CHAPTER - 6

PROFIT ANALYSIS OF
RELIABILITY MODELS SUBJECT
TO DEGRADATION AND
INSPECTION AT DIFFERENT
LEVELS OF DAMAGES WITH NO
OPERATION IN ABNORMAL
WEATHER
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1. Introduction

It is a known fact that the single-unit systems are frequently used in every sphere of life due to their inherent reliability and common man’s affordability. Recently, the reliability models of the systems operating under controlled weather conditions have been studied by many scholars and reliability engineers including Chander and Bansal [2005] and Chander [2007] considering the idea of inspection to reveal the possibility of on-line feasibility of repair. It is commonly assumed that unit works as new after repair and there is no effect of weather on operation of the system. In fact, such assumptions can not be true for each system. Since it is not always possible to keep the weather conditions under control which may fluctuate due to changing environment and other disasters. Malik and Barak [2007] have analyzed a single-unit system working under two weather conditions – normal and abnormal. Also, a completely failed unit after repair by an ordinary server does not work as new and so it may be considered as degraded. Mokaddis et al. [1997] and Malik et al. [2008] have probed the systems subject to degradation. Furthermore, sometimes the operation of a system can not be permitted in abnormal weather as precautionary measures to avoid the excessive damage to the system.

While incorporating the concepts of inspection at different levels of damages, no operation in abnormal weather and degradation, two reliability models are suggested in this chapter to extend the work reported in the previous chapter. The unit may fail completely either directly from normal mode or via partial failure. And, the unit is considered as degraded after repair at complete failure while it works as new after repair at partial failure. In model 1, server inspects the partially failed unit to see the possibility of on-line repair and the degraded unit at its failure to examine the feasibility of repair. However, in model 2, the inspection of the unit is also carried out at its complete failure...
to check the feasibility of repair. If on-line repair of the unit is not possible, it is repaired in down state. Further, the unit is replaced by new one in case its repair is not feasible so that unnecessary expanses on repair may be avoided. The inspection and repair are stopped in abnormal weather.

The random variables are assumed as independent and uncorrelated. The distributions of failure time and time to change of weather conditions follow negative exponential while that of inspection and repair times are arbitrary. The switch devices are perfect. The system is observed at suitable regenerative epochs by using regenerative point technique to obtain various measures of its effectiveness such as mean sojourn times, mean time to system failure (MTSF), steady state availability, busy period and expected number of visits by the server. The profit function to each model is also derived. To depict the behaviour of MTSF and profit of the models graphically, the numerical results for a particular case are obtained.

2. Notation

\( E \) : Set of regenerative states

\( O \) : Unit is operative and in normal mode

\( \overline{WO} \) : Unit is good but waiting for operation due to abnormal weather

\( r_1/r_2/\lambda \) : Constant failure rate of the unit from normal mode to partial failure mode / partial failure mode to complete failure / normal mode to complete failure

\( \lambda_1 \) : Constant failure rate of the degraded unit

\( \beta/\beta_1 \) : Constant rate of change of weather from normal to abnormal / abnormal to normal weather

\( p/q \) : Probability that repair of the degraded unit is not feasible / feasible

\( PUr/\overline{PUr} \) : Unit is partially failed and operative but under repair / under repair but not operating due to abnormal weather
FU_r/FU_r : Unit is completely failed under repair in normal weather/ abnormal weather

DO/D WO : The degraded unit is operative under normal weather / waiting for operation due to abnormal weather

DFUi/DFUi : The degraded unit is failed and under inspection in normal weather / in abnormal weather

PFUi/PFUi : Unit is partially failed and operative but under inspection in normal weather / under inspection in abnormal weather but not operating

PUrd/PUrd : Unit is under repair in down state in normal weather / abnormal weather

FU_i/FU_i : Unit is completely failed and under inspection in normal weather / abnormal weather

g(t)/G(t) : pdf / cdf of repair time of the unit at complete failure and partial failure

g_1(t)/G_1(t) : 
g_2(t)/G_2(t) : pdf / cdf of repair time of the partial failed unit

f(t)/F(t) : pdf / cdf of inspection time of the unit at partial failure and complete failure

f_1(t)/F_1(t) : 
h(t)/H(t) : pd / cdf of inspection time of the degraded unit

q_{ij}(t)/ Q_{ij}(t) : pdf / cdf of passage time from regenerative state i to regenerative state j or to a failed state j visiting state k once in (0,t].

M_0(t) : Probability that the system is up initially in state S_i\in E is up at time t without visiting to any other regenerative state

W_i(t) : Probability that the server is busy in state S_i, upto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states.
The unconditional mean time taken by the system to transit from any regenerative state $S_i$ when it (time) is counted from epoch of entrance into that state $S_j$. Mathematically, it can be written as

$$m_{ij} = \int_0^\infty t d[Q_q(t)] = -q_q^*(0).$$

The mean sojourn time in state $S_i$ which is given by $\mu_i = E(T) = \int_0^\infty P(T > t) dt = \sum m_{ij}$, where $T$ denotes the time to system failure.

Symbol for Laplace-Stieltjes convolution/Laplace convolution

Symbol for Laplace Steiltjes Transform / Laplace Transform

Symbol for derivative of the function

Fig. 1: Model 1
3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

\[ p_{ij} = Q_y(\infty) = \int q_y(t)dt \text{ as} \]

**For Model 1**

\[ p_{01} = \frac{\beta_1}{(\lambda + \beta + r_1)}, \quad p_{03} = \frac{\lambda}{(\lambda + \beta + r_1)} \]
\[ p_{05} = \frac{\beta}{(\lambda + \beta + r)} \], \quad p_{12} = xf^*(r_2 + \beta), \\
\[ p_{1,13} = xf^*(r_2 + \beta), \quad p_{13} = \frac{r_2}{(r_2 + \beta)} \left[ 1 - f^*(r_2 + \beta) \right], \]
\[ p_{16} = \frac{\beta}{(r_2 + \beta)} \left[ 1 - f^*(r_2 + \beta) \right], \quad p_{20} = g^*_1(r_2 + \beta), \]
\[ p_{23} = \frac{r_2}{(r_2 + \beta)} \left[ 1 - g^*_1(r_2 + \beta) \right], \quad p_{27} = \frac{\beta}{(r_2 + \beta)} \left[ 1 - g^*_1(r_2 + \beta) \right], \]
\[ p_{34} = g^*(\beta), \quad p_{38} = 1 - g^*(\beta), \]
\[ p_{49} = \frac{\beta}{(\lambda_1 + \beta)}, \quad p_{4,10} = \frac{\lambda}{(\lambda_1 + \beta)}, \quad p_{50} = 1, \]
\[ p_{61} = \left[ 1 - f^*(\beta) \right], \quad p_{6,12} = xf^*(\beta), \]
\[ p_{67} = y^* f^*(\beta), \quad p_{72} = g^*_1(\beta), \]
\[ p_{75} = 1 - g^*_1(\beta), \quad p_{83} = g^*(\beta), \]
\[ p_{89} = 1 - g^*(\beta), \quad p_{94} = 1, \]
\[ p_{10.0} = qh^*(\beta), \quad p_{10.3} = ph^*(\beta), \]
\[ p_{10,11} = \left[ 1 - h^*(\beta) \right], \quad p_{11.5} = qh^*(\beta), \]
\[ p_{11.8} = ph^*(\beta), \quad p_{11,10} = \left[ 1 - h^*(\beta) \right], \]
\[ p_{13.0} = g^*_2(\beta), \quad p_{13,12} = \left[ 1 - g^*_2(\beta) \right], \]
\[ p_{12.13} = g^*_2(\beta), \quad p_{12.5} = \left[ 1 - g^*_2(\beta) \right] \] (3.1)

