CHAPTER II
GENERALISED MODELS FOR RUMOUR
WITH REALISATION PERIOD

2.1 INTRODUCTION

The diffusion of rumour is a well studied social phenomenon as the review of literature presented in chapter I suggests. Most of the studies have been performed with only two categories of individuals - ignorant and spreader - Daley and Kendall (1965), Gray and Broembsen (1974), Bartholomew (1976a) and Karmeshu (1980b,c). Sharma et al (1983) improved upon by taking up the third category of individuals - stifler. But as the review in the preceding chapter reveals, it does not provide the general solution of probability generating function and probability that transmission does not start upto time ‘T’ in stochastic model of the phenomenon. The study howsoever improving also lacked the detailed presentation of deterministic model. These gaps in the evolution of the models are bridged in the present study.

The population under study is homogeneous and closed consisting of some finite units, say (n+1). It consists of three categories - ignorant, knower and spreader. The difference between the ‘knower’ and ‘stifler’ is that the ‘knower’ becomes spreader after the realisation of rumour, which inevitably takes place; whereas the ‘stifler’ may become a spreader or may stifle the rumour. Information or rumour is transmitted to the ignorants through contacts with the spreader. The ignorant is an individual who does not know the rumour and the spreader is an individual who is responsible for the diffusion of rumour. The ignorant who receives the rumour becomes knower, and after realising, the knower becomes spreader. The process of diffusion continues until all have heard the rumour, i.e., until the transmission ceases.
2.2 STOCHASTIC MODEL

Suppose at time \( t=0 \), we have a closed homogeneous mixed population of \((n+1)\) individuals and assume for simplicity that the process of diffusion starts at time \( t=0 \) with 'one' spreader and 'n' ignorants.

The model is studied with the help of the following assumptions:

(i) Throughout the process, the total size of the population remains unchanged, i.e., no immigration or emigration is possible. If \( X(t) \), \( Y(t) \) and \( Z(t) \) denote respectively the number of ignorants, knowers and spreaders at time \( t \), then \( X(t) + Y(t) + Z(t) = n+1 \).

(ii) Initially by our supposition, we have \( X(0) = n, Y(0) = 0, Z(0) = 1 \).

(iii) Once an individual becomes spreader, he remains spreader till the transmission ceases.

(iv) The chance of occurrence of a new contact of 'ignorant' with 'spreader' is proportional to both the number of ignorants and the number of spreaders. It '\( \alpha \)' is the contact rate, then the chance of contact between a particular ignorant and a particular spreader in time \( '\Delta t' \) is \( \alpha X(t)Z(t)\Delta t + o(\Delta t) \). It follows that the probability that \( 'Y(t)' \) gets an increment of 'one unit' in time \( '\Delta t' \) is \( \alpha X(t)Z(t)\Delta t + o(\Delta t) \).

(v) The probability of addition of one unit to \( 'Z(t)' \) in time \( '\Delta t' \) can be taken to be \( \beta Y(t)Z(t)\Delta t + o(\Delta t) \), where '\( \beta \)' is the inner-development rate of knowers due to contact with spreaders.

(vi) We now write \( P_{x,y,z}(t) \) for the probability that there are 'x' ignorants, 'y' knowers and 'z' spreaders at time 't', i.e.,

\[
P_{x,y,z}(t) = \text{Prob.} \left[ X(t) = x, Y(t) = y, Z(t) = z \right].
\]
The process is governed by the following equations:

\[ P_{x,z}(t+\Delta t) = \left[ 1 - \{ \alpha xz + \beta (n+1-x-z)z \} \Delta t \right] P_{x,z}(t) + \right. \\
\left. + \alpha (x+1) z \Delta t P_{x+1,z}(t) + \beta (n+2-x-z)(z-1) \Delta t P_{x,z-1}(t) \right. \\
(0 \leq x+y \leq n, 0 \leq x \leq n, 0 \leq y \leq n, 1 \leq z \leq n+1) \quad (2.1) \\

and

\[ P_{n,1}(t+\Delta t) = (1-\alpha n \Delta t) P_{n,1}(t) \quad (2.2) \]

These give the following differential-difference equations:

\[ P'_{x,z}(T) = -[xz+\lambda (n+1-x-z)z] P_{x,z}(T) +(x+1)z P_{x+1,z}(T) + \right. \\
\left. + \lambda (n+2-x-z)(z-1) P_{x,z-1}(T) \right. \\
(0 \leq x+y \leq n, 0 \leq x \leq n, 0 \leq y \leq n, 1 \leq z \leq n+1) \quad (2.3) \\

and

\[ P'_{n,1}(T) = -n P_{n,1}(T) \quad (2.4) \]

where \( T = \alpha t \), \( \lambda = \beta / \alpha \)

and the initial condition being \( P_{n,1}(0) = 1 \).

By using Laplace-transform, eq. (2.3) and eq. (2.4) result in

\[ P^*_{x,z}(s) = \frac{(x+1)z P^*_{x+1,z}(s) + \lambda (n+2-x-z)(z-1) P^*_{x,z-1}(s)}{s+xz+\lambda (n+1-x-z)z} \]
\[ (0 \leq x+y \leq n, 0 \leq x \leq n, 0 \leq y \leq n, 1 \leq z \leq n+1) \quad (2.5) \]
and

\[ P^*_{x-1,1}(s) = \frac{1}{s+n} \]  

(2.6)

where \( P^*_{x,a}(s) \) denote the laplace-transform of \( P_{x,a}(T) \).

Now we calculate the probability that up to time "T", no ignorant has become spreader, i.e., the transmission does not start by taking different values of ignorants and knowers by keeping spreaders fixed, i.e., by keeping \( z=1 \) and \( x+y=n \).

Put \( x=n-1 \), \( y=1 \) and \( z=1 \) in eq. (2.5), we get

\[ P^*_{n-1,1}(s) = \frac{nP^*_{n-1,1}(s)}{s+n + (\lambda - 1)} \]

\[ = \frac{n}{[s+n][s+n + (\lambda - 1)]} \]  

(2.7)

Put \( x=n-2 \), \( y=2 \) and \( z=1 \) in eq. (2.5), we obtain

\[ P^*_{n-2,1}(s) = \frac{(n-1)P^*_{n-1,1}(s)}{s+n + 2(\lambda - 1)} \]

\[ = \frac{n(n-1)}{[s+n][s+n + (\lambda - 1)][s+n + 2(\lambda - 1)]} \]  

(2.8)

Applying the same procedure and solving recursively, we find
\[ P_{n-r,1}(s) = \frac{n(n-1)(n-2) \ldots (n-r+1)}{[s+n][s+n+(\lambda - 1)] [s+n+2(\lambda - 1)] \ldots [s+n+r(\lambda - 1)]} \]

\[(1 \leq r \leq n) \quad (2.9)\]

After taking laplace inverse of eq. (2.7), we have

\[ P_{n-1,1}(T) = L^{-1}\{P_{n-1,1}(s)\} \]

\[ = nL^{-1}\left[ \frac{1}{(s+n)(s+n+\lambda-1)} \right] \]

Applying laplace convolution theorem, we get

\[ P_{n-1,1}(T) = n \int_0^T e^{-(s+n+\lambda-1)u} e^{-n(T-u)} \, du \]

\[ = n e^{-nT} \int_0^T e^{-(\lambda-1)u} \, du \]

\[ = \frac{n}{\lambda-1} [e^{-nT} - e^{-(s+n+\lambda-1)T}] \quad (2.10) \]

