\[ + (1-p_{11,10})[\epsilon_2 - p_{24} \epsilon_3] \]
\[ + [p_{02} + p_{01}(p_{12}^{(3,6)} + p_{12}^{(3,6,9)} + p_{110}^{(3,6)})] p_{24} p_{41}^{(8)}(\epsilon_{10}+ \epsilon_{11}) \]
\[ + p_{01}p_{20} p_{110}^{(3,6)} [ \epsilon_{10}- \epsilon_{11}] \]  (7.2.94)

**Busy Period Analysis of Repairman**

By probabilistic arguments we have the following recursive relations:

\[ B_0(t) = q_{01}(t) \circ B_1(t) + q_{02}(t) \circ B_2(t) \]
\[ B_1(t) = W_1(t) + q_{10}(t) \circ B_0(t) + q_{10}^{(3)}(t) \circ B_0(t) \]
\[ + q_{12}^{(3,6)}(t) \circ B_2(t) + q_{12}^{(3,6,9)}(t) \circ B_2(t) + q_{110}^{(3,6)}(t) \circ B_{10}(t) \]
\[ B_2(t) = W_2(t) + q_{20}(t) \circ B_0(t) + q_{24}(t) \circ B_4(t) + q_{22}^{(5)}(t) \circ B_2(t) \]
\[ B_4(t) = W_4(t) + q_{42}(t) \circ B_2(t) + q_{42}^{(8)}(t) \circ B_2(t) + q_{41}^{(8)}(t) \circ B_{10}(t) \]
\[ + q_{42}^{(8,9)}(t) \circ B_2(t) \]
\[ B_{10}(t) = W_{10}(t) + q_{10,11}(t) \circ B_{11}(t) \]
\[ B_{11}(t) = W_{11}(t) + q_{11,2}(t) \circ B_2(t) + q_{11,2}^{(9)}(t) \circ B_2(t) + q_{11,10}(t) \circ B_{10}(t) \]  (7.2.95)-(7.2.100)
where \( W_1(t) = e^{-b_1 t} g(t) + (b_1 e^{-b_1 t} \otimes e^{-(b_2 + \lambda_1) t} g(t) \\
+ (b_1 e^{-b_1 t} \otimes b_2 e^{-(b_2 + \lambda_1) t} \otimes 1) g(t) \\
+ (b_1 e^{-b_1 t} \otimes \lambda_1 e^{-(b_2 + \lambda_1) t} \otimes e^{-(b_2 + \lambda_1) t} g(t) \\
+ (b_1 e^{-b_1 t} \otimes \lambda_1 e^{-(b_2 + \lambda_1) t} \otimes b_2 e^{-(b_2 + \lambda_1) t} \otimes 1) g(t) \\
W_2(t) = e^{-(\lambda + \lambda_1) t} \tilde{g}(t) + (\lambda_1 e^{-(\lambda + \lambda_1) t} \otimes 1) \tilde{g}(t) \\
W_4(t) = e^{-b_1 t} \tilde{g}(t) + (b_1 e^{-b_1 t} \otimes e^{-(b_2 + \lambda_1) t} \tilde{g}(t) \\
+ (b_1 e^{-b_1 t} \otimes b_2 e^{-(b_2 + \lambda_1) t} \otimes 1) \tilde{g}(t) \\
W_{10}(t) = \tilde{g}(t) \\
W_{11}(t) = e^{-(b_2 + \lambda_1) t} \tilde{g}(t) + (b_2 e^{-(b_2 + \lambda_1) t} \otimes 1) \tilde{g}(t) \quad (7.2.101)-(7.2.105)

Taking L.T. of above equations in (7.2.101)-(7.2.105) and letting \( s \to 0 \), we have

\( W_1^*(0) = \varepsilon_1 \) (already specified)

\( W_2^*(0) = \varepsilon_2 \) (already specified)

\( W_4^*(0) = \varepsilon_4 \) (already specified)

\( W_{10}^*(t) = K_1 \) (already specified)