It can be verified that
\[ p_{01} + p_{13} + p_{15} = p_{12} + p_{1,13} + p_{16} + p_{13} = p_{20} + p_{27} + p_{23} = p_{34} + p_{38} = p_{49} + p_{4,10} = p_{50} = p_{61} + p_{67} + p_{6,12} = p_{72} + p_{75} = p_{83} + p_{89} = p_{94} = p_{10.0} + p_{10.3} + p_{10,11} = p_{11.5} + \]
\[ p_{11,8} + p_{11,10} = p_{12,13} + p_{12,5} = p_{13,0} + p_{13,12} = 1 \]  \tag{3.2}

The mean sojourn times \( (\mu_i) \) is the state \( S_i \) are

\[ \mu_0 = \int_0^\infty P(T > t)dt = \frac{1}{\lambda + \beta + r_i}, \quad \mu_1 = \frac{1}{r_2 + \beta} \left[ 1 - f^* (r_2 + \beta) \right], \]

\[ \mu_2 = \frac{1}{r_2 + \beta} \left[ 1 - g^*_i (r_2 + \beta) \right], \quad \mu_3 = \frac{1}{\beta} \left[ 1 - g^* (\beta) \right], \]

\[ \mu_4 = \frac{1}{\beta + \lambda}, \quad \mu_5 = 1, \]

\[ \mu_6 = \frac{1}{\beta_i} \left[ 1 - f^* (\beta_i) \right], \quad \mu_7 = \frac{1}{\beta} \left[ 1 - g^* (\beta) \right], \]

\[ \mu_8 = \frac{1}{\beta} \left[ 1 - g^* (\beta) \right], \quad \mu_9 = 0, \]

\[ \mu_{10} = \frac{1}{\beta} \left[ 1 - h^* (\beta) \right], \quad \mu_{11} = \frac{1}{\beta_i} \left[ 1 - h^* (\beta_i) \right], \]

\[ \mu_{12} = \frac{1}{\beta_i} \left[ 1 - g^*_2 (\beta_i) \right], \quad \mu_{13} = \frac{1}{\beta} \left[ 1 - g^*_2 (\beta) \right] \]  \tag{3.3}

For Model 2

The transition probabilities \( p_{01}, p_{03}, p_{05}, p_{12}, p_{11,13}, p_{13}, p_{16}, p_{20}, p_{22}, p_{27}, p_{50}, p_{61}, p_{67}, p_{612}, p_{72}, p_{75}, p_{12,5}, p_{12,13}, p_{13,0}, p_{13,12} \) same as defined in model 1 and remaining are

\[ p_{34} = b f^*_i (\beta), \quad p_{30} = a f^*_i (\beta), \]

\[ p_{38} = \left[ 1 - f^*_i (\beta) \right], \quad p_{49} = 1 - g^* (\beta), \]

\[ p_{4,10} = g^* (\beta), \quad p_{83} = \left[ 1 - f^*_i (\beta_i) \right], \]

\[ p_{85} = a f^*_i (\beta_i), \quad p_{89} = b f^*_i (\beta_i), \]

\[ p_{9,11} = g^* (\beta_i), \quad p_{84} = \left[ 1 - g^* (\beta_i) \right], \]

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The mean sojourn time $\mu_i$ of the state $S_i$ are given by

$$\mu_3 = \frac{1}{\beta} \left[ 1 - f_i^*(\beta) \right], \quad \mu_4 = \frac{1}{\beta} \left[ 1 - g^*(\beta) \right],$$

$$\mu_8 = \frac{1}{\beta_i} \left[ 1 - f_i^*(\beta_i) \right], \quad \mu_9 = \frac{1}{\beta_i} \left[ 1 - g^*(\beta_i) \right],$$

$$\mu_{10} = \frac{1}{\lambda_i + \beta}, \quad \mu_{11} = 1,$$

$$\mu_{14} = \frac{1}{\beta} \left[ 1 - h^*(\beta_i) \right], \quad \mu_{15} = \frac{1}{\beta_i} \left[ 1 - h^*(\beta_i) \right]$$

(3.5)

and the remaining $\mu_i$ for $i = 0, 1, 2, 5, 6, 7, 11, 12, 13$ are same as derived for model I.

It can be verified that

$$p_{30} + p_{34} + p_{38} = p_{49} + p_{4.10} = p_{83} + p_{85} + p_{89} = p_{9,11} + p_{94} = p_{10,14} + p_{10.11} = p_{11.10} = p_{14.0} + p_{14.14}$$

$$+ p_{14.15} = p_{15.5} + p_{15.9} + p_{15.14} = 1$$

(3.6)

4. **Reliability and Mean Time to System Failure (MTSF)**

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state $i$ to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:
For Model

\[ \phi_0(t) = Q_{01}(t) \phi_1(t) + Q_{05}(t) \phi_5(t) + Q_{03}(t) \]

\[ \phi_1(t) = Q_{12}(t) \phi_2(t) + Q_{16}(t) \phi_6(t) + Q_{1,13}(t) \phi_{13}(t) + Q_{13}(t) \]

\[ \phi_2(t) = Q_{20}(t) \phi_9(t) + Q_{27}(t) \phi_7(t) + Q_{23}(t), \]

\[ \phi_5(t) = Q_{50}(t) \phi_0(t) \]

\[ \phi_6(t) = Q_{61}(t) \phi_1(t) + Q_{67}(t) \phi_7(t) + Q_{6,12}(t) \phi_{12}(t) \]

\[ \phi_7(t) = Q_{72}(t) \phi_2(t) + Q_{75}(t) \phi_5(t), \]

\[ \phi_{12}(t) = Q_{12,5}(t) \phi_5(t) + Q_{12,13}(t) \phi_{13}(t) \]

\[ \phi_{13}(t) = Q_{13,0}(t) \phi_0(t) + Q_{13,12}(t) \phi_{12}(t) \] (4.1)

Taking LST of above relations (4.1) and solving for \( \tilde{\phi}_0(s) \)

Using this, we have

\[ R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s} \] (4.2)

The reliability of the system models can be obtained by taking Laplace inverse transform of (4.2).