From eq. (2.8), after taking laplace inverse, we obtain

\[ P_{n-2,1}(T) = L^{-1}\{P_{n-2,1}(s)\} \]

\[ = n(n-1)L^{-1}\left[ \frac{1}{[s+n][s+n+(\lambda - 1)] [s+n+2(\lambda - 1)]} \right] \]

By using eq. (2.10) and applying laplace convolution theorem, it gives
Applying the same procedure and after taking inversion of eq. (2.9), we obtain

\[ P_{n-2,1}(T) = \frac{n(n-1)}{(\lambda - 1)} \int_0^T [e^{-\lambda u} - e^{-e^{-(n+\lambda-1)u}}][e^{-\lambda u - (\lambda - 1)(T-u)}]du \]

\[ = \frac{n(n-1)}{(\lambda - 1)} e^{-\lambda T} \int_0^T [e^{\lambda u} - e^{-(\lambda - 1)u}]du \]

\[ = \frac{n(n-1)}{(\lambda - 1)^2} [e^{-\lambda T} - 2e^{-\lambda u + (\lambda - 1)T} + e^{-\lambda u + 2(\lambda - 1)T}] \]

\[ = \frac{n(n-1)}{(\lambda - 1)^2} \sum_{m=0}^{2} (-1)^m \sum_{r=0}^{m} C_r e^{-(n+m\lambda-m)T} \]

(2.11)

After inversion eq. (2.6) results in

\[ P_{n-r,1}(T) = \frac{n(n-1)(n-2)\ldots(n-r+1)}{(\lambda - 1)^r} \sum_{m=0}^{r} (-1)^m C_m e^{-(n+m\lambda-m)T} \]

(1 ≤ r ≤ n) \hspace{1cm} (2.12)

After inversion eq. (2.6) results in

\[ P_{r,1}(T) = e^{-\alpha T} \]

(2.13)

The probability that no ignorant has become spreader, i.e., the transmission does not start up to time "T" is given by

\[ P(T) = \sum_{r=0}^{n} P_{n-r,1}(T) \]

(2.14)
THE PROBABILITY-GENERATING FUNCTION

To obtain the probability-generating function, we derive a partial differential equation for $P(s_1, s_2, T)$. Let us define the joint probability-generating function for the state probabilities by

$$P(s_1, s_2, T) = \sum_{x, z} p_{x, z}(T) s_1^x s_2^z$$

(2.15)

It implies

$$\frac{\partial P(s_1, s_2, T)}{\partial T} = \sum_{x, z} s_1^x s_2^z \frac{\partial p_{x, z}(T)}{\partial T}$$

(2.16)

Multiplying eq. (2.3) by $s_1^x s_2^z$ and taking sum over all ‘x’ and ‘z’ then using eq. (2.16), we find

$$\frac{\partial P(s_1, s_2, T)}{\partial T} = \sum_{x, z} [-xz - \lambda(n + 1 - x - z)z] s_1^x s_2^z p_{x, z}(T) + \sum_{x, z} (x + 1)z s_1^x s_2^{z+1} p_{x, z+1}(T) + \sum_{x, z} \lambda(n + 2 - x - z)(z - 1) s_1^x s_2^z p_{x, z-1}(T)$$

(2.17)

After simplification of eq. (2.17), we obtain

$$\frac{\partial P(s_1, s_2, T)}{\partial T} = (-s_1 s_2 + \lambda s_1 s_2 + s_2 - \lambda s_1 s_2^z) \frac{\partial^2 P(s_1, s_2, T)}{\partial s_1 \partial s_2} +$$
\[ +(-n \lambda s_2 + n \lambda s_1^2) \frac{\partial P(s_1, s_2, T)}{\partial s_2} \]

\[ + (\lambda s_2^2 - \lambda s_1^2) \frac{\partial^2 P(s_1, s_2, T)}{\partial s_2^2} \]

(2.18)

It is the partial differential equation subject to the initial condition

\[ P(s_1, s_2, 0) = s_1^n s_2 \]

(2.19)

We solve eq. (2.18) by using the technique of separation of variables.

Let

\[ P(s_1, s_2, T) = S_1(s_1)S_2(s_2)T'(T) \]

(2.20)

where \( S_1, S_2 \) and \( T' \) are functions of \( s_1, s_2 \) and \( T \) only.

From eq. (2.20), we have

\[ \frac{\partial P(s_1, s_2, T)}{\partial s_1} = S_1'(s_1)S_2(s_2)T'(T) = S_1's_2T' \]

(2.21)

\[ \frac{\partial^2 P(s_1, s_2, T)}{\partial s_1 \partial s_2} = S_1's_2' T' \]

(2.22)

\[ \frac{\partial P(s_1, s_2, T)}{\partial s_2} = S_1s_2'T' \]

(2.23)
\[
\frac{\partial^2 P(s_1, s_2, T)}{\partial s_2^2} = S_1 S_2'' T''
\]  

(2.24)

\[
\frac{\partial P(s_1, s_2, T)}{\partial T} = S_1 S_2 T''
\]  

(2.25)

By using eq. (2.22) to eq. (2.25) in the eq. (2.18), we have

\[
S_1 S_2 T'' = (-s_1 s_2 + \lambda s_1 s_2 + s_2 - \lambda s_1 s_2)S_1' S_2' T'
\]

\[+ (-n \lambda s_2 + n \lambda s_2^2)S_1 S_2'' T'
\]

\[+ (\lambda s_2^2 - \lambda s_2^3)S_1 S_2'' T'
\]

(2.26)

After dividing both sides of eq. (2.26) by \(S_1 S_2 T''\) and neglecting terms of \(s_2^2\) and higher order because initially the number of spreaders are very small, it implies

\[
\frac{T''}{T'} = (-s_1 s_2 + \lambda s_1 s_2 + s_2)\frac{S_1' S_2'}{S_1 S_2} - n \lambda s_2^2 \frac{S_2'}{S_2} = -k, \quad \text{say (2.27)}
\]

where \(k\) is a suitable constant.

From the first and third elements of eq. (2.27), we find

\[
T' = e^{kT}
\]

(2.28)

After rearranging the second and third elements of eq. (2.27), we get

\[
(-s_1 + \lambda s_1 + 1)\frac{S_1'}{S_1} = n \lambda - \frac{k S_2}{S_2 S_2}' = -1, \quad \text{say (2.29)}
\]
where ‘I’ is some suitable constant.

From eq. (2.29), after simplification we immediately obtain

\[ S_i \propto (1 - s_i + \lambda s_i)^{\frac{1}{1 - \lambda}} \]  

(2.30)

and

\[ S_2 \propto \frac{T}{1 + e^T} \]  

(2.31)

Now we know that \( P(s_1, s_2, T) \) must be a polynomial of degree ‘n’ in ‘s_1’ and ‘n+1’ in ‘s_2’. So \( \frac{i}{1 - \lambda} \) must be a non-negative integer ‘g’ (say) in the range \( 0 \leq g \leq n \) and \( \frac{1}{1 + \lambda} \) must be a non-negative integer ‘h’ (say) in the range \( 1 \leq h \leq n+1 \). The permissible eigenvalues are accordingly

\[ k_g = h(1+n\lambda) \quad (0 \leq g \leq n) \]  

(2.32)

So the general solution of eq. (2.18) can thus be written as

\[ P(s_1, s_2, T) = \sum_{g=0}^{n} \sum_{h=1}^{n+1} c_{gh} e^{-h(1+n\lambda)T} (1 - s_1 + \lambda s_1)^{g-1} s_2^h \]  

(2.33)

where the constants ‘\( c_{gh} \)’ can be determined from the initial condition given by eq. (2.19) and is accordingly given by

\[ c_{gh} = \frac{c_g}{(\lambda - 1)^h s^h C_1} \]  

(2.34)
The individual probabilities ‘\( P_{XZ}(T) \)’ can now be obtained from eq. (2.33) by picking out the coefficients of \( s_1s_2' \).

