CHAPTER 3

PROFIT ANALYSIS OF RELIABILITY MODELS SUBJECT TO DEGRADATION WITH NO REPAIR ACTIVITY IN ABNORMAL WEATHER
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1. Introduction

The study of maintained systems in respect of environmental conditions has drawn the attention of various workers in the discipline of reliability engineering. Recently, single-unit reliability models operating under controlled weather conditions have been probed by the scholars including Chander and Bansal [2005] and Chander [2007] with different types of failure and repair policies. Since it is not always possible to keep the environmental conditions under control. Therefore, in chapter 2, the reliability models of a single-unit system operating under two weather conditions – normal and abnormal have been analyzed stochastically. In these models, it is assumed that operation, inspection and repair of the unit are not possible in abnormal weather. But under a real fact that the system may be increased down time if it is not permitted to do work and is not repaired in abnormal weather. So, it becomes necessary to study the reliability and economic measures of a system when its operation and repair activities are allowed in abnormal weather.

In view of the above, the present chapter deals with the profit analysis of two single-unit reliability models in which unit remains operative in abnormal weather. In each model, the unit may fail completely either directly from normal mode or via partial failure. There is a single server who attends the system immediately whenever needed. In model 1, the repair of the unit is done at complete failure while in model 2, the unit is repaired both at its partial and complete failure. The unit becomes degraded after repair at complete failure, however, it works as new after repair at partial failure. The server inspects the degraded failed unit to examine the feasibility of repair. If repair of the degraded unit is not feasible, it is replaced by new one in order to avoid the unnecessary expansions on repair. The repair and inspection of the unit are stopped in abnormal weather.
All random variables are taken as independent and uncorrelated. It is assumed that the distribution of failure time and time to change in weather conditions follow negative exponential, whereas, the inspection and repair time distributions are arbitrary. The switch devices are perfect. Some reliability and economic measures such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), steady state availability, busy period of the server and expected number of visits by the server are obtained by using regenerative point technique. Finally, the profit function is derived for each model to carry out the economic analysis. Graphs are drawn to compare MTSF and profit of the models for a particular case.

2. Notation

\begin{align*}
E & : \quad \text{Set of regenerative states} \\
O & : \quad \text{Unit in operative and in normal mode} \\
\bar{O} & : \quad \text{Unit is operative in abnormal weather} \\
r_1/r_2/\lambda & : \quad \text{Constant failure rate of the unit from normal mode to partial failure mode / partial failure mode to complete failure / normal mode to complete failure} \\
\lambda_1 & : \quad \text{Constant failure rate of the degraded unit} \\
\beta/\beta_1 & : \quad \text{Constant rate of change of weather from normal to abnormal / abnormal to normal weather} \\
p/q & : \quad \text{Probability that repair of the degraded unit is not feasible / feasible} \\
PFO/PFO/ & : \quad \text{Unit is partially failed and operative in normal weather / in} \\
PUr/PFOWr & : \quad \text{abnormal weather / operative and under repair in normal weather /} \\
& \hspace{1cm} \text{operative but waiting for repair due to abnormal weather} \\
FUr/FWr & : \quad \text{Unit is completely failed under repair / waiting for repair due to} \\
& \hspace{1cm} \text{abnormal weather} \\
DO/D\bar{O} & : \quad \text{The degraded unit is operative in normal weather / in abnormal} \\
& \hspace{1cm} \text{weather}
\end{align*}
DFU\textsubscript{i}/DFW\textsubscript{i} : The degraded unit is failed and under inspection / waiting for inspection due to abnormal weather

g(t)/G(t),g\_i(t)/G\_i(t) : pdf / cdf of repair time of the unit at complete failure / partial failure

h(t)/H(t) : pdf / cdf of inspection time of the degraded unit

q\_ij(t), Q\_ij(t) : pdf, cdf of first passage time from regenerative state i to regenerative state j or to a failed state j visiting state k once in \((0,t]\).

M\_i(t) : Probability that the system is up initially in state \(S_i \in \mathbb{E}\) is up at time \(t\) without visiting to any other regenerative state

W\_i(t) : Probability that the server is busy in state \(S_i\) upto time \(t\) without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states.

