CHAPTER - 4

PROFIT ANALYSIS OF RELIABILITY MODELS SUBJECT TO DEGRADATION WITH NO OPERATION IN ABNORMAL WEATHER
Chapter 4

PROFIT ANALYSIS OF RELIABILITY MODELS SUBJECT TO DEGRADATION WITH NO OPERATION IN ABNORMAL WEATHER

1. Introduction

In the previous chapters, profit analysis of reliability models has been made subject to weather conditions under the following assumptions:

(i) Unit is repaired at all possible levels of damages

(ii) Unit becomes degraded after repair at complete failure

(iii) The degraded unit is replaced by new one if its repair is not feasible

Furthermore