\( W_{11}^*(t) = \varepsilon_{11} \) (already specified) \quad (7.2.106)-(7.2.110)

Now taking L.T. of equations in (7.2.95)-(7.2.100) and solving them for \( B_0^*(s) \), we have

\( B_0^*(s) = \frac{N_2(s)}{D_1(s)} \) \quad (7.2.111)

where

\( N_2(s) = W_1^*(s)q_{01}^*(s)((1-q_{22}^{(5)})(1-q_{10,11}^*(s))q_{11,10}^*(s) \\
- q_{24}^{(8)}(s)(q_{42}^*(s) + q_{42}^{(8)}(s) + q_{42}^{(8,9)}(s))(1-q_{10,11}^*(s))q_{11,10}^*(s) \\
+ q_{4,10}^{(8)}(s)(q_{10,11}^*(s)(q_{11,2}^*(s) + q_{11,2}^{(9)}(s)))]) \)
\[ + W_2(s)[q_{01}(s)((q_{12}(3,6,9)(s) + q_{12}(3,6)(s))(1 - q_{10,11}(s)q_{11,10}(s)) - q_{11,10}(3,6)(s)(q_{10,11}(s)q_{11,10}(s)q_{11,10}(s)) + q_{02}(s)(1 - q_{10,11}(s)q_{11,10}(s))] \]
\[ + W_4(s)q_{01}(s)q_{24}(s)((q_{12}(3,6,9)(s) + q_{12}(3,6)(s))(1 - q_{10,11}(s)q_{11,10}(s)) - q_{12}(3,6)(s)(q_{10,11}(s)q_{11,10}(s)) + W_4(s)q_{02}(s)q_{24}(s) \]
\[ (1 - q_{10,11}(s)q_{11,10}(s)) + W_4(s)q_{01}(s)q_{24}(s)q_{410}(3)(s) \]
\[ (q_{12}(3,6,9)(s) + q_{12}(3,6)(s))(1 - q_{11,10}(3,6)(s))(1 - q_{10,11}(3,6)(s)) \]
\[ - q_{24}(s)(q_{42}(s) + q_{42}(3)(s) + q_{42}(8,9)(s)) \]
\[ + W_4(s)q_{02}(s)q_{24}(s)q_{410}(3)(s) + W_4(s)q_{10}(s)q_{11,10}(s)q_{01}(s) \]
\[ [(q_{12}(3,6,9)(s) + q_{12}(3,6)(s))q_{24}(s)q_{410}(3)(s) + q_{11,10}(3,6)(s) \]
\[ (1 - q_{12}(5)(s)) - q_{24}(s)(q_{42}(s) + q_{42}(3)(s) + q_{42}(8,9)(s)) \]
\[ + W_4(s)q_{10}(s)q_{11,10}(s)q_{02}(s)q_{24}(s)q_{410}(3)(s) \]
\[ \text{(7.2.112)} \]

and \( D_1(s) \) is already specified in equation (7.2.94).

In steady state, the total fraction of the time for which the system is under repair is given by

\[ B_0 = \lim_{s \to 0} (s B_0*(s)) = \frac{N_2}{D_1} \]
\[ \text{(7.2.113)} \]

where

\[ N_2 = p_{01}p_{20} [\epsilon_1(1 - p_{11,10}) + p_{11,10}(3,6)(K_1 + \epsilon_{11})] \]
\[ + [p_{02} + p_{01}(p_{12}(3,6) + p_{12}(3,6,9) + p_{11,10}(3,6))] \]
\[ [(1 - p_{11,10})(\epsilon_2 + p_{24} \epsilon_4) + p_{24}p_{410}(3)(K_1 + \epsilon_{11})] \]
\[ \text{(7.2.114)} \]

and \( D_1 \) is already specified in equation (7.2.94)