\(m_{ij}\) : The unconditional mean time taken by the system to transit from any regenerative state \(S_i\) when it (time) is counted from epoch of entrance in to that state \(S_j\). Mathematically, it can be written as

\[
m_{ij} = \int_0^t q\_i(t) dt = -q\_i'(0).
\]

\(\mu_i\) : The mean sojourn time in state \(S_i\) which is given by \(\mu_i = E(T) = \int_0^\infty P(T > t) dt = \sum m_{ij}\), where \(T\) denotes the time to system failure.

\(\mathbb{S}/\mathbb{C}\) : Symbol for Laplace-Stieltjes convolution/Laplace convolution

\(\sim/\ast/\prime\) : Symbol for Laplace Steiltjes Transform (LST) / Laplace Transform (LT) / Symbol for derivative of the function

The transition states for each model are regenerative. The possible transitions between states along with transition rates for the models 1 and 2 are shown in figures 1 and 2 respectively. The transition diagram for model 2 is same as that of model 1 except states \(S_1\) and \(S_5\). In model 2, \(S_1 = \text{PUR}\) and \(S_5 = \text{PFOWr}\) and there is also a transition from state \(S_1\) to \(S_0\) with repair time distribution \(g_1(t)\).
State Transition Diagrams

Fig. 1: Model 1

Fig. 2: Model 2
3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-
zero elements

\[ p_0 = Q_0(\infty) = \int q_0(t) \, dt \quad \text{as} \]

For Model 1

\[ p_{01} = \frac{r_1}{(\lambda + \beta + r_1)}, \quad p_{02} = \frac{\lambda}{(\lambda + \beta + r_1)}, \quad p_{04} = \frac{\beta}{(\lambda + \beta + r_1)}, \]

\[ p_{12} = \frac{r_2}{(r_2 + \beta)}, \quad p_{15} = \frac{\beta}{(r_2 + \beta)}, \quad p_{23} = g^*(\beta), \]

\[ p_{26} = 1 - g^*(\beta), \quad p_{37} = \frac{\lambda}{(\lambda + \beta)}, \quad p_{38} = \frac{\beta}{(\lambda + \beta)}, \]

\[ p_{40} = \frac{\beta}{\lambda + \beta_1 + r_1}, \quad p_{45} = \frac{r_1}{\lambda + \beta_1 + r_1}, \quad p_{46} = \frac{\lambda}{\lambda + \beta_1 + r_1}, \]

\[ p_{51} = \frac{\beta_1}{r_2 + \beta_1}, \quad p_{56} = \frac{r_2}{r_2 + \beta_1}, \quad p_{62} = p_{98} = 1. \]

\[ p_{73} = \frac{\beta_1}{\beta_1 + \lambda_1}, \quad p_{79} = \frac{\lambda}{\beta_1 + \lambda_1}, \quad p_{80} = qh^*(\beta). \]

\[ p_{82} = ph^*(\beta), \quad p_{89} = 1 - h^*(\beta) \quad (3.1) \]

It can be verified that

\[ p_{01} + p_{02} + p_{04} = p_{12} + p_{15} = p_{23} + p_{26} = p_{37} + p_{38} = p_{40} + p_{45} + p_{46} = p_{51} + p_{56} = \]

\[ p_{73} + p_{79} = p_{80} + p_{82} + p_{89} = 1 \quad (3.2) \]

The mean sojourn times \( (\mu_i) \) is the state \( S_i \) are

\[ \mu_0 = \int_0^x P(T > t) \, dt = \frac{1}{\lambda + \beta + r_1}, \]

\[ \mu_1 = \frac{1}{r_2 + \beta}, \quad \mu_2 = \frac{1}{\beta} [1 - g_1^*(\beta)]. \]
\[ \mu_3 = \frac{\lambda_i}{\lambda_i + \beta}, \quad \mu_4 = \frac{1}{\lambda + \beta_i + r_i}, \]
\[ \mu_5 = \frac{1}{\beta_i + r_2}, \quad \mu_7 = \frac{1}{\lambda_2 + \beta_i}, \]
\[ \mu_8 = \frac{1}{\beta_i} [1 - h^*(\beta)], \quad \mu_6 = \mu_9 = \frac{1}{\beta_i} \]  

(3.3)