### 2.3 INTERPRETATIONS: \( P(T) \)

Using eq. (2.14), we have computed the values of the probabilities ‘\( P(T) \)’ for various values of ‘\( \lambda \)’ when the population size \( n=5,10 \). The results given in Tabs. 2.1 and 2.2 have been graphed in Figs. 2.1 and 2.2. The following observations have been drawn:

(i) For a fixed ‘\( \lambda \)’, the probability \( 1-P(T) \) of occurrence of new transmissions increases with time.

(ii) As ‘\( \lambda \)’ increases, \( 1-P(T) \) also increases with time.

(iii) As expected the probability of occurrence of new transmission is greater when the population size is ‘10’ as compared to the case when it is ‘5’. This result also holds for any two values of the population size.

Interestingly the above conclusions are the same as anticipated and thus the validity of the model is confirmed.

### 2.4 DETERMINISTIC MODEL

To deal with the deterministic model, we consider that \( \alpha x(t)z(t)\Delta t \) and \( \beta y(t)z(t)\Delta t \) are the exact values of transitions in the small interval ‘\( \Delta t \)’ of time.

We get the following equations which govern the system:

\[
x(t+\Delta t) = x(t) - \alpha xz\Delta t \quad (2.35)
\]

\[
y(t+\Delta t) = y(t) + \alpha xz\Delta t - \beta yz\Delta t \quad (2.36)
\]
\[ z(t+\Delta t) = z(t) + \beta yz \Delta t \quad (2.37) \]

with initial conditions \( x(0) = n-1, y(0)=0, z(0)=1 \)

Equations (2.35) to (2.37) give the following differential equations those govern the system:

\[ \frac{dx}{dt} = -\alpha xz \quad (2.38) \]

\[ \frac{dy}{dt} = \alpha xz - \beta yz \quad (2.39) \]

\[ \frac{dz}{dt} = \beta yz \quad (2.40) \]

Dividing eq. (2.40) by eq. (2.38), we get

\[ \frac{dz}{dx} = -\frac{y}{\rho x} \quad (2.41) \]

where

\[ \rho = \frac{\alpha}{\beta} \quad (2.42) \]

We know that

\[ x+y+z = n \]

It implies
\[ \frac{dx}{ds} = -(1 + \frac{dy}{ds}) \] (2.43)

By comparing eq. (2.41) and (2.43), we find

\[ \frac{dy}{dx} = \frac{y - \rho x}{\rho x} \]

It gives

\[ \rho x = (y - \rho x) \frac{dx}{dy} \] (2.44)

Put \( x = vy \) (2.45)

\[ \Rightarrow \frac{dx}{dv} = v + y \frac{dx}{dy} \] (2.46)

The eq. (2.44) after using eq. (2.45) and eq. (2.46) can be written as

\[ \rho vy = (y - \rho vy) \left( v + y \frac{dv}{dy} \right) \]

It gives

\[ \frac{dy}{y} = \frac{1 - \rho v}{v(\rho + \rho v - 1)} \, dv \] (2.47)

After using method of partial fractions and integration, the eq. (2.47) results in
\[
\log y = \frac{1}{\rho - 1} \left[ \log \left( \frac{v}{\rho + \rho v - 1} \right) \right] - \log(\rho + \rho v - 1) + \log C
\]

(2.48)

Replace \( v = \frac{x}{y} \), it gives

\[
\frac{1}{\rho - 1} \log x + \log C = \frac{\rho}{\rho - 1} \log(\rho y + \rho x - y)
\]

(2.49)

The constant 'log C' can be found by applying initial condition \( x = n-1, \ y=0 \) and is accordingly given by

\[
\log C = \log \left[ \frac{e^\rho}{\rho^\rho (n - 1)} \right]
\]

(2.50)

The eq. (2.49) after simplification and putting the value of 'log C' from eq. (2.50) takes the form

\[
y = \frac{1}{\rho - 1} \left[ \frac{1}{x^A} - \rho x \right]
\]

(2.51)

where

\[
A = \rho(n - 1)^{\frac{\rho - 1}{\rho}}
\]

(2.52)

Put the value of 'y' from eq. (2.51) in eq. (2.38), we have
\[
\frac{dx}{dt} = -\alpha x \left[ n - x - \frac{1}{\rho - 1} \left( \frac{1}{x} A - px \right) \right]
\]

\[
= Bx + Cx^2 + Dx^{\frac{1}{\rho - 1}}
\]

(2.53)

where

\[
B = -n \alpha
\]

\[
C = -\frac{\alpha}{\rho - 1}
\]

\[
D = \frac{A \alpha}{\rho - 1}
\]

(2.54)

After solving eq. (2.53) by Picard Method, the approximate solution for 'x(t)' is given by

\[
x(t) = (n - 1) \left( 1 + \alpha t + \frac{\alpha^2 t^2}{2} \right) - \frac{\alpha^3}{6\rho} (n - 1)^2 t^3
\]

(2.55)

The eq. (2.51) after using eq. (2.55) produces the approximate solution for 'y(t)' and is accordingly written as

\[
y(t) = \frac{\rho(n - 1)E \left[ L_{\rho-1}^{(3)} \right] - 1}{(\rho - 1)}
\]

(2.56)

Now as we know z = n-x-y, thus by putting the values from eq. (2.55) and (2.56), we immediately obtain
The rumour curve which gives the rate \( w(t) \) at which new transmission takes place is given by

\[
\begin{align*}
  w(t) &= \frac{dz}{dt} \\
  &= \frac{\beta p(n-1)}{(p-1)} \left[ nE^{\frac{1}{p-1}} - 1 \right] + \frac{(n-1)}{(p-1)} E^{\frac{p-2}{p-1}} - 1 \left[ 1 - \rho E^{\frac{1}{p-1}} \right] \\
  &= \frac{\beta p(n-1)}{(p-1)} \left[ nE^{\frac{1}{p-1}} - 1 \right] + \frac{(n-1)}{(p-1)} E^{\frac{p-2}{p-1}} - 1 \left[ 1 - \rho E^{\frac{1}{p-1}} \right]
\end{align*}
\]  

(2.58)

where

\[
E = 1 - \alpha t + \frac{\alpha^2 t^2}{2} - \frac{\alpha^3(n-1)t^3}{6p}
\]  

(2.59)

### 2.5 INTERPRETATIONS: DETERMINISTIC MODEL

The results given in Tabs. 2.3 to 2.11 have been graphed in Figs. 2.3 to 2.11. The following observations have been drawn from these:

(i) Behaviour of \( x(t) \), \( y(t) \) and \( z(t) \) with time:

We have drawn graphs for \( x(t) \), \( y(t) \) and \( z(t) \) by giving different values to population size ‘\( n \)’, contact rate ‘\( \alpha \)’ and inner-development rate ‘\( \beta \)’. It is observed from the Figs. 2.3 to 2.6 that ‘\( x(t) \)’ continuously decreases with time, ‘\( y(t) \)’ first increases with time then after a certain stage it decreases with time and ‘\( z(t) \)’ continuously increases...
with time. These facts are as anticipated because when transmission starts in the population of ignorants with only one spreader, then first ignorants become knowers due to contact rate \( \alpha \) and so there is an increase in the number of knowers and decrease in the number of ignorants. After a certain time, knowers become spreaders due to inner-development rate \( \beta \) and so there must be decrease in the number of knowers and increase in the number of spreaders and ultimately population consists of spreaders only.