**Expected Number of Visits by Repairman**

By probabilistic arguments we have the following recursive relations:

\[ V_0(t) = Q_{01}(t) [1 + V_1(t)] + Q_{02}(t) [1 + V_2(t)] \]
\[ V_3(t) = Q_{10}(t) V_0(t) + Q_{10}(3)(t) V_0(t) + Q_{10}(3,7)(t) V_0(t) \]
Taking L.S.T. of equations in (7.2.115)-(7.2.120) and solving them for $V_0^{**}(s)$, we have

$$V_0^{**}(s) = \frac{N_4(s)}{D_1(s)} \quad (7.2.121)$$

where

$$N_4(s) = (q_{01}^{*}(s) + q_{02}^{*}(s))(1 - q_{22}^{(5)}(s))(1 - q_{10,11}^{*}(s))q_{11,10}^{*}(s))$$

$$- (q_{01}^{*}(s) + q_{02}^{*}(s))q_{24}^{*}(s)[(q_{42}^{*}(s) + q_{42}^{(8)}(s)) + q_{42}^{(8,9)}(s)]$$

$$(1 - q_{10,11}^{*}(s)q_{11,10}^{*}(s)) + q_{41,10}^{(8)}(s)(q_{11,2}^{*}(s) + q_{11,2}^{(9)}(s))q_{10,11}^{*}(s))$$

and $D_1(s)$ is already specified in equation (7.2.91).

In steady state, the number of visits per unit time is given by

$$V_0 = \lim_{t \to \infty} \frac{[V_0(t)]}{t} = \lim_{s \to 0} [s \cdot V_0^{**}(s)] = \frac{N_4}{D_1} \quad (7.2.123)$$

where

$$N_4 = (1 - p_{11,10})p_{20} \quad (7.2.124)$$

and $D_1$ is already specified in equation (7.2.94)

**Cost-Benefit Analysis**

The expected total profit incurred to the system in steady-state is given by

$$P_2 = C_0A_0 - C_1B_0 - C_2V_0 \quad (7.2.125)$$

where
$C_0 = $ revenue per unit up time of the system
$C_1 = $ cost per unit time for which the repairman is busy
$C_2 = $ cost per unit visit of the repairman.

**Particular Case**

For graphical representation, the following particular cases is considered. Let us assume that $g(t) = \alpha e^{-\alpha t}$, $g_1(t) = \beta e^{-\beta t}$ and the remaining distributions are the same as in the general case. Therefore, we have

\[
\begin{align*}
p_{01} &= \lambda/(\lambda + \lambda_1), \quad p_{02} = \lambda_1/(\lambda + \lambda_1), \quad p_{10} = \alpha/(b_1 + \alpha), \\
p_{10}^{(3)} &= (\alpha b_1)/((b_1 + \alpha)(b_2 + \lambda_1 + \alpha)), \quad p_{10}^{(3,7)} = (b_1 b_2)/((b_1 + \alpha)(b_2 + \lambda_1 + \alpha)) \\
p_{12}^{(3,6)} &= (\alpha b_1 \lambda_1)/((b_1 + \alpha)(b_2 + \lambda_1 + \alpha)^2) \\
p_{10}^{(3,6)} &= (b_1 \lambda_1^2)/((b_1 + \alpha)(b_2 + \lambda_1 + \alpha)^2) \\
p_{12}^{(3,6,9)} &= (b_1 b_2 \lambda_1)/((b_1 + \alpha)(b_2 + \lambda_1 + \alpha)^2) \\
p_{20} &= \alpha/(\alpha + \lambda + \lambda_1), \quad p_{24} = \lambda/(\alpha + \lambda + \lambda_1), \quad p_{25} = \lambda_1/(\alpha + \lambda + \lambda_1) \\
p_{22}^{(5)} &= \lambda_1/(\lambda_1 + \lambda + \alpha), \quad p_{42} = \alpha/(b_1 + \alpha), \quad p_{410}^{(8)} = (b_1 \lambda_1)/((b_1 + \alpha)(b_2 + \lambda_1 + \alpha)) \\
p_{42}^{(8)} &= (\alpha b_1)/((b_2 + \lambda_1 + \alpha)(b_1 + \alpha)) \\
p_{42}^{(8,9)} &= (b_1 b_2)/((b_2 + \lambda_1 + \alpha)(b_1 + \alpha)) \\
p_{10,11} &= 1, \quad p_{11,2} = \alpha/(b_2 + \lambda_1 + \alpha) \\
p_{11,2}^{(9)} &= b_2/(b_2 + \lambda_1 + \alpha), \quad p_{11,10} = \lambda_1/(\lambda_1 + b_2 + \alpha) \\
\mu_0 &= 1/(\lambda + \lambda_1), \quad \mu_1 = 1/(\alpha + b_1), \quad \mu_2 = 1/(\alpha + \lambda + \lambda_1) \\
\mu_4 &= 1/(\alpha + b_1), \quad \mu_{10} = K_1, \quad \mu_{11} = 1/(b_2 + \lambda_1 + \alpha) \\
K_1 &= 1/\beta, \quad K = 1/\alpha \quad (7.2.126)-(7.2.153)
\end{align*}
\]