**For Model 2**

The transition probabilities \( p_{01}, p_{02}, p_{04}, p_{23}, p_{26}, p_{37}, p_{38}, p_{40}, p_{45}, p_{46}, p_{51}, p_{56}, p_{62}, p_{40}, p_{73}, p_{79}, p_{80}, p_{82}, p_{89}, p_{98} \) are same as defined in model 1 and remaining are

\[ p_{10} = g_i \left( r_2 + \beta \right), \quad p_{12} = \frac{r_2}{r_2 + \beta} \left[ 1 - g_i \left( r_2 + \beta \right) \right], \]
\[ p_{15} = \frac{\beta}{r_2 + \beta} \left[ 1 - g_i \left( r_2 + \beta \right) \right] \]  

(3.4)

The mean sojourn time \( \mu_i \) of the state \( S_i \) is given by

\[ \mu_1 = \frac{1}{r_2 + \beta} \left[ 1 - g_i \left( r_2 + \beta \right) \right] \]  

(3.5)

and the remaining \( \mu_i \) for \( i = 0, 2-9 \) are same as defined for model 1.

### 4. Reliability and Mean Time to System Failure (MTSF)

Let \( \phi_i(t) \) be the cdf of first passage time from regenerative state \( i \) to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for \( \phi_i(t) \):

**For Model 1**

\[ \phi_0(t) = \sum \phi_0(t) + \sum \phi_3(t) + \phi_0(t), \]
\[ \phi_1(t) = \sum \phi_1(t) + \phi_3(t), \]
\[ \phi_4(t) = \sum \phi_4(t) + \phi_0(t) + \sum \phi_3(t) + \phi_4(t). \]
\[ \phi_5(t) = Q_5(t) \leq \phi_1(t) + Q_{56}(t) \]  

(4.1)

**For Model 2**

The expressions for \( \phi_i(t) \) (for \( i = 0,4,5 \)) are same as defined for model 1 and remaining is

\[ \phi_1(t) = Q_{10}(t) \leq \phi_0(t) + Q_{15}(t) \leq \phi_5(t) + Q_{12}(t) \]  

(4.2)

Taking LST of above relations (4.1) and (4.2) and solving for \( \tilde{\phi}_0(s) \).

Using this, we have

\[ R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s} \]  

(4.3)

The reliability of the system models can be obtained by taking Laplace inverse transform of (4.3).

The mean time to system failure (MTSF) is given by

**For Model 1**

\[ \text{MTSF}(T_1) = \frac{\mu_1 V_5 + \mu_3 V_4 V \left\{ \mu_0 \left(1 - 2 p_{04} p_{46}\right) + p_{04} \left(1 - p_2 p_{46}\right) \mu_4 \right\} V V_1}{V V_1 - p_{10} V_5} \]  

(4.4)

**For Model 2**

\[ \text{MTSF}(T_2) = \frac{\mu_1 V_5 + \mu_3 V_4 V \left\{ \mu_0 \left(1 - 2 p_{04} p_{46}\right) + p_{04} \left(1 - p_2 p_{46}\right) \mu_4 \right\} V V_1}{V V_1 - p_{10} V_5} \]  

(4.5)

\[ V = (1 - p_{15} p_{51}) \], \quad V_1 = (1 - p_{04} p_{10}) \], \quad V_4 = (p_{01} p_{15} + p_{04} p_{45}) \], \quad V_5 = (p_{01} + p_{04} p_{45} p_{51}) \]

5. **Steady State Availability**

Let \( A_i(t) \) be the probability that the system is in up-state at instant \( 't' \) given that the system entered regenerative state \( i \) at \( t = 0 \). The recursive relations for \( A_i(t) \) are given as
For Model 1

\[ A_i(t) = M_i(t) + \sum_{j=1}^{6} q_{ij}(t) A_j(t) \]  \hspace{1cm} (5.1)

where

\[ j = 1,2,4; 2,5; 3,6; 0,5,6; 1,6; 2; 3,9; 0,2,9; 8 \text{ for } i = \{0 - 9\} \text{ respectively} \]

and \( M_i(t) = 0 \) for \( i = \{2, 6, 8, 9\} \)

while \( M_0(t) = e^{-\gamma t}, M_1(t) = e^{-(\lambda_1 + \lambda_2) t}, M_2(t) = e^{-(\lambda_1 + \lambda_3) t}, M_3(t) = e^{-(\lambda_1 + \lambda_4) t} \),