(ii) Effects on \( x(t) \), \( y(t) \) and \( z(t) \) with variations in population size \( 'n' \):

The Figs. 2.3 and 2.4 have been drawn for \( x(t) \), \( y(t) \) and \( z(t) \) taking \( \alpha = .2, \beta = .1 \) for \( n = 5, 20 \) respectively. We see from these two figures that as population size \( 'n' \) increases for fixed \( \alpha \) and \( \beta \), the duration of the process decreases. This is so because as \( 'n' \) increases then the chance of contacts between ignorants and spreaders also increases and there are more number of pairs of ignorants and spreaders. Due to this phenomenon, the ignorants become knowers with a faster rate and then knowers become spreaders in a less time due to inner-development rate \( \beta \) and the process ends at that time when there are only spreaders in the population. It may be observed from the Fig. 2.3 (\( n = 5, \alpha = .2, \beta = .1 \)) that the process ceases at time \( '5.8' \) and in Fig. 2.4 (\( n = 20, \alpha = .2, \beta = .1 \)), the process ceases at time \( '3.6' \). This fact is also true for other values of \( 'n' \), \( \alpha \) and \( \beta \) as can be observed from Fig. 2.5 and Fig. 2.6.

It is, therefore, concluded that as the population size \( 'n' \) increases with a particular pair of \( \alpha \) and \( \beta \), then the process takes less time to end.

We further observe from the Fig. 2.3 (\( n = 5, \alpha = .2, \beta = .1 \)) that \( y(t) \) increases upto time \( '4.6' \) and attains a maximum value nearly equal to \( '2.0' \) units at time \( '4.6' \) and then after, it decreases and at last become 'zero'. The Fig. 2.4 (\( n = 20, \alpha = .2, \beta = .1 \)) indicates that \( y(t) \) increases upto time \( '3.0' \) and attains a maximum value nearly equal to \( '9.5' \) units and then it decreases and becomes 'zero' after certain time. So we infer that as the population size increases for fixed \( \alpha \) and \( \beta \), then the number of knowers also increases and attain a greater value during a smaller period of interval. The above facts
also hold good for other pairs of ‘α’ and ‘β’ as can be observed from Figs. 2.5 and 2.6. These observations are obviously true because as the population size increases, then there is a faster conversion of ignorants into knowers and the maximum value of the knowers increases and is attained in a smaller interval of time.

(iii) Effects of variations of ‘α’ and ‘β’ on \( x(t) \), \( y(t) \) and \( z(t) \):

The effects of variations of ‘α’ and ‘β’ on \( x(t) \), \( y(t) \) and \( z(t) \) may be noted from the Fig. 2.7 (n=20 with different pairs of ‘α’ and ‘β’). It is observed from the Fig. 2.7 that as the contact rate ‘α’ (keeping ‘β’ fixed) increases, then ‘y(t)’ approaches to ‘zero’ in a smaller duration of time and its graph is more peaked. However, in the other case if we increase the inner-development rate ‘β’ (keeping ‘α’ fixed) then also ‘y(t)’ becomes ‘zero’ more rapidly but the graph in this case is less peaked. The reason is obvious as in the former case the conversion from ignorants to knowers is faster whereas in the later case it is from knowers to spreaders. It may be observed from the Fig. 2.7 (n=20) that ‘y(t)’ becomes ‘zero’ at time ‘4.7’ when \( α = .1, β = .2 \) whereas it becomes ‘zero’ at time ‘3.4’ when \( α = .1, β = .6 \) and it is ‘zero’ at time ‘2.1’ when \( α = .2, β = .6 \). The above fact is also true for other values of population size ‘n’. Thus we conclude that ‘y(t)’ approaches to ‘zero’ in a shorter duration as ‘α’ or ‘β’ any one increases keeping other fixed. These inferences are also applicable in the case of \( x(t) \) and \( z(t) \) i.e. ‘x(t)’ becomes zero and ‘z(t)’ takes maximum values in a smaller period.

Further from the Fig. 2.7 (n=20) it may observed that ‘y(t)’ attains a maximum value ‘4.75’ units (when \( α = .1, β = .2 \)), ‘2.213’ units (when \( α = .1, β = .6 \)) and ‘3.55’ units (when \( α = .2, β = .6 \)). We therefore deduce that by increasing ‘β’ with fixed ‘α’ and ‘n’, the maximum value of ‘y(t)’ decreases and by increasing ‘α’ with fixed ‘β’ and ‘n’, the maximum value of ‘y(t)’ increases. These conclusions seem to be true because as inner-development rate ‘β’ increases, then knowers become spreaders quickly and due to this phenomenon maximum value of knowers decreases but if contact rate ‘α’ increases.
then ignorants become knowers with a faster rate and so maximum value of knowers increases. These results which are true for \( n = 20 \) in Fig. 2.7 also hold good for other values of population size ‘\( n \)’.

(iv) Behaviour of transmission rate ‘\( w(t) \)’ with time:

Transmission curves have been drawn in Figs. 2.8 to 2.11 for different values of population size ‘\( n \)’, contact rate ‘\( \alpha \)’ and inner-development rate ‘\( \beta \)’. It is noted from the Figs. 2.8 to 2.11 that ‘\( w(t) \)’ first increases up to a certain time and after this it decreases. This fact is self true because first knowers increase and so there is a faster conversion of knowers into spreaders thereby resulting in the increase in transmission rate. Correspondingly the decrease in number of knowers after a certain time resulting in the decrease in transmission rate. Another important inference is that all the processes \( x(t) \), \( y(t) \), \( z(t) \) and \( w(t) \) terminates at the same time which justify the validity of the model.

(v) Effect on transmission rate ‘\( w(t) \)’ with variations in population size ‘\( n \)’:

The Fig. 2.8 (\( \alpha = .1, \beta = .2 \)) shows that as ‘\( n \)’, the population size increases, the transmission rate ‘\( w(t) \)’ increases and due to this increase the process ends earlier. It is due the fact that the chance of contacts between ignorants and spreaders increases with increase in ‘\( n \)’ and thus it increases ‘\( w(t) \)’ and due to increase in ‘\( w(t) \)’, the individual get the rumour in a shorter duration and hence the process ends earlier. The following table gives the maximum value and the termination time of the process ‘\( w(t) \)’ for different value of ‘\( n \)’:

<table>
<thead>
<tr>
<th>Population size ‘( n )’:</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Value of ‘( w(t) )’:</td>
<td>4.6684</td>
<td>8.3246</td>
<td>13.0564</td>
<td>18.8356</td>
<td>33.5562</td>
<td>52.4896</td>
</tr>
<tr>
<td>Termination time of ‘( w(t) )’:</td>
<td>5.2</td>
<td>4.7</td>
<td>4.4</td>
<td>4.2</td>
<td>3.8</td>
<td>3.6</td>
</tr>
</tbody>
</table>
The same behaviour can be judged from the Fig.2.9 (α=.1, β=.6).