Using the above equations in (7.2.126) - (7.2.153), (7.2.76), (7.2.92), (7.2.113) and (7.2.123), we get the expressions for MTSF and profit for this particular case.
Failure Rate $A$ vs MTSF

Fig. 7.6

(Variation in Failure Rate $A$)

Failure Rate $A$

$A = 0.01$ $A = 0.03$

MTSF

(15 25 35 45 55)

$\lambda = 0.3$, $\lambda = 0.4$, $\lambda = 0.5$
Figure 7.7

Profit (P_a) vs. \( b \)

(Variation in \( \alpha \) of \( b^\alpha \))

\( \alpha = 1, \gamma = 1, \lambda = 0.8, \lambda = 0.9, C_i = 500, C_i = 150, C_i = 75 \)
Failure Rate

Variation in Failure Rate

Failure Rate (A) vs Profit (P* )

Fig. 7.8

(\text{Variation in Failure Rate})
Fig. 7.9

Cost per visit (C₉) vs Variation in Failure Rate (W)

Cost per visit (C₉) vs Diff. of Profits (ΔP)

(ΔP) = C₉ - C₀

Cost per visit (C₉) vs Diff. of Profits (ΔP)
Graphical Interpretation

Figs. 7.6 and 7.8 show the relationship between MTSF w.r.t. failure rate ($\lambda_1$) and profit ($P_2$) w.r.t. failure rate ($\lambda_1$) respectively. It is clear from these figures that as the failure rate increases, MTSF as well as profit ($P_2$) decreases. Here variation is taken in failure rate ($\lambda$) which is $\lambda = 0.01$, $\lambda = 0.02$ and $\lambda = 0.03$ for MTSF and $\lambda = 2$, $\lambda = 2.1$ and $\lambda = 2.2$ for profit and as it increases there is decrease in MTSF as well as in the profit.

Fig. 7.7 shows the behaviour of profit ($P_2$) w.r.t. rate ($b_1$) (where $b_1$ is the constant rate of sub-units to become operative from rest). It is obvious from the figure that as the rate ($b_1$) increases, profit ($P_2$) decreases. Also as variation is taken in the rate ($b_2$) (where $b_2$ is the constant rate of sub-units to come to rest from operating state) which is $b_2 = 0.1$, $b_2 = 0.5$ and $b_2 = 1$ in this case and as it increases, profit ($P_2$) decreases.

Comparative Study of Profits for Model 1 and Model 2

From Figs. 7.9 and 7.10 we interpret that as the cost per visit of repairman ($C_2$) and rate ($b_1$) increases difference of profits ($P_2 - P_1$) also increases. Following conclusions can be drawn:

(i) If value of $C_2$ and $b_1$ are such that $(P_2 - P_1) > 0$, then model 2 is better than model 1.

(ii) If value of $C_2$ and $b_1$ are such that $(P_2 - P_1) = 0$, then model 1 and model 2 are equally good.

(iii) If value of $C_2$ and $b_1$ are such that $(P_2 - P_1) < 0$, then model 1 is better than model 2.