\( M_4(t) = e^{-(\lambda_1 + \lambda_5) t}, M_5(t) = e^{-(\lambda_1 + \lambda_6) t} \), \( M_6(t) = e^{-(\lambda_1 + \lambda_7) t} \), \( M_7(t) = e^{-\lambda_5 t} \)

For Model 2

\( j = 1,2,4; 0,2,5; 3,6; 0,5,6; 1,6; 2; 3,9; 0,2,9; 8 \text{ for } i = \{0 - 9\} \text{ respectively and } M_i(t) \text{ for } i = \{0, 3, 4, 5, 7\} \text{ are same as defined as model I and remaining is} \)

\[ M_1(t) = e^{-\gamma_1 t} \]

Taking LT of above relations (5.1) and solving for \( A_0'(s) \). The steady-state availability can be determined as

\[ A_0(\infty) = \lim_{s \to 0} sA_0'(s) \]  \hspace{1cm} (5.2)

For Model 1

\[ A_0 = \frac{N_{11}}{D_{11}} \text{, where} \]

\[ N_{11} = p_{80} (1-p_{37}p_{73})p_{23}[V(\mu_0+p_{04}\mu_4)+\mu_1 V_5+\mu_5(p_{01}p_{15}+p_{04}p_{45})]+(1-p_{80})V_4p_{23}(\mu_3+p_{37}\mu_7).K \]

\[ V = (1-p_{15}p_{51}), \quad K = [p_{12} V_5+V(p_{02}+p_{04}p_{46})+p_{56}(p_{01}p_{15}+p_{04}p_{45})] \]

\[ D_{11} = [p_{23} \{V_2(p_{80}ZV_{11}+p_{51}Z_1)\{p_{15} V_1 V_3+p_{15}(K_1+p_{82} V_1)\}+p_{12}p_{04}p_{45}p_{80}\}+p_{04}(1-p_{45})p_{80} V Z_2 
+Z_3(p_{82} V_{11}+p_{80} V_6)+p_{01}Z_4 K_2+V_4 p_{56} p_{62} p_{80} Z_5+p_{80} V_4 K_1+p_{04} p_{45} p_{80}(p_{12}+p_{56}) Z_6)+p_{37} 
+ p_{23} Z_7 V V_1 V_3+p_{80} \{V_6+p_{04}p_{46} V\}]+p_{23} \{V_2 \mu_3+p_{37} p_{79} \mu_7\} \{V V_4 p_{82}+p_{80} V_6\}+V V_1 V_2 V_3 
+ p_{26} Z] \]
\[ V_1 = (1 - p_{04}p_{40}). \]
\[ V_3 = (1 - p_{09}). \]
\[ K_1 = (p_{02} + p_{04}p_{46}). \]
\[ z = (p_8 + p_9), \]
\[ Z_2 = (p_0 + p_4). \]
\[ Z_4 = (p_0 + p_4). \]
\[ Z_6 = (p_0 + p_4). \]
\[ Z_8 = (p_2 + p_6). \]

**For Model 2**

\[ A_0 = \frac{N_{21}}{D_{21}}, \text{ where } N_{21} = N_{11}, \text{ and } \]
\[ D_{21} = D_{11} + p_{10}\{V_5\{p_{23}\{V_2(Z_4V_3 - p_{08}V\} - p_{37}p_{75}Z_3V_5\} - p_{26}V_5V_3Z_8\} + p_{23}\{V_2\{p_{04}p_{51}\}
\]
\[ \{p_{45}(\mu_4 + \mu_5)V_3 - p_{52}(p_{45}Z_2 - Z_1)\} - p_{52}VV_1V_5 - p_{01}p_{82}Z_4\} \cdot \{V_2\mu_3 + p_{37}p_{79}t_7\}V_5\} \]

6. **Busy Period Analysis**

Let \( B_i(t) \) be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered regenerative state \( i \) at \( t = 0 \). The recursive relations for \( B_i(t) \) are as follows:

**For Model 1**

\[ B_i(t) = W_i(t) + \sum_{i,j} q_{ij}(t) \mu B_j(t) \]

where

\( j = 1, 2, 4; 2.5; 3.6; 0.5, 6; 1.6; 2; 3.9; 0.2, 9; 8 \) for \( i = \{0 - 9\} \) respectively and \( W_i(t) = 0 \) for \( i = \{0, 1, 3, 4, 5, 6, 7, 9\} \)

while
\[ W_2(t) = e^{-\eta(t)}G(t), \quad W_8(t) = e^{-\eta(t)}H(t) \]