The mean time to system failure (MTSF) of both the models is same which is given by

\[
\text{MTSF (T)} = \frac{\pi_{12,13} + \pi_{1,13} + \pi_{13,12} + \pi_{12,13} + \pi_{13,12}}{V_8(V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7 + V_8)}
\]

where

\[ V = (1 - p_{05}), \quad V_1 = (1 - p_{16}p_{61}), \quad V_2 = (1 - p_{27}p_{72}), \]

\[ V_3 = (1 - p_{13,12}p_{12,13}), \quad V_4 = (1 - p_{12,13}p_{13,0}), \quad V_5 = (1 - p_{13,0} + p_{12,13}p_{12,5}), \quad V_6 = (1 - p_{13,0} + p_{12,13}p_{12,5}) \]
5. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant ‘t’ given that the system entered regenerative state $i$ at $t = 0$. The recursive relations for $A_i(t)$ are given as

For Model 1

$$A_i(t) = M_i(t) + \sum_{j} q_{ij}(t) A_j(t)$$

(5.1)

where

$$j = 1,3,5;2,3,6,13;0,3,7;4,8;9,10;0;1,7,12;2,5;3,9;4;0,3,11;5,8,10;5,13;0,12$$

for $i = \{0 - 13\}$ respectively and $M_i(t) = 0$ for $i = 3,5-13$

while

$$M_0(t) = e^{-(\gamma + \beta) t}, \quad M_1(t) = e^{-(\gamma + \beta) t} F(t).$$

$$M_2(t) = e^{-(\gamma + \beta) t} G(t), \quad M_4(t) = e^{-(\beta + \gamma) t}$$

For Model 2

$$j = 1,3,5;2,3,6,13;0,3,7;0,4,8;9,10;0;1,7,12;2,5;3,5,9;4,11;11.14;10;5,13;0,12;0,4,15$$

and $5,9,14$ for $i = \{0 - 15\}$ respectively and $M_i(t) = 0$ for $i = \{3-9,11-15\}$ and remaining $M_i(t)$

$$M_{10}(t) = e^{-(\gamma + \beta) t}$$

Taking LT of above relations (5.1) and solving for $A_0(s)$.

The steady-state availability is given by

$$A_0(\infty) = \lim_{s \to 0} sA_0(s)$$

(5.2)

For Model 1

$$A_0 = \frac{N_{11}}{D_1}, \text{ where}$$

$$N_{11} = \begin{bmatrix} V_8 \{V_3 V_4 V_7 p_{4,10}(p_{34} L_1 + p_{34} L_2) \} V_2 (p_{01} V_4 + p_{01} V_0) + V_7 V_8 p_{34} \mu_4 \\
\{p_{01} V_1 V_2 + p_{01} p_{13} V_2 + p_{01} p_{23} V_0 \} \end{bmatrix}$$

$$L_1 = (p_{10} p_{11} + p_{10} p_{11} p_{11}), \quad L_2 = (p_{38} p_{10} + p_{10} p_{11} p_{11}).$$
\[ V_3 = (1 - p_{38}p_{83}) , \quad V_4 = (1 - p_{49}p_{94}) , \]
\[ V_7 = (1 - p_{10,11}) , \quad V_9 = (p_{12} + p_{16}p_{67}p_{72}) , \]
\[ L_4 = (p_{10,11}p_{11,5}p_{50}+p_{10,0}) \]

\[ D_{11} = \left[ (X_8V_3+X_3V_8)V_4V_7+(X_4V_7+X_7V_4)V_3V_8 \right] \left( L \right) + \left[ -X_8\left( p_{34}L_1+p_{89}p_{94}L_2 \right) + p_{34}V_8 \right] \left\{ L_1 (\mu_3 + \mu_4 + \mu_10) + p_{10,11}p_{11,8}p_{83}(\mu_8 + \mu_{11}) \right\} + p_{89}p_{94}V_8 \left\{ (\mu_8 + \mu_4 + \mu_4)L_2 + p_{38}p_{10,3}(\mu_3 + \mu_10) + p_{10,11} \right. \\
\left. p_{11,8}(\mu_{10} + \mu_{11}) \right\} p_{4,10}L + V_8 \left[ V_3V_4V_7-p_{4,10}(p_{34}L_1+p_{89}p_{94}L_2) \right] \left[ V(X_2V_3+X_3V_2)+XV_2V_3+ \right. \\
\left. (\mu_0 + \mu_1 + \mu_2) \left\{ p_{01}p_{12}(p_{20}+p_{27}p_{75}) \right\} \right] p_{01}p_{12}p_{27}p_{75}(\mu_5 + \mu_7) + p_{01}p_{16}p_{67} \left\{ (\mu_0 + \mu_1 + \mu_2 + \mu_6 + \mu_7) \\
p_{20}p_{72}+p_{75}(\mu_0 + \mu_1 + \mu_5 + \mu_6 + \mu_7) \right\} + \left[ -X_8L_3L_4+\{ L_3\mu_3+p_{38}p_{80}(\mu_8 + \mu_0) \} L_4V_8 + \{ L_4\mu_{10} + \\
p_{10,11}p_{11,5}(\mu_5 + \mu_{11}) \} L_2V_8 \right] \left[ p_{03}V_1V_2+p_{01}\{ p_{13}V_2+P_{12}p_{23}V_9 \} \right] + (V_4L_4) \left[ p_{03}V_1(\mu_0V_2- \right.
\left. X_2)-p_{03}X_1V_2+p_{01} \{ p_{12}p_{23}(\mu_0 + \mu_1 + \mu_2) - p_{13} \{ X_2+p_{13}V_2(\mu_0 + \mu_1) + p_{16}p_{67}p_{72}p_{23}(\mu_0 + \mu_1 + \mu_2 + \mu_6 + \mu_7) \} \} + p_{4,10} \{ p_{13}L_2 + p_{80}p_{94}L_2 \} + p_{34} \{ L_1 (\mu_3 + \mu_10) + p_{83}p_{10,11}p_{11,8}(\mu_8 + \mu_{11}) \} \right] + p_{89} \\
p_{38} \{ (\mu_8 + \mu_9)L_2 + p_{38}p_{10,3}(\mu_5 + \mu_{10}) + p_{10,11}p_{11,8}(\mu_{10} + \mu_{11}) \} \right\} - V_3(X_7V_4+X_4V_7) - X_3V_4V_7 \\
\left[ V_2 \{ p_{01}p_{13}L_5+p_{01}p_{16}p_{6,12}L_6 \} \right] + \left[ -V_7V_3V_4+\{ p_{34}L_1+p_{80}p_{94}L_2 \} p_{4,10} \right] \{ X_2p_{01} \{ p_{1,13}L_5+ \\
p_{16}p_{6,12}L_6 \} - p_{01}V_2 \{ p_{1,13} \{ (\mu_0 + \mu_1 + \mu_3) L_5 + p_{13,12}p_{12,5}(\mu_12 + \mu_13 + \mu_5) \} + p_{16}p_{6,12} \{ \mu_0 + \mu_1 + \mu_6 + p_{12,13}p_{13,0}(\mu_12 + \mu_13) + p_{12,5}(\mu_12 + \mu_5) \} \} \} , \quad V_6 = (p_{30}+p_{38}p_{85}) , \quad X = p_{05}(\mu_0 + \mu_5) .
\]

\[ X_1 = p_{16}p_{61}(\mu_1 + \mu_6) , \quad X_2 = p_{27}p_{72}(\mu_2 + \mu_7) , \]
\[ X_3 = p_{38}p_{83}(\mu_3 + \mu_8) , \quad X_4 = p_{49}p_{94}(\mu_4 + \mu_9) , \]
\[ X_5 = p_{14,15}p_{15,14}(\mu_4 + \mu_5) , \quad X_7 = p_{10}p_{11}(\mu_10 + \mu_{11}) , \]
\[ X_8 = p_{12,13}p_{13,12}(\mu_12 + \mu_{13}) , \quad L_3 = (p_{34}+p_{38}p_{89}p_{94}) , \]
\[ L = \left\{ VV_1V_2-p_{01}p_{12}(p_{20}+p_{27}p_{75}) - p_{01}p_{16}p_{67}(p_{72}p_{20}p_{75}) \right\} \]