(vi) Effects of variations of ‘α’ and ‘β’ on the behaviour of transmission rate ‘w(t)’:

The following tables give the maximum value and the termination time of the process ‘w(t)’ for different values of ‘α’ and ‘β’:

(a) Population size (n) : 20

<table>
<thead>
<tr>
<th>Values of ‘α, β’</th>
<th>Maximum value of ‘w(t)’</th>
<th>Termination time of ‘w(t)’</th>
</tr>
</thead>
<tbody>
<tr>
<td>α=.1, β=.2</td>
<td>8.3246</td>
<td>4.7</td>
</tr>
<tr>
<td>α=.1, β=.6</td>
<td>9.8624</td>
<td>3.4</td>
</tr>
<tr>
<td>α=.2, β=.6</td>
<td>18.2932</td>
<td>2.1</td>
</tr>
</tbody>
</table>

(b) Population size (n) : 50

<table>
<thead>
<tr>
<th>Values of ‘α, β’</th>
<th>Maximum value of ‘w(t)’</th>
<th>Termination time of ‘w(t)’</th>
</tr>
</thead>
<tbody>
<tr>
<td>α=.1, β=.2</td>
<td>52.4896</td>
<td>3.6</td>
</tr>
<tr>
<td>α=.1, β=.6</td>
<td>61.7491</td>
<td>2.6</td>
</tr>
<tr>
<td>α=.2, β=.6</td>
<td>114.6764</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The transmission rate ‘w(t)’ has been drawn in Fig. 2.10 (n = 20) and Fig. 2.11 (n = 50) for different values of ‘α’ and ‘β’. It is seen from above tables and the Figs. 2.10 and 2.11 that an increase in the value of ‘α’ or ‘β’ by keeping the other fixed for a particular value of the population size ‘n’ results in the increase of the maximum value of the transmission rate ‘w(t)’ and due to this, the duration of the process ‘w(t)’ becomes smaller. The above conclusions also remain same for other values of the population size ‘n’, the contact rate ‘α’ and the inner-development rate ‘β’. These results are self explanatory because when ‘α’ increases (for fixed β) or ‘β’ increases (for fixed α), then
the maximum value of \( w(t) \) also increases due to faster conversion of ignorants into knowers in the former case and knowers into spreaders in the later case. So due to increase in the maximum value of \( w(t) \), the population come to know about the rumour and information in a shorter duration of time and this is the reason for the early termination of the transmission rate.

### 2.6 MASTER EQUATION SOLUTION

To find the rate equations for the moments and steady-state solution of the phenomenon, we set up the master equation for the problem and use the method of system-size expansion of the master equation introduced by van Kampen (1961).

Now the eq. (2.3) can be written as

\[
P'_{x,z}(t) = -\alpha x z P_{x,z}(t) + \beta (n+1-x-z)z P_{x,z}(t) + \alpha (x+1)z P_{x+1,z}(t)
\]

\[+ \beta [n+1-x-(z-1)] [z-1] P_{x,z-1}(t)\]

In terms of \( y \) and \( z \), we can write it as

\[
P'_{y,z}(t) = -\alpha (n+1-y-z)z P_{y,z}(t) - \beta y z P_{y,z}(t)
\]

\[+ \alpha [n+1-(y-1)-z]z P_{y-1,z}(t) + \beta (y+1)(z-1) P_{y+1,z-1}(t)\]

where \( X(t)+Y(t)+Z(t)=n+1 \) (2.60)

Let for simplicity we write, \( X(t)+Y(t)+Z(t)=N \), where \( n+1=N \) (2.61)

Thus eq. (2.60) can now be written as

\[
P'_{y,z}(t) = -\alpha (N-y-z)z P_{y,z}(t) - \beta y z P_{y,z}(t)
\]
To write eq. (2.62) in the form of ‘master equation’, we define the difference operators ‘E’ and ‘F’, such that

\[ F^+ p_y(t) = p_{y+1}(t) \]

and

\[ E^+ p_x(z) = p_{y+1}(z) \]

With the help of using these operators, the eq. (2.62) can be written in the form of ‘Master Equation’ as follows:

\[ P'_{y,z}(t) = [F^+ -1] \alpha [N-y-z] p_{y,z}(t) + [F -1] \beta y z p_{y,z}(t) \]  

Multiply both sides by N, we get

\[ NP'_{y,z}(t) = [\{F^+ -1\} \alpha (N-y-z) + \{F -1\} \beta y z] N p_{y,z}(t) \]  

To solve this ‘master equation’, we use the method of system-size expansion of the master equation. Since the mean values of the stochastic variables y(t) and z(t) in the asymptotic regime are generally expected to be of order ‘N’ and the corresponding fluctuations to be of order \( N^2 \), thus we transform the stochastic variables y(t) and z(t) to new variables m(t) and n(t) by setting

\[ y(t) = N \psi(t) + N^{\frac{1}{2}} m(t) \]
and

\[
z(t) = N^\psi(t) + N^\phi n(t) \tag{2.66}
\]

where smooth functions \(\psi(t)\) and \(\phi(t)\) represent the time development of the ‘deterministic aspects’ of the phenomenon while \(m(t)\) and \(n(t)\) are stochastic variables governing the fluctuations in \(y(t)\) and \(z(t)\). Thus we are now interested in studying the new probability distribution \(\Pi(m,n;t)\) of the stochastic variables \(m(t)\) and \(n(t)\), where

\[
\Pi(m,n;t) = P(y,z;t) \mid J \mid
\]

where

\[
J = \begin{vmatrix}
\frac{\partial y}{\partial m} & \frac{\partial y}{\partial n} \\
\frac{\partial z}{\partial m} & \frac{\partial z}{\partial n}
\end{vmatrix} = \begin{vmatrix}
N^\psi & 0 \\
0 & N^\phi
\end{vmatrix} = N
\]

Therefore

\[
\Pi(m,n;t) = NP(y,z;t)
\]

\[
= NP[N^\psi(t) + N^\phi m(t), N^\phi(t) + N^\psi n(t); t]
\]

(2.67)

Further, we have

\[
[F^\psi - 1]P(y,z) = F^\psi P(y,z) - P(y,z)
\]

\[
= P(y\pm 1,z) - P(y,z)
\]
\begin{align*}
&= P(y,z) + (\pm 1) \frac{\partial P(y,z)}{\partial y} + \frac{(\pm 1)^2}{2} \frac{\partial^2 P(y,z)}{\partial y^2} \\
&\pm \ldots \ldots - P(y,z) \\
&= \left[ \pm \frac{\partial}{\partial y} + \frac{1}{2} \frac{\partial^2}{\partial y^2} \pm \ldots \right] P(y,z)
\end{align*}

(2.68)

But

\[
\frac{\partial P(y,z)}{\partial y} = N^{-\frac{1}{2}} \frac{\partial P(y,z)}{\partial m}
\]

and

\[
\frac{\partial^2 P(y,z)}{\partial y^2} = N^{-1} \frac{\partial^2 P(y,z)}{\partial m^2}
\]

By using these relations, the eq. (2.68) can be written as

\[
[F^{x_1} - 1] P(y,z;t) = \left[ \pm N^{-\frac{1}{2}} \frac{\partial}{\partial m} + \frac{1}{2} N^{-1} \frac{\partial^2}{\partial m^2} \pm \ldots \right] P(y,z;t)
\]

(2.69)

Similarly

\[
[E^{x_1} - 1] P(y,z;t) = \left[ \pm N^{-\frac{1}{2}} \frac{\partial}{\partial n} + \frac{1}{2} N^{-1} \frac{\partial^2}{\partial n^2} \pm \ldots \right] P(y,z;t)
\]

and

\[
[FE^{x_1} - 1] P(y,z;t) = FE^{3} P(y,z;t) - P(y,z;t)
\]
Further, we have

\[ NP(y,z,t) = \Pi(m,n,t) \]

\[
\Rightarrow N \frac{dP(y,z,t)}{dt} = \frac{\partial \Pi}{\partial m} \frac{dm}{dt} + \frac{\partial \Pi}{\partial n} \frac{dn}{dt} + \frac{\partial \Pi}{\partial t} \frac{dt}{dt}
\]

\[
= \frac{\partial \Pi}{\partial m} \left(-N^2 \frac{d\psi}{dt}\right) + \frac{\partial \Pi}{\partial n} \left(-N^2 \frac{d\phi}{dt}\right) + \frac{\partial \Pi}{\partial t}
\]