**For Model 2**

\[ j = 1, 2, 4; 0, 2, 5; 3, 6; 0, 5, 6; 1, 6; 2; 3, 9; 0, 2, 9; 8 \]

for \( i = \{0 - 9\} \) respectively and \( W_j(t) \) for \( (i = 2, 8) \) are same as defined as model 1 and remaining is \( W_1 = e^{-(\eta_1 + \delta_1)}G_i(t) \).

Taking LT of above relations (6.1) and solving for \( B_0'(s) \). The busy period of the server can be obtained as

\[ B_0(\infty) = \lim_{s \to 0} sB_0'(s) \tag{6.2} \]

**For Model 1**

\[ A_0 = \frac{N_{11}}{D_{11}}, \text{ where} \tag{6.3} \]

\[ N_{12} = (\mu_2 V_3 + \mu_8) V_2K \]

and \( D_{11} \) is already specified.

**For Model 2**

\[ B_0 = \frac{N_{22}}{D_{21}}, \text{ where} \tag{6.4} \]

\[ N_{22} = \mu_1 p_80 V_2 p_{21} V_5 + (\mu_2 V_3 + \mu_8) K V_2 \]

and \( D_{21} \) is already specified.

### 7. Expected Number of Visits by the Server

Let \( N_i(t) \) be the expected number of visits by the server in \((0, t]\) given that the system entered the regenerative state \( i \) at \( t = 0 \). The recursive relations for \( N_i(t) \) are given as

\[ N_i(t) = \sum_{j} Q_{ij}(t) \left[ K + N_j(t) \right] \tag{7.1} \]
For Model 1

\[ j = 1, 2, 4; 2.5; 3.6; 0.5, 6; 1.6; 2; 3.9; 0.2, 6; 1, 6; 2; 3.9; 0, 2, 9; 8 \text{ for } i = \{0 - 9\} \text{ respectively} \]
and
\[ K = \begin{cases} 
0 \text{ for } i = \{1, 2, 4-9\} \\
1 \text{ for } i = 0, j = 2 \text{ and } j = 2; i = 3, j = 8 
\end{cases} \]

For Model 2

\[ j = 1, 2, 4; 0.2, 5; 3.6; 0.5, 6; 1.6; 2; 3.9; 0.2, 9; 8 \text{ and } 14 \text{ for } i = \{0-9\} \text{ respectively} \]
and
\[ K = \begin{cases} 
0 \text{ for } i = \{1, 2, 4-9\} \\
1 \text{ for } i = 0, j = 1 \text{ and } j = 2; i = 3, j = 8 
\end{cases} \]

Taking LST of above relations (7.1) and solving for \( \hat{N}_0(s) \). The expected number of visits per unit time are given by
\[ N_0(\infty) = \lim_{s \to 0} s\hat{N}_0(s) \quad (7.2) \]

For Model 1

\[ N_0 = \frac{N_{13}}{D_{11}}, \text{ where} \quad (7.3) \]
\[ N_{13} = (p_{01} + p_{02})p_{80}V_1V_2 + p_{38}p_{23}V_3.K \]
and \( D_{11} \) is already defined.

For Model 2

\[ N_0 = \frac{N_{23}}{D_{21}}, \text{ where} \quad (7.4) \]
\[ N_{23} = N_{13} \text{ and } D_{21} \text{ is already specified.} \]

8. Profit Analysis

Profit incurred to the system model in steady state can be evaluated as
\[ P_i = K_0A_0 - K_1B_0 - K_2N_0 \text{ (i = 1, 2), where} \quad (8.1) \]
\[ K_0 = \text{Revenue per unit up-time of the system} \]
$K_1 =$ Cost per unit time for which server is busy

$K_2 =$ Cost per unit visit by the server

9. Particular Case

Suppose $g(t) = \alpha e^{-\beta t}$, $h(t) = \theta e^{-\mu t}$,

$g_i(t) = \alpha_1 e^{-\mu t}$.