For Model 2

\[ A_0 = \frac{N_{21}}{D_{21}} , \quad \text{where} \quad (5.4) \]
\[ N_{21} = \left\{ V_8 \{ V_5 V_3 V_4 - p_{10,14} (p_{4,10} L_7 + p_{9,11} L_8) \} \{ V_2 V_3 (V_1 \mu_0 + \mu_1 p_01) + p_01 V_9 \} + V_5 V_8 \mu_{10} \{ p_01 p_13 V_2 + p_03 V_1 V_2 + p_01 p_23 V_9 \} \{ p_34 (p_{49} p_{9,11} + p_{4,10}) + p_38 p_89 (p_{9,11} + p_{94} p_{4,10}) \} \right\} \\
V_5 = (1 - p_{15,14} p_{14,15}), \quad L_7 = (p_{14,4} + p_{14,15} p_{15,9} p_{94}), \]

\[ L_8 = (p_{94} p_{14,4} + p_{14,15} p_{15,9}) \]

\[ D_{21} = \left\{ V_4 V_3 V_7 - p_{10,14} (p_{4,10} L_7 + p_{9,11} L_8) \} \{ X_8 L_9 + V_8 \{ V_1 X_2 + X_2 V_1 \} (V V_3 - p_{03} V_6) + V_1 V_2 \{ X V_3 + V_3 p_{03} \{ (\mu_0 + \mu_1) V_6 + p_{38} p_{85} (\mu_5 + \mu_8) \} \} + p_01 \{ p_20 \{ V_9 (\mu_0 V_3 + X_3) + V_3 \{ V_9 (\mu_1 + \mu_2) + p_{16} p_{67} p_{72} (\mu_6 + \mu_7) \} \} + p_{12} p_{23} \{ (\mu_0 + \mu_1 + \mu_2 + \mu_3) V_6 + p_{38} p_{85} (\mu_5 + \mu_8) \} \} \} + \left\{ p_{12} p_{27} + p_{16} p_{67} \{ (\mu_1 + \mu_2) V_3 + X_3 \} + p_{12} p_{27} (\mu_1 + \mu_2) + p_{16} p_{67} (\mu_1 + \mu_6) \right\} + p_{16} p_{67} p_{72} p_{23} V_6 \{ (\mu_0 + \mu_1 + \mu_2 + \mu_6 + \mu_7) + p_{38} p_{85} (\mu_5 + \mu_8) \} \} \right\} + V_8 L_9 J_1 + \left\{ \left\{ V_8 \{ M_{14} (\mu_1 + \mu_2) + p_{38} p_{89} M_{2} \} + p_{34} \{ \mu_3 M_2 + p_{4,10} \mu_4 M_2 + p_{40} p_{9,11} (\mu_9 + \mu_{11}) \} + p_{38} p_{89} (\mu_5 + \mu_8) M_3 + p_{4,10} p_{94} (\mu_4 + \mu_9) + p_{9,11} (\mu_9 + \mu_{11}) \right\} \right\} M_4 + V_8 M_{10,14} (p_{34} M_2 + p_{38} p_{89} M_4) \{ p_{03} \{ p_{34} V_1 V_2 - (X_1 V_2 + X_2 V_1) \} + p_{01} \{ p_{23} (\mu_0 + \mu_1 + \mu_2) (p_{12} + p_{16} p_{67} p_{72} + p_{13} (\mu_0 + \mu_1) + p_{13} X_2 + p_{16} p_{67} p_{72} p_{23} (\mu_6 + \mu_7)) \} - J_1 V_1 V_3 \{ p_{11,13} L_5 + p_{16} p_{6,12} L_6 \} + V_4 V_5 V_7 - p_{10,14} (p_{34} L_7 + p_{9,11} L_8) \} \{ (X_2 V_3 - V_1 X_2) p_{01} (p_{11,13} L_5 + p_{16} p_{6,12} L_6) \} + p_{01} V_2 V_3 \{ p_{11,13} (\mu_0 + \mu_1 + \mu_3) L_5 + p_{11,12} p_{12,5} (\mu_1 + \mu_5) \} + p_{16} p_{6,12} \left\{ (\mu_0 + \mu_1 + \mu_6 + p_{12,13} p_{13,0} (\mu_1 + \mu_3) + p_{12,5} (\mu_1 + \mu_5) \right\} \right\}. L_9 = [V_1 V_2 \{ V V_3 - p_{30} V_8 \} - p_{01} V_3 \{ p_{20} V_9 + p_{12} p_{27} p_{75} - p_{01} p_{13} V_2 V_6 - p_{01} p_{16} p_{67} p_{75} V_3 - p_{01} V_6 p_{23} V_9 \}]

\[ M = (p_{14,0} + p_{14,15} p_{15,5}), \quad M_2 = (p_{4,10} + p_{49} p_{9,11} p_{11,10}), \]

M_3 = (p_{94} p_{4,10} + p_{9,11} p_{11,10}), \quad M_4 = [p_{03} V_1 V_2 + p_{01} p_{23} V_9 + p_{01} p_{13} V_5],

\[ J_1 = \left\{ V_4 (X_7 V_5 + V_5 X_5) + V_7 V_5 X_4 + p_{10,14} \{ (\mu_1 + (\mu_4 + \mu_4) L_7 + p_{14,15} p_{15,9} p_{9,11} + (\mu_0 + \mu_5)) \} + p_{9,11} \{ (\mu_9 + \mu_1) L_8 + p_{49} p_{14,4} (\mu_4 + \mu_4) + p_{14,15} p_{15,9} (\mu_4 + \mu_5) \} \right\} \]

6. Busy Period Analysis

Let B_i(t) be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for B_i(t) are as follows:
\[ B_i(t) = W_i(t) + \sum_{j} q_{ij}(t)B_j(t), \]  

(6.1)