Put \( t = \frac{dr}{N} \), we get \( dt = \frac{dr}{N} \)

\[
= \frac{\partial \Pi}{\partial m} \left(-N^2 \frac{d\psi}{d\tau}\right) + \frac{\partial \Pi}{\partial n} \left(-N^2 \frac{d\phi}{d\tau}\right) + N \frac{\partial \Pi}{\partial \tau}
\]

(2.71)

Thus from eq. (2.64), after using eqs. (2.69) to (2.71), we get
\[ N \frac{\partial \Pi}{\partial \tau} - \frac{\partial \Pi}{\partial m} N^{-\frac{1}{2}} \frac{d\psi}{d\tau} - \frac{\partial \Pi}{\partial n} N^{-\frac{1}{2}} \frac{d\phi}{d\tau} = \left[ \left\{ -N^{-\frac{1}{2}} \frac{\partial}{\partial m} + 1 \right\} \frac{1}{2} N^{-1} \frac{\partial^2}{\partial m^2} \ldots \right] \left[ \alpha \left( N - N^2 \psi - N^2 m - N \phi - N^2 n \right) \left( N \phi + N^2 n \right) \right] \\
+ \left\{ N^{-\frac{3}{2}} \frac{\partial}{\partial m} - N^{-\frac{3}{2}} \frac{\partial}{\partial n} \right\} \left[ \beta \left( N \psi + N^2 m \right) \left( N \phi + N^2 n \right) \right] \\
+ \frac{1}{2} \left[ N^{-1} \frac{\partial^2}{\partial m^2} - 2 N^{-1} \frac{\partial^2}{\partial m \partial n} + N^{-1} \frac{\partial^2}{\partial n^2} \right] \left[ \beta \left( N^2 \psi \phi + N^2 \psi n + N^2 m \phi + N mn \right) \right] + \ldots ] \Pi \\
= \left[ \left\{ -N^{-\frac{1}{2}} \frac{\partial}{\partial m} + 1 \right\} \frac{1}{2} N^{-1} \frac{\partial^2}{\partial m^2} \ldots \right] \left[ \alpha \left( N^{-\frac{1}{2}} \phi + N^2 \phi n - N^2 \psi n - N^2 \phi n - N^2 \phi n - N^2 n \right) \right] \\
+ \left\{ N^{-\frac{3}{2}} \frac{\partial}{\partial m} - N^{-\frac{3}{2}} \frac{\partial}{\partial n} \right\} \left[ \beta \left( N^2 \psi \phi + N^2 \psi n + N^2 m \phi + N mn \right) \right] \\
+ \frac{1}{2} \left[ N^{-1} \frac{\partial^2}{\partial m^2} - 2 N^{-1} \frac{\partial^2}{\partial m \partial n} + N^{-1} \frac{\partial^2}{\partial n^2} \right] \left[ \beta \left( N^2 \psi \phi + N^2 \psi n + N^2 m \phi + N mn \right) \right] + \ldots ] \Pi \\
(2.72) \\

Now comparing the coefficients of \(-\frac{\partial \Pi}{\partial n} N^{-\frac{1}{2}}\) on both sides of eq. (2.72), we get

\[ \frac{d\psi}{d\tau} = \alpha \phi - \alpha \phi \psi - \alpha \phi^2 - \beta \psi \phi \]

\[ = \alpha \phi (1 - \psi - \phi) - \beta \psi \phi \]

\[ = \alpha \phi \chi - \beta \psi \phi \]

(2.73)
where $\chi = 1 - \psi - \phi$

Comparing the coefficients of $-\frac{\partial \Pi}{\partial n}N^2$ on both sides of eq.(2.72), we get

$$\frac{d\phi}{d\tau} = \beta \psi \phi$$ \hspace{1cm} (2.74)

and

$$\frac{d\chi}{d\tau} = \frac{d}{d\tau}(1 - \phi - \psi) = -\frac{d\phi}{d\tau} - \frac{d\psi}{d\tau} = -\alpha \phi \chi$$ \hspace{1cm} (2.75)

Further after comparing the coefficients of 'N' on both sides of eq.(2.72), we get

$$\frac{\partial \Pi(m, n; \tau)}{\partial \tau} = \left[ -\frac{\partial}{\partial m} \{\alpha n - \alpha \psi n - \alpha \phi m - \alpha \phi n - \beta \psi n - \beta \phi m\} \right.$$  

$$-\frac{\partial}{\partial n} \{\beta \psi n + \beta \phi m\} + \frac{1}{2} \frac{\partial^2}{\partial m \partial n} \{\alpha \phi - \alpha \psi \phi - \alpha \phi^2 + \beta \psi \phi\}$$  

$$+\frac{1}{2} \frac{\partial^2}{\partial n^2} \beta \psi \phi - \frac{\partial^2}{\partial m \partial n} \beta \psi \phi \} \Pi(m, n; \tau)$$  

$$= \left[ -\frac{\partial}{\partial n} (A_{11}n + A_{12}m) - \frac{\partial}{\partial m} (A_{21}n + A_{22}m) \right.$$  

$$+\frac{1}{2} \left( D_{11} \frac{\partial^2}{\partial n^2} + 2D_{12} \frac{\partial^2}{\partial m \partial n} + D_{22} \frac{\partial^2}{\partial m^2} \right) \} \Pi(m, n; \tau)$$  

(2.76)

where
Now as obvious in view of the uniform and sufficiently rapid decreasing characteristics of the functions \( \Pi(m,n;\tau) \), \( \frac{\partial \Pi(m,n;\tau)}{\partial m} \) and \( \frac{\partial \Pi(m,n;\tau)}{\partial n} \) as \( m,n \to \pm \infty \), i.e., \( \Pi \frac{\partial \Pi}{\partial m} \) and \( \frac{\partial \Pi}{\partial n} \to 0 \) as \( m,n \to \pm \infty \), we can easily understand that for all positive integers \( (s,r) \), the functions \( m^n n^r \Pi \), \( m^n r^s \frac{\partial \Pi}{\partial m} \), \( m^n r^{r-1} \frac{\partial \Pi}{\partial n} \to 0 \) as \( m,n \to \pm \infty \). Now by using these characteristics of the functions, we can deduce rate equations for the moments from eq. (2.76) after multiply it by \( m^n n^r \) (for \( s+r=1 \) or \( 2 \)) and then integrating over the entire range of \( 'm' \) and \( 'n' \). It is as follows:

(i) Multiply eq. (2.76) by \( n \) (for \( s=0, r=1 \)) and integrate over the entire range of \( 'm' \) and \( 'n' \), we get

\[
L.H.S. = \int \int n. \frac{\partial \Pi(m,n;\tau)}{\partial \tau} dm dn
\]

\[
= \int \frac{\partial}{\partial \tau} n. \Pi(n;\tau) dn \int \Pi(m;\tau) dm
\]
\[
\frac{d}{d\tau} < n > ;
\]
(2.77)

because

\[
\int_m \Pi(m;\tau)dm = 1
\]