The following results are obtained:

For Model 1

$$MTSF(T_1) = \frac{\left[ r_1 Y_1 x_1 x_2 + \beta r_1 (x_2 + x_1) x_1 x_2 + Y_1 \left\{ (x_1 x_2 - 2\beta \lambda) x_2 + \beta (x_2 - 2\lambda) X_1 \right\} \right]}{Y_1 Y_4 x_1 x_2}$$

(9.1)

For Model 2

$$MTSF(T_2) = \frac{\left[ r_2 Y_1 x_1 x_2 + \beta r_1 (x + x_1) x_1 x_2 + Y_2 \left\{ (x_1 x_2 - 2\beta \lambda) x_2 + \beta (x_2 - 2\lambda) X_1 \right\} \right]}{x_1 x_2 \{ Y_2 Y_4 - r_1 \alpha_1 Y_3 \}}$$

(9.2)

$$x = (\beta + r_2 + \alpha_1) , \quad x_1 = (\lambda + \beta + \mu) .$$

$$x_2 = (\lambda + \beta_1 + \mu) , \quad x_3 = (r_2 + \alpha_1) .$$

$$x_8 = (r_2 + \beta_1) . \quad Y_3 = x_1 x_3 + \beta \beta_1 ,$$

$$Y_5 = x_1 x_8 - \beta \beta_1 , \quad Y_4 = x_1 x_2 - \beta \beta_1 .$$

$$Y_2 = x x_3 - \beta \beta_1 .$$

Also,

Availability ($A_0$) = $\frac{N_{11}}{D_{11}}$ (for model 1).

$$A_0 = \frac{N_{21}}{D_{21}}$$ (for model 2)  \hspace{1cm} (9.3)
Busy Period (B_0) = \frac{N_{12}}{D_{11}} \text{ (for model 1)}.

(B_0) = \frac{N_{22}}{D_{21}} \text{ (for model 2)} \tag{9.4}

Expected Number of Visits (N_0) = \frac{N_{11}}{D_{11}} \text{ (for model 1)}.

(N_0) = \frac{N_{21}}{D_{21}} \text{ (for model 2)} \tag{9.5}

where,

\begin{align*}
N_{11} &= A_{30} \alpha \beta_1 x_8 \{q Y_1 \{Y_5(x_2+\beta_1)+r_1 Y_3+r_1 \beta(x_8+x_2)\}+(x_5+\beta_1)\{r_1 r_2 \{x_2 x_3+\beta(x_8+x_1+\beta_1)\}\}
+Y_5 \lambda(x_2+\beta_1)\}\}. \\
X_4 &= \lambda_1+\beta, \quad X_5 = \lambda_1+\beta_1, \\
Y_1 &= (x_4 x_5-\beta \beta_1), \quad X_6 = (\theta+\beta), \\
X_7 &= \alpha+\beta, \quad A_{30} = x_1 x_2 x_3 x_4 x_5 x_6 x_7.
\end{align*}

\begin{align*}
N_{12} &= A_{30} \beta_1 Y_1 (\theta+\alpha) x_8 \{r_1 r_2 x_2 x_3+Y_5 \lambda(x_2+\beta_1)+r_1 r_2 \beta(x_8+x_1+\beta_1)\}\}. \\
A_{13} &= A_{30} x_8 \alpha \beta_1 \theta[x_2(\lambda+r_1) Y_1 Y_5+\lambda_1 \{r_1 r_2 \{x_2 x_3+\beta(x_8+x_1+\beta_1)\}+Y_5 \lambda(x_2+\beta_1)\}]
\end{align*}