**For Model 1**

where

\[
j = 1, 3, 5; 2, 3, 6, 13; 0, 3, 7; 4, 8, 9, 10; 0, 1, 7, 12; 2, 5, 3, 9; 4, 0, 3, 11; 5, 8, 10; 5, 13; 0, 12 \text{ for } i = \{0 - 13\} \]

respectively and \( W_i(t) = 0 \) for \( i = \{0, 4, 5, 9\} \) while remaining are

\[
W_1(t) = e^{-(\lambda + \mu) t} F(t), \quad W_2(t) = e^{-(\lambda + \mu) t} G(t), \\
W_3(t) = e^{-\lambda t} G(t), \quad W_6(t) = e^{-\lambda t} F(t), \\
W_7(t) = e^{-\beta t} G(t), \quad W_8(t) = e^{-\beta t} G(t), \\
W_{10}(t) = e^{-\beta t} H(t), \quad W_{11}(t) = e^{-\beta t} H(t), \\
W_{12}(t) = e^{-\beta t} G_2(t), \quad W_{13}(t) = e^{-\beta t} G_2(t)
\]

**For Model 2**

\[
j = 1, 3, 5; 2, 3, 6, 13; 0, 3, 7; 0, 4, 8, 9, 10; 0, 1, 7, 12; 2, 5, 3, 9; 4, 11, 14, 10; 5, 13; 0, 12; 0, 4, 15 \text{ and } 5, 9, 14 \text{ for } i = \{0 - 15\} \]

respectively and

\[
W_i(t) = 0 \text{ for } i = \{0, 5, 10, 11\} \text{ while remaining are}
\]

\[
W_1(t) = e^{-(\lambda + \mu t)} F(t), \quad W_2(t) = e^{-(\lambda + \mu t)} G(t), \\
W_3(t) = e^{-\lambda t} F(t), \quad W_6(t) = e^{-\lambda t} G(t), \\
W_7(t) = e^{-\beta t} G(t), \quad W_8(t) = e^{-\beta t} G(t), \\
W_9(t) = e^{-\beta t} H(t), \quad W_{10}(t) = e^{-\beta t} H(t), \\
W_{12}(t) = e^{-\beta t} G_2(t), \quad W_{13}(t) = e^{-\beta t} G_2(t), \\
W_{14}(t) = e^{-\beta t} H(t), \quad W_{15}(t) = e^{-\beta t} H(t)
\]

Taking LT of above relations (6.1) and determine \( B'_0(s) \) for each model. Using this, we get in the long run, the time for which server is busy as
\[ B_0(\infty) = \lim_{s \to 0} B_s^*(s) \quad (6.2) \]

**For Model 1**

\[ B_0 = \frac{N_{12}}{D_{11}}, \text{ where} \quad (6.3) \]

\[ N_{12} = p_{4.10} \{ V_8 [(V_3 V_7-L_1 L_3) \{ p_{01} V_2 \mu_1 + p_{01} V_9 \mu_2 + p_{01} p_{16} V_2 \mu_6 + p_{01} (p_{16} p_{07} + p_{12} p_{27}) \mu_7 \} + \{ \mu_8 (p_{34} p_{10.11} p_{11.8} + p_{38} V_7) + L_3 (\mu_{10} + \mu_{11} p_{10.11}) \} \{ p_{01} p_{23} V_9 + V_2 (p_{03} V_1 + p_{01} p_{13}) \} ] + V_2 (V_3 V_7-L_1 L_3) p_{01} \{ p_{1.13} (\mu_{13} + p_{13.12} \mu_2) + p_{16} p_{6.12} (\mu_{12} + \mu_{13} p_{12.13}) \} + \mu_3 V_7 V_8 \{ p_{01} p_{23} V_9 + V_2 (p_{03} V_1 + p_{01} p_{13}) \} ] \]

and \( D_{11} \) is already defined.

**For Model 2**

\[ B_0 = \frac{N_{22}}{D_{21}}, \text{ where} \quad (6.4) \]

\[ N_{22} = p_{4.10} \{ V_8 \{ V_4 V_5 \{ p_{4.10} p_{1.7} + p_{9.11} L_8 \} \} \{ V_3 \{ p_{01} \mu_1 V_2 + p_{01} V_9 \mu_2 + p_{01} p_{16} V_2 \mu_6 + p_{01} (p_{12} p_{27} + p_{16} p_{7}) \mu_7 \} + \{ p_{01} p_{23} V_9 + V_2 (p_{03} V_1 + p_{01} p_{13}) \} (p_{38} \mu_4 + \mu_3 ) \} + V_8 \{ p_{01} p_{23} V_9 + V_2 (p_{03} V_1 + p_{01} p_{13}) \} \{ \mu_9 (p_{34} p_{4.10} p_{14.15} p_{15.9} + p_{34} p_{49} + p_{38} p_{89}) V_9 \} + (\mu_{14} + \mu_{15} p_{14.15}) \{ (p_{34} p_{49} + p_{38} p_{89}) p_{9.11} p_{11.10} + p_{34} p_{4.10} \} + p_{34} \mu_4 V_5 \} + V_2 p_{01} \{ p_{1.13} (\mu_{13} + p_{13.12} \mu_2) + p_{16} p_{6.12} (\mu_{12} + p_{12.13} \mu_3) \} \]

and \( D_{21} \) is already defined.

**7. Expected Number of Visits by the Server**

Let \( N_i(t) \) be the expected number of visits by the server in \((0, t]\) given that the system entered the regenerative state \( i \) at \( t = 0 \). The recursive relations for \( N_i(t) \) are given as

\[ N_i(t) = \sum_{j \neq i} Q_{ij}(t) \mathcal{S}[K + N_j(t)] \quad (7.1) \]

**For Model 1**

\( j = 1,3,5;2,3,6,13;0,3,7;4,8;9,10;0;1,7,12;2,5;3,9;4;0,3,11;5,8,10;5,13;0,12 \) for \( i = \{0-13\} \) respectively and
\[ K = \begin{cases} 
0 & \text{for } i = \{1, 2, 3, 5-13\} \\
1 & \text{for } i = 0, j = 1, 3 \text{ and } i = 4, j = 10
\end{cases} \]

For Model 2
\[ j = 1, 3, 5; 2, 3, 6, 13; 0, 3, 7, 0, 4, 8, 9, 10; 0, 1, 7, 12; 2, 5; 3, 5, 9; 4, 11; 11, 14; 10; 5, 13; 0, 12; 0, 4, 15 \text{ and } 5, 9, 14 \text{ for } i = \{0 - 15\} \text{ and } \]
\[ K = \begin{cases} 
0 & \text{for } i = \{1-9, 11-15\} \\
1 & \text{for } i = 0, j = 1, 3 \text{ and } i = 10, j = 14
\end{cases} \]

Taking LST of above relations (7.1) and solving for \( \tilde{N}_0(s) \).

The expected number of visits per unit time are given by
\[
N_0(\infty) = \lim_{s \to 0} s \tilde{N}_0(s) \tag{7.2}
\]

For Model 1
\[
N_0 = \frac{N_{13}}{D_{11}} \text{, where} \tag{7.3}
\]
\[
N_{13} = [(p_{01} + p_{03})V_4V_5V_7+ p_{10,14}(p_{4,10}L_7+p_{9,11}L_8)] V_8V_1V_2V_3 + p_{4,10}V_7V_8L_3 \{p_{03}V_1V_2 + p_{01}p_{13}V_2 + p_{01}p_{23}V_9\}.
\]

and \( D_{11} \) is already defined.