Now the first term of R.H.S. of eq. (2.76) imply:

\[
- \int_n \int_m \frac{\partial}{\partial n} (A_{11} n + A_{12} m) \cdot \Pi(m, n; \tau)dm \, dn
\]

\[
= - \int_n \frac{\partial}{\partial n} A_{11} n \cdot \Pi(n; \tau)dn \int_m \Pi(m; \tau)dm
\]

\[
- \int_n \frac{\partial}{\partial n} A_{12} \Pi(n; \tau)dn \int_m m \cdot \Pi(m; \tau)dm
\]

But \[
\int_m \Pi(m; \tau)dm = 1
\]

\[
= - [A_{11} n^2 \cdot \Pi(n; \tau)]_0^\infty + \int_n A_{11} n \cdot \Pi(n; \tau)dn
\]

\[
- \left[ n A_{12} \Pi(n; \tau) \right]_0^\infty - \int_n A_{12} \Pi(n; \tau)dn \int_m m \cdot \Pi(m; \tau)dm
\]

But \[
\int_n \Pi(n; \tau)dn = 1
\]
Similarly from the others terms of R.H.S. of eq. (2.76) after simplifications, we have

\[
\int \int m \frac{\partial}{\partial m} \{A_{21} n + A_{22} m\} \Pi(m,n;\tau) dm dn = 0
\]

(2.79)

and

\[
\int \int n \left[ D_{11} \frac{\partial^2}{\partial n^2} + 2D_{12} \frac{\partial^2}{\partial m \partial n} + D_{22} \frac{\partial^2}{\partial m^2} \right] \Pi(m,n;\tau) dm dn = 0
\]

(2.80)

Thus from eqs. (2.77) to (2.80), we find

\[
\frac{d}{dt} <n> = A_{11} <n> + A_{12} <m>
\]

(2.81)

(ii) Multiply eq. (2.76) by m (for s=1, r=0) and integrate over the entire range of m and n, after simplification, we get

\[
\frac{d}{dt} <m> = A_{21} <n> + A_{22} <m>
\]

(2.82)

(iii) Multiply eq. (2.76) by n^2 (for s=0, r=2) and integrate over the entire range of m and n, it gives after simplification

\[
\frac{d}{dt} <n^2> = 2A_{11} <n^2> + 2A_{12} <mn> + D_{11}
\]

(2.83)
(iv) Multiply eq. (2.76) by \(mn\) (for \(s=1, r=1\)) then integrate over the entire range of \(m\) and \(n\), after simplification it presents

\[
\frac{d}{dt} <mn> = A_{21}<n^2> + (A_{11} + A_{22})<mn>
+ A_{12}<m^2> + D_{12}
\]

(2.84)

(v) Multiply eq. (2.76) by \(m^2\) (for \(s=2, r=0\)) and integrate over the entire range of \(m\) and \(n\), after simplification of it, we get

\[
\frac{d}{dt} <m^2> = 2A_{21}<mn> + 2A_{22}<m^2> + D_{22}
\]

(2.85)

The eqs. (2.81) to (2.85) are known as 'rate equations' for moments.

Now the eqs. (2.81) and (2.82), can be written in the form

\[
\frac{d}{d\tau} \begin{bmatrix} <n> \\ <m> \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} <n> \\ <m> \end{bmatrix}
\]

(2.86)

Put \(M = \begin{bmatrix} <n> \\ <m> \end{bmatrix}\) and \(B = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\)

We have

\[
\frac{dM}{d\tau} = BM
\]

\[
\Rightarrow \frac{dM}{M} = B d\tau
\]
After taking integration of both sides and simplification, we get

\[ M = e^{\beta t}c, \text{ where } c \text{ is constant of integration and can be determined from the initial conditions.} \]

At \( \tau = 0 \), we have

\[ M_{a \tau=0} = c \]

Thus

\[ c = M_{a \tau=0} = \begin{bmatrix} < n(0) > \\ < m(0) > \end{bmatrix} \]

Therefore, it gives

\[ M = e^{\beta t} \begin{bmatrix} < n(0) > \\ < m(0) > \end{bmatrix} \]

or

\[
\begin{bmatrix}
< n(\tau) > \\
< m(\tau) >
\end{bmatrix} = e^{\beta t} \begin{bmatrix}
< n(0) > \\
< m(0) >
\end{bmatrix}
\]

(2.87)

Thus from eq. (2.87) the mean values, viz., \( <n(\tau)> \) and \( <m(\tau)> \) can be obtained at any time \( \tau \) if \( <n(0)> \) and \( <m(0)> \) are known.

### 2.7 STEADY-STATE SOLUTION

In the steady-state of the system, we have \( \frac{dv}{dt} = 0 \) and \( \frac{ds}{dt} = 0 \). Thus from eqs. (2.73) and (2.74), we find
\[ \alpha \phi \chi - \beta \phi \psi = 0 \]  \hspace{1cm} (2.88)

and

\[ \beta \psi \phi = 0 \]  \hspace{1cm} (2.89)

After adding eqs. (2.88) and (2.89), we have

\[ \alpha \phi \chi = 0 ; \text{ where } \chi = 1 - \psi - \phi \]  \hspace{1cm} (2.90)

Similarly in the steady-state, the eqs. (2.83) to (2.85) can be written in the form

\[
\begin{bmatrix}
2A_{11} & 2A_{12} & 0 & \langle n^2 \rangle \\
A_{21} & A_{11} + A_{22} & A_{12} & \langle mn \rangle \\
0 & 2A_{21} & 2A_{22} & \langle m^2 \rangle \\
\end{bmatrix} = -
\begin{bmatrix}
D_{11} \\
D_{12} \\
D_{22}
\end{bmatrix}
\]

or we write

\[
\begin{bmatrix}
\langle n^2 \rangle \\
\langle mn \rangle \\
\langle m^2 \rangle \\
\end{bmatrix} = -
\begin{bmatrix}
D_{11} \\
D_{12} \\
D_{22}
\end{bmatrix}
\]  \hspace{1cm} (2.91)

where \([A]\) is known as matrix of coefficient. Further the determinant of the coefficient matrix \([A]\) after simplification is given by

\[ |A| = 4(A_{11}A_{22} - A_{12}A_{21})(A_{11} + A_{22}) \]  \hspace{1cm} (2.92)

For the existence and the stability of the second moments, \(|A|\) should be non zero and the real parts of the eigenvalues of the matrix \([A]\) should be negative.
Now from eq. (2.90) for $\alpha > 0$, there are two possibilities:

(i) $\phi = 0$

(ii) $\chi = 0$

The first possibility implies that finally there are no spreader in the population and thus the relative fractions of ignorants and knowers satisfy the relation $\chi + \psi = 1$. But it happens only when $\beta = 0$.

The second possibility implies that eventually no ignorants are left in the population. The relative fractions of knowers and spreaders are thus related through the relation $\phi + \psi = 1$. In this case when $\chi = 0$, eqs. (2.88) and (2.89) both reduces to the

$$\beta \phi \psi = 0$$

$$\Rightarrow \beta \phi (1 - \phi) = 0$$

For $\beta > 0$, it implies either $\phi = 0$ or $\phi = 1$. But ‘$\phi$’ can not be zero for $\beta > 0$ thus it implies $\phi = 1$. Hence it states that finally ‘$\psi$’ is also ‘zero’, i.e., there are spreaders only in the population for $\alpha$ and $\beta > 0$ which is obvious from the nature of the phenomenon.