\begin{align*}
D_{11} &= [x_8 R_1 \beta Y_5 Y_4 \{Y_1 R_4 \alpha(x_6+\beta_1)+\theta Y_1 R_5(x_7+\beta_1)+\alpha \beta_1^2 \theta(x_4+x_5) R_3\}+\alpha \beta_1^2 (x_8+x_3) \theta \\
& \quad R_1 R_7 Y_1 \{\beta Y_5 Y_4+r_1 \{(\lambda x_2+\beta)q+p Y_4\}+r_1 r_2 q\}+\alpha \lambda q Y_1 Y_5 R_2(x_8 R_1 \beta+\beta_1+x_8 x_2^2)+\theta R_2 x_2 \\
& \quad Y_1 \alpha(x_8+x_1) \{r_1 r_2 x_2 x_3 p+\beta_1 q r_2 x_2\}+\alpha \beta_1^2 \theta Y_1(x_1+x_2) Y_2 R_2 x_8+\alpha \beta_1 \theta(x_6+x_7) Y_1 x_8 R_1 R_9 \\
& \quad [p \{Y_5 Y_4+r_1 r_2 x_2 x_3\}+q \{r_1 r_2 \{\beta x_2+\lambda(\beta_1-x_8)\}+Y_5(x_2+\beta)\}]+\alpha \beta Y_1 \{q r_1 r_2 (x_3+\beta_1) \\
& \quad (x_8+x_2) x_8 R_1 R_7+x_8 R_2 \beta_1(x_1+x_2) r_1 r_2 q(x_3+x_8)+\lambda Y_5 q-p Y_5 \beta_1\}+\alpha \{\beta_1 x_4(x_5+\beta_1)+
\beta_1 x_5 Y_1\} \{Y_5 Y_4+r_1 r_2 x_2 x_3\} p+q \{Y_5(x_2+\beta)+r_1 r_2 \beta(x_8+x_2+\beta_1)\}] x_8 R_1 R_3
\end{align*}

\begin{align*}
N_{21} &= A_{30} \alpha \theta \beta_1 \{q Y_1 \{Y_2(x_2+\beta_1)+r_1 Y_3+r_1 \beta(x+x_2)\}+(x_5+\beta_1)\{r_1 r_2 \{x_2 x_3+\beta(x+x_1+\beta_1)\}\}
+Y_2 \lambda(x_2+\beta_1)\}\}, \\
A_{31} &= x A_{30}
\end{align*}
10. Conclusion

There is a decline in the values of mean time to system failure (MTSF) and profit of the system models with the increase of abnormal weather rate ($\beta$) and direct failure rate $\lambda$ for fixed values of other parameters as shown in figures 3 to 6. However, their values increase as and when normal weather rate ($\beta_1$) increases. It may also be noted that system model 2 has more values of MTSF and profit. And, the difference of MTSF of the system models becomes more with the increase of normal weather rate ($\beta_1$) while the difference of profit reduces to its minimum value at $\beta_1 = .3$, $\lambda = .001$, $p = .4$, $q = .6$, $\alpha = 1$, $\alpha_1 = 1.2$, $\theta = 20$, $r_1 = 0.3$, $r_2 = 0.5$, $\lambda_1 = 0.003$, $k_0 = 5000$, $k_1 = 450$, $k_2 = 150$ with the increase of $\beta$. On the basis of the results obtained for a particular case, it is analyzed that
(i) the idea to repair the system at its partial failure is much economically beneficial as compared to repair at its complete failure.

(ii) the system which is working in abnormal weather can be made more reliable and profitable either by repairing it whenever needed or by controlling the weather conditions.
Fig. 3: Graph between MTSF v/s Abnormal Weather Rate

\[ p=0.4, \ q=0.6, \ \alpha=1, \ \lambda_1=0.003, \ theta=20, r_1=0.3, \ r_2=0.5 \]

Fig. 4: Graph between MTSF v/s Abnormal Weather Rate

\[ p=0.4, \ q=0.6, \ \alpha=1, \ \alpha_1=1.2, \ \lambda_1=0.003, \ theta=20, r_1=0.3, \ r_2=0.5 \]
Fig. 5: Graph between Profit v/s Abnormal Weather Rate

\[ p=0.4, q=0.6, \alpha=1, \lambda_1=0.003, \theta=20, r_1=0.3, r_2=0.5, k_0=5000, k_1=450, k_2=150 \]

\[ \lambda=0.001, \beta_1=0.3 \]
\[ \lambda=0.005, \beta_1=0.1 \]
\[ \lambda=0.001, \beta_1=0.1 \]

Fig. 6: Graph between Profit v/s Abnormal Weather Rate

\[ p=0.4, q=0.6, \alpha=1, \alpha_1=1.2, \lambda_1=0.003, \theta=20, r_1=0.3, r_2=0.5, k_0=5000, k_1=450, k_2=150 \]

\[ \lambda=0.005, \beta_1=0.1 \]
\[ \lambda=0.001, \beta_1=0.3 \]
\[ \lambda=0.001, \beta_1=0.1 \]