For Model 2
\[
N_0 = \frac{N_{23}}{D_{21}} \text{, where} \tag{7.4}
\]
\[
N_{23} = [(p_{01} + p_{03})V_4V_5V_7+ p_{10,14}(p_{4,10}L_7+p_{9,11}L_8)] V_8V_1V_2V_3 + V_5V_8p_{10,14} \{p_{01}p_{13}V_2 + p_{03}V_1V_2 + p_{01}p_{23}V_9\} \{p_{34}(p_{49}p_{9,11}+p_{4,10})+p_{38}p_{89}(p_{9,11}+p_{94}p_{4,10})\}
\]

and \( D_{11} \) is already defined.

8. Profit Analysis

Profit incurred to the system models in steady state can be evaluated as
\[
P_i = K_0A_0 - K_1B_0 - K_2N_0 \ (i = 1, 2), \text{ where} \tag{8.1}
\]
$K_0$ = Revenue per unit up-time of the system

$K_1$ = Cost per unit time for which server is busy

$K_2$ = Cost per unit visit by the server

and $A_0, B_0, N_0$ are already defined.

### 9. Particular Case

Suppose $g(t) = \alpha e^{-\mu t}$, $h(t) = \theta e^{-\mu t}$.

$g_1(t) = \alpha e^{-\mu t}$, $g_2(t) = \alpha_2 e^{-\mu t}$,

$f(t) = \delta e^{-\mu t}$, $f_1(t) = \delta e^{-\mu t}$

The following results are obtained

$$MTSF(T) = \frac{\left[ \begin{array}{c} r_1 \beta R_4 \left\{ y_\delta \left( Y_1 + T_2 \beta + T_6 \beta \right) + R_5 \left( \beta R_2 + r_1 \alpha_1 \psi \left( T_6 + \beta \right) \right) + K_1 \beta_1^2 r_1 R_2 \left( T_{12} + T_{13} \right) \right\} \\
T_{13} x_\delta \left( T_6 + \beta \right) + r_2 T_6 \right\} + r_2 x_\delta \left( \beta r_2 + \beta T_6 \right) + \alpha_2 K R_4 \left( \beta T_6 T_{12} + \beta \beta, x_\delta T_{13}^2 \right) \right]}{\beta A_1 \left\{ R_3 \left( R_1 + r_1 \right) - r_1 y_\delta \alpha_1 \left\{ \left( T_7 + \beta \right) T_6 + \beta \left( T_2 + \beta \right) \right\} - r_2 R_3 x_\delta \alpha_2 \left( T_{12} + \beta \right) \left( T_7 + \beta \right) \right\} }$$

where

$R_2 = \left( T_1 T_2 - \beta \beta_1 \right)$, $R_3 = \left( T_{12} T_{13} - \beta \beta_1 \right)$,

$Y_1 = \left( T_6 T_7 + \beta \beta_1 \right)$, $R_4 = \left( T_7 T_6 - \beta \beta_1 \right)$,

$A_5 = TT_1 T_2 T_6 T_{12} T_{13}$, $T = \left( \lambda + \beta + \eta \right)$,

$T_1 = \left( \beta + r_2 + \alpha_1 \right)$, $T_2 = \left( \beta + r_2 + \delta \right)$,

$T_6 = \left( \beta_1 + \delta \right)$, $T_7 = \left( \alpha_1 + \beta \right)$,

$T_{12} = \left( \beta_1 + \alpha_2 \right)$, $T_{13} = \left( \beta + \alpha_2 \right)$,

$K_1 = TT_1 T_2 T_6 T_7$.

Also,
Availability \( A_0 \) = \( \frac{N_{11}}{D_{11}} \) (for model 1),

\( (A_0) = \frac{N_{21}}{D_{21}} \) (for model 2) \hspace{1cm} (9.2)

Busy Period \( B_0 \) = \( \frac{N_{12}}{D_{11}} \) (for model 1),

\( (B_0) = \frac{N_{22}}{D_{21}} \) (for model 2) \hspace{1cm} (9.3)

Expected Number of Visits \( N_0 \) = \( \frac{N_{13}}{D_{11}} \) (for model 1),

\( (N_0) = \frac{N_{31}}{D_{21}} \) (for model 2) \hspace{1cm} (9.4)

where,

\[ N_{11} = \left[ \lambda_1\{R_8M_5\{R_2(R_1+r_1T_6)+r_1y\delta R_6\}\}+R_7R_8T_8\alpha\{R_2(R_1+r_1T_6)+r_1y\delta R_6\}\right]A_{50} \]

\[ N_{12} = \lambda_1[R_8[H_1r_1M_7+\{\beta(\alpha\beta+R_7)+\alpha(T_8+\beta)(T_{11}+\beta)\}M_6]+H_1r_1R_2\delta\{T_6(T_{12}+\beta)+\beta \]

\[ (T_{13}+\beta_1)\}+R_3R_7R_8M_6]A_{50} \]

\[ N_{13} = \lambda_1R_8[(r_1+\lambda)M_5R_1R_2+\lambda_7\alpha(T_8+\beta)M_6]A_{50} \]

\[ N_{21} = \left[ \lambda_1\{R_8M_5\{R_2R_9(r_1+r_1T_6)+r_1y\delta R_6\}\}+R_7R_8M_6\alpha\beta_1\{R_8(T_8+\beta)+\beta(T_3+\beta_1)\}\right]A_{51} \]

\[ N_{22} = \lambda_1[R_8[H_1r_1r_1M_7+M_6(T_{15}+\beta)]+R_8M_6\{b\delta_1\{\alpha\beta\theta T_{15}+\beta(T_3+T_{15})R_9\}+(T_{11}+\beta) \]

\[ b\delta_1\alpha\{(T_3+T_{15})+T_8T_{15}\}+\beta_1R_9T_8T_{15}\}+r_1R_2R_9H_1x\delta\{(T_{12}+\beta)T_6+\beta(T_{13}+\beta_1)\}]A_{51} \]

\[ N_{23} = \lambda_1R_8[(r_1+\lambda)M_5R_1R_2R_9+\lambda_7M_6\alpha\beta_1\{(T_8+\beta)T_{15}+\beta(T_{3}+\beta_1)\}]A_{51} \]

\[ D_{11} = \left(\lambda_2\alpha\beta_1^2\beta[H_1H_2+\alpha(T_8+\beta)\theta_0(T_{12}+\beta)H_4]\right)+[R_8\{(\beta(T_4+\beta_1)R_3R_7K_4+\lambda_1\beta\beta_1^2\{(T_3+T_8) \]

\[ K_3R_7+(T_{10}+T_{11})R_3K_3\}H_3\}+\lambda_1\beta\beta_1^2\{K_7T_1T_6(T_2+T_7)\{H_1(\beta_1+\alpha_1)R_1-H_2R_1\}+(T_1+T_6) \]