In this case the eigen values of the matrix $[A]$ are give by

$$\lambda_1 = A_{11} + A_{21} = A_{12} + A_{22} = -\alpha < 0$$

(2.93)

$$\lambda_2 = A_{11} - A_{12} = A_{22} - A_{21} = -\beta < 0$$

(2.94)
Now

\[ \lambda_1 + \lambda_2 = A_{11} + A_{22} \]

and

\[ \lambda_1 \lambda_2 = (A_{11} + A_{21})(A_{11} - A_{12}) \]

\[ = A_{11}(A_{11} - A_{12}) + A_{22}A_{11} - A_{21}A_{12} \]

\[ = A_{11}(A_{22} - A_{21}) + A_{22}A_{11} - A_{21}A_{12} \]

\[ = A_{11}A_{22} - A_{21}A_{12} \]

Thus from eq. (2.92), we have

\[ |A| = 4(A_{11}A_{22} - A_{12}A_{21})(A_{11} + A_{22}) \]

\[ = 4\lambda_1 \lambda_2(\lambda_1 + \lambda_2) = -4\alpha\beta(\alpha + \beta) < 0 \quad (2.95) \]

Thus

\[ |A| \neq 0 \]

The eqs. (2.93) to (2.95) imply the existence of second moments. Also we find that in this case when \( \phi \neq 1 \), we have

\[ D_{11} = D_{22} = D_{12} = 0 \quad (2.96) \]
It means that in the steady-state, all the diffusion coefficients become zero, i.e., the transmission ceases as obvious.

Further from eq. (2.91), in this case, we find

\[ <m^2> = <mn> = <n^2> = 0 \quad (2.97) \]

i.e. the second moments and \(<mn>\) are zero.
TABLE 2.1
The values of ‘P(T)’ at different times (0.1 to 1.4) for n=5.

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<th>λ=3.5</th>
<th>λ=4.5</th>
<th>λ=5.0</th>
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The values of ‘P(T)’ at different times (0.1 to 0.9) for n=10.

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TABLE 2.4

The values of ‘x(t)’, ‘y(t)’ and ‘z(t)’ at different times (0.0 to 3.6) for n=20 and α=.2, β=.1.

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TABLE 2.5
The values of ‘x(t)’, ‘y(t)’ and ‘z(t)’ at different times (0.0 to 2.0) for n=50 and 
α=.2, β=.3.

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The values of ‘x(t)’, ‘y(t)’ and ‘z(t)’ at different times (0.0 to 1.6) for n=100 and 
α=.2, β=.3.

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The values of \( y(t) \) at different times for \( n=20 \) and for different values of \( \alpha \) and \( \beta \).

| TIME | \( y(t) \)  
| \( \alpha=0.1, \beta=0.2 \) | \( y(t) \)  
| \( \alpha=0.1, \beta=0.6 \) | \( y(t) \)  
| \( \alpha=0.2, \beta=0.1 \) | \( y(t) \)  
| \( \alpha=0.2, \beta=0.6 \) |
|---|---|---|---|---|
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.3 | 0.547915 | 0.5217 | 1.095523 | 1.043402 |
| 0.6 | 1.064373 | 0.980057 | 2.127831 | 1.999743 |
| 0.9 | 1.564928 | 1.398601 | 3.131604 | 2.895502 |
| 1.2 | 2.060197 | 1.768671 | 4.135279 | 3.546036 |
| 1.4 | 2.390185 | 1.971503 | 4.814575 | 3.645007 |
| 1.5 | 2.555215 | 2.054215 | 5.158637 | 3.533317 |
| 1.8 | 3.048381 | 2.202859 | 6.208508 | 2.314883 |
| 1.9 | 3.21095 | 2.212936 | 6.562731 | 1.576267 |
| 2.0 | 3.371733 | 2.200769 | 6.917324 | 0.680451 |
| 2.1 | 3.530009 | 2.165436 | 7.270478 | 0.0 |
| 2.4 | 3.980486 | 1.918733 | 8.291895 | |
| 2.7 | 4.368011 | 1.47751 | 9.142008 | |
| 3.0 | 4.645956 | 0.881009 | 9.494392 | |
| 3.3 | 4.749812 | 0.158261 | 8.273036 | |
| 3.4 | 4.731593 | 0.0 | 6.74979 | |
| 3.5 | 4.680934 | | 1.591073 | |
| 3.6 | 4.59374 | | 0.0 | |
| 3.9 | 4.066722 | | | |
| 4.2 | 3.028314 | | | |
| 4.5 | 1.303994 | | | |
| 4.6 | 0.541097 | | | |
| 4.7 | 0.0 | | | |
TABLE 2.8

The values of 'w(t)' at different times for $\alpha=.1$, $\beta=.2$ and for different values of 'n'.

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TABLE 2.9
The values of ‘w(t)’ at different times for α=.1, β=.6 and for different values of ‘n’.

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# TABLE 2.10

The values of ‘w(t)’ at different times for \( n=20 \) and for different values of ‘\( \alpha \)’ and ‘\( \beta \)’.

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TABLE 2.11
The values of ‘w(t)’ at different times for n=50 and for different values of ‘α’ and ‘β’.

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<tr>
<td>3.6</td>
<td>0.0</td>
<td></td>
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</tr>
</tbody>
</table>
FIGURE 2.1
Graph-Relation Between 'T' and 'P(T)'

\[ n = 5 \]

\[ \lambda = 0.5 \quad \lambda = 1.5 \quad \lambda = 2.5 \quad \lambda = 3.5 \quad \lambda = 4.5 \quad \lambda = 5.0 \]
FIGURE 2.2
Graph-Relation Between 'T' and 'P(T)'

\[ n = 10 \]

\[ \lambda = 0.5 \quad \lambda = 1.5 \quad \lambda = 2.5 \quad \lambda = 3.5 \quad \lambda = 4.5 \quad \lambda = 5.0 \]
FIGURE 2.3
Graph-Number of 'x(t)', 'y(t)' & 'z(t)'
as a function of time 't'

n = 5
α = 0.2, β = 0.1
FIGURE 2.4
Graph-Number of 'x(t)', 'y(t)' & 'z(t)' as a function of time 't'

- x(t) + y(t) + z(t)

n = 20

α = 0.2, β = 0.1
FIGURE 2.5
Graph-Number of 'x(t), y(t)' & 'z(t)'
as a function of time 't'

\[ n = 50 \]
\[ \alpha = 0.2, \beta = 0.3 \]
FIGURE 2.6
Graph-Number of 'x(t)', 'y(t)' & 'z(t)' as a function of time 't'

- $x(t)$
- $y(t)$
- $z(t)$

$n = 100$

$\alpha = 0.2, \beta = 0.3$
FIGURE 2.7
Graph-Number of 'y(t)' as a function of time 't'

- I: $\alpha = 0.1, \beta = 0.2$
- II: $\alpha = 0.1, \beta = 0.6$
- III: $\alpha = 0.2, \beta = 0.1$
- IV: $\alpha = 0.2, \beta = 0.6$
FIGURE 2.8
Graph-Transmission rate 'w(t)'
as a function of time 't'
FIGURE 2.9
Graph-Transmission rate 'w(t)'
as a function of time 't'

\[ \alpha = 0.1, \beta = 0.6 \]

\[ n = 35 \quad n = 40 \quad n = 45 \quad n = 50 \]
FIGURE 2.10
Graph-Transmission rate 'w(t)'
as a function of time 't'

- I $\alpha = 0.1, \beta = 0.2$
- II $\alpha = 0.1, \beta = 0.6$
- III $\alpha = 0.2, \beta = 0.1$
- IV $\alpha = 0.2, \beta = 0.6$
FIGURE 2.11
Graph-Transmission rate 'w(t)'
as a function of time 't'

I $\alpha = 0.1, \beta = 0.2$
II $\alpha = 0.1, \beta = 0.6$
III $\alpha = 0.2, \beta = 0.3$
IV $\alpha = 0.2, \beta = 0.6$