\[ K_6\{H_1(\beta+\alpha_1)R_2-H_2\alpha_2R_2\}+R_8+\lambda_1\beta_1(T_1+\beta_1)R_1R_2R_8K_8+KH_1R_8\lambda_1\beta_1\gamma_1\delta_1\{(T+T_1) \]

\[ T_2+TT_1\}+T_6(T_7+\beta+\beta_1)\}+\lambda_1r_1y\delta_1\alpha_1(T_6^2(T_7+\beta_1)R_8H_1K_7K_8+\lambda_1\beta_1\alpha_1\gamma_1p\delta(T_6+T_7)R_8 \]

\[ H_1K_7K_8(T_2+\beta_1)+\lambda_1r_1\delta_1\alpha_1\gamma_1\delta(T+T_1)\}+TT_1\}R_8H_1T_2^2K+KH_1\beta_1\alpha_1\gamma_1\{T_3+T_4 \]

\[ R_4K_9+\{R_4T_8(1+\beta_1(T_8+T_{11}))T_{10}\}F_1+\alpha_\lambda_1p\theta_0R_8K_1H_3\{K_4+T_8\}\beta_1T_4T_8\}T_8+T_11 \]

\[ K_9+\beta_1(T_{10}+T_3)T_11+(T_{10}+T_{11})T_3\}F_1]+\alpha_\lambda_1q\theta_0H_4[\beta_1\{R_8(T_3+\beta_1)\{2T_10T_{11}+\beta_1(T_10+T_{11}) \]
\[ M_5 = \{ R_3 R_7 \alpha \theta \{ R_4 + \beta (T_3 + T_{11}) \}. \]

\[ M_6 = \{ R_2 (\lambda R_4 + r_1 r_2 T_6) + r_1 r_2 y \delta R_6 \}. \]

\[ M_7 = \{ R_2 T_6 + y \delta R_6 + \beta R_2 + \beta (T_2 + T_6) \}. \]

\[ A_{50} = \beta_1 K_1 T_3 T_4 T_8 F_8 T_{12} T_{13}. \]

\[ A_{51} = A_{50} T_{14} T_{15}. \]

\[ D_{21} = \{ \beta R_8 (R_3 R_7 (T_4 + \beta_1) K_5 + \beta_1^2 \lambda_1 \{ R_7 K_3 (T_3 + T_8) + (T_{10} + T_{11}) R_3 \}) + \alpha \lambda_1 p \theta R_8 \{ \beta_1 \{ (T_3 + T_4) K_1 F_3 R_5 + \{ R_2 T_8 T_{11} + \beta_1 T_1 T_10 (T_8 + T_{11}) \} + \beta_2 \{ \{ (T_8 + T_{10}) \beta_1 + T_8 T_{10} \} (T_3 + T_{11}) F_6 T_{11} + \beta_1 (T_3 + T_8) T_{10} T_{11} + (T_{10} + T_{11}) T_3 T_8 \} F_7 \} \} H_4 F_4 F_4 H_7 [\beta_1^2 \{ (T_{12} + T_{13}) H_4 K_2 + (T_1 + T_6) R_2 R_2 \} \left\{ R_9 (\lambda + r_1) - \lambda a \delta_1 (T_{15} + \beta) \right\} K_6 + (T_2 + T_7) \{ R_1 R_9 - a \delta_1 (T_{15} + \beta) r_1 r_2 T_6 + r_1 \} K_7 T_1 T_6 + R_1 R_2 \ K K_8 \{ \lambda R_1 a \delta_1 (T_{14} + \beta) + \beta (T_1 + \beta_1) K_9 \} R_7 + R_8 K K_8 T H_7 [a \delta_1 \{ T_{15} + \beta \} \beta_1 T_{15} + \beta (T_{15} + \beta_1) T_{14} \} \left\{ \lambda R_1 R_2 + r_1 r_2 (T_2 + T_6) + \beta \delta R_6 \right\} + \beta_2 \{ (T_14 + T_{15}) \{ R_1 R_2 (\lambda + r_1) - r_1 \alpha \lambda y \delta \{ R_6 + \beta (T_2 + T_6) \} \} + r_1 R_8 H_7 [y \delta \{ (T + T_1) T_2 + T T_1 \} \{ \beta_1 R_4 \{ T_6 (T_7 + \beta) + \beta_1 \} \alpha_1 + (T_1 + \beta) r_2 a \delta_1 R_6 \} + \beta_1 F_4 T_2 \{ r_2 T_7 \} \} \left\{ r_2 a \delta_1 (r_1 y \delta \alpha_1 R_5) \} + R_8 F_4 K K_8 H_7 \beta y \delta r_1 \{ (T_6 + T_7) \} \{ \beta_1 R_9 \{ T_15 + \beta_1 \} r_2 a \delta_1 \} + \alpha_1 R_3 \{ \{ T_6 + \beta_1 \} \} (T_2 + T_6) \} + T_6 (T_7 + \beta_1) \} + \alpha_2 \delta_1 \lambda_1 q \theta R_8 H_8 (T_{10} + \beta_1) \} \{ \beta_1 \{ T_{13} + T_{14} \} T_4 + T_4 T_{15} \} K_2 T_3 + \{ \beta_2 \{ (T_3 + T_8) + T_3 (T_8 + \beta_1) \} F_4 F_7 F_8 \} + \{ (T_10 + \beta_1) q \theta R_8 H_4 F_4 \} \{ \beta_1 \{ \lambda R_1 R_2 K K_8 + \beta_1 \} \{ T_1 + T_6 \} M_2 R K_6 (T_2 + T_7) \} T_1 T_6 K_2 \{ \lambda R_1 + r_1 r_2 T_6 \} + \beta_1 r_2 [y \delta \{ (T + T_1) T_2 + T T_1 \} \} R_6 + \beta_2 \beta_1 y \delta K_4 (T_6 + T_7) \} + \{ (T + T_1) R_2 T_2 T_2 K \} G_1 \{ \beta_1 \{ R_2 R_5 R_9 F_4 \{ (T_4 + \beta_1) R_3 K_4 - \beta_1^2 \{ (T_3 + T_8) \} K_3 \} + \beta_2 \{ (T_3 + T_8) + T_3 (T_8 + \beta_1) \} F_4 F_6 T_2 T_7 \} + \{ (T_1 + T_{15}) R_2 R_2 T_7 T K K_8 \} \} - R_2 R_2 F_4 \{ \alpha \lambda_1 p \theta
\[ G_1[\beta_7 R_6 K_4 F_5 + \beta_1 T_3 F_7 \{ R_6 T_8 T_{11} + \beta_1 T_{10} (T_8 + T_{11}) \} + \beta F_5 F_9 G_3 (T_3 + T_{11}) \{ (T_8 + \beta_1) T_4 + T_8 \beta_1 \} + \beta_1 K_1 G_2 \{ (T_3 + T_{10}) T_{11} + (T_{10} + T_{11}) T_3 \} ] + \alpha_2 r_1 x \delta H_2 [ \beta_1 \{ (T + T_6) T_2 + T T_6 \} (T_{13} + \beta_1) F_2 T_2 (T_6 + \beta) + K_2 \{ \beta_1 \{ (T_{13} + \beta_1) T_{12} + T_{13} (T_{12} + \beta_1) \} + \beta \{ \beta_1^2 (T_{12} + T_{13}) + T_{13} (T_{12} + \beta_1) \} \}] \]

\[ T_3 = (\alpha + \beta), \quad T_4 = (\lambda_1 + \beta), \]
\[ T_8 = (\alpha + \beta_1), \quad T_{10} = (\theta + \beta), \]
\[ T_{11} = (\theta + \beta_1), \quad T_{14} = (\beta + \delta_1), \]
\[ T_{15} = (\beta_1 + \delta_1), \quad R_7 = (T_{10} T_{11} - \beta_1), \]
\[ R_3 = (T_3 T_8 - \beta_1), \quad R_4 = (T_{11} T_8 - \beta_1), \]
\[ R_5 = (T_{11} T_3 + \beta_1), \quad R_6 = (T_6 T_7 + \beta_1), \]
\[ R_9 = (T_{14} T_{15} - \beta_1), \quad R_8 = (T_{12} T_{13} - \beta_1), \]
\[ A_5 = (K_1 T_{12} T_{13}), \quad K_1 = T T_1 T_2 T_6 T_7, \]
\[ K_2 = K_1 T_3 T_4 T_{10} T_{11}, \quad K_3 = A_5 T_4 T_{12} T_{13}, \]
\[ K_4 = A_5 T_{12} T_{13} T_3 T_8, \quad K_5 = A_5 T_3 T_8 T_{11}, \]
\[ K_6 = T_2 T_3 T_4 T_{12} T_{13} T_7 T_8 T_{10} T_{11}, \quad F_8 = T_{10} T_{11}, \]
\[ K_7 = F_8 T_4 T_{12} T_{13} T T_3 T_8, \quad K = F_8 T_3 T_8 T_7 T_{12} T_{13} T_6 T_4, \]
\[ K_8 = T_1 T_2, \quad K_9 = F_8 T_8 T_{12} T_{13}, \]
\[ F_1 = T_3 T_7 T_{12} T_{13}, \quad F_2 = F_8 T_3 T_8 T_{12} T_{13} T_4 T_7, \]
\[ F_3 = T K_8 T_7 T_3, \quad F_4 = T_{14} T_{15}, \]
\[ F_5 = F_8 T_3 T_{12} T_{13}, \quad F_9 = T_6 T_1, \]
\[ F_6 = K_1 T_3 T_{12} T_{13} T_4, \quad F_7 = K_1 T_{12} T_{13} T_4, \]
\[ G_2 = T_{12} T_{13} T_4, \quad G_3 = T_2 T_7, \]
\[ H_1 = \{ R_3 R_7 - \alpha \theta \{ R_4 + \beta (T_3 + T_{11}) \} \}, \quad H_2 = \alpha q \theta \{ R_4 + \beta (T_{10} + \beta) \}, \]
\[ H_3 = \{ R_1 R_2 (\lambda + \tau_1) - R_1 y \delta \alpha_1 \{ T_6 (T_7 + \beta) + \beta (T_2 + \beta) \} \}, \]
\[ H_4 = \{ \lambda R_1 R_2 + \tau_1 \tau_2 (y \delta R_6 + R_2 T_0) \}, \quad H_5 = \alpha q \theta R_8 (T_8 + T_3) (T_{10} + \beta_1). \]
H_7 = \lambda_1 H_1, \quad H_8 = [R_2 \{\lambda R_1 + r_1 r_2 T_6\} + r_1 r_2 y R_6] T_{15},

H_9 = \beta \delta_1 \alpha \lambda_1 (T_3 + \beta_1)(T_6 + \beta).

G_1 = \alpha_2 (T_13 + \beta_1) R_{15} x \delta (T_6 + \beta).

H_6 = [R_1 R_2 \{R_9 (\lambda + r_1) - \lambda a \delta_1 (T_{15} + \beta)\} r_1 R_9 a y R_6 - r_1 r_2 R_2 (T_{14} + \beta_1) - r_1 a_1 y R_9 \beta (T_2 + T_6) - r_1 r_2 y a \delta_1 (T_{15} + \beta) R_6]

10. Conclusion

There is no change in mean time to system failure (MTSF) of the system models by introducing the concept of inspection at complete failure of the unit to reveal the feasibility of repair. Figure 3 indicates that MTSF goes on increasing with the increase of abnormal weather rate (\beta) for fixed values of other parameters. However, it decreases as and when direct failure rate (\lambda) and normal weather rate (\beta_1) increase. It may also be noted that the effect of normal weather rate (\beta) on MTSF is more than that of direct failure rate (\lambda). The profit of models keeps on decreasing with the increase of abnormal weather rate (\beta) and direct failure rate (\lambda) as shown in figures 4 and 5. But system models become more profitable if normal weather rate (\beta_1) increases. And, model 2 is always profitable over model 1. On the basis of the results obtained for a particular case, it is concluded that

(i) The idea of inspection at complete failure of the unit to see the feasibility of repair is useful economically.

(ii) The system models can be made more profitable by controlling the weather conditions.

(iii) The idea of stopping the operation of the system and continuing the repair activities of the unit in abnormal weather is economically beneficial.
Graph between MTSF v/s Abnormal Weather Rate

\[ p=0.4, q=0.6, \alpha=1, \alpha_1=1.2, \lambda_1=0.003, \theta=20, r_1=0.3, r_2=0.5, \delta=20, \alpha_2=2.2, x=0.6, y=0.4 \]

\[ \lambda=0.001, \beta_1=0.1 \]

\[ \lambda=0.005, \beta_1=0.1 \]

\[ \lambda=0.001, \beta_1=0.3 \]

Fig. 3: Abnormal Weather Rate (\( \beta \))

Graph between Profit v/s Abnormal Weather Rate

\[ p=0.4, q=0.6, \alpha=1, \alpha_1=1.2, \lambda_1=0.003, \theta=20, r_1=0.3, r_2=0.5, \delta=20, \alpha_2=2.2, k_0=5000, k_1=450, k_2=150, x=0.6, y=0.4 \]

\[ \lambda=0.005, \beta_1=0.1 \]

\[ \lambda=0.001, \beta_1=0.1 \]

\[ \lambda=0.001, \beta_1=0.3 \]

Fig. 4: Abnormal Weather Rate (\( \beta \))
Graph between Profit vs Abnormal Weather Rate

$p=0.4, q=0.6, \alpha=1, \alpha_1=1.2, \lambda_1=0.003.$
$\theta=20, \ r_i=0.3, \ r_i=0.5, \ a=0.4, \ b=0.6.$
$\delta_i=20, \delta_i=15, \alpha_2=2.2, k_0=5000, k_1=450.$
$k_2=150, \chi=0.6, \eta=0.4$

Fig. 5: Abnormal Weather Rate ($\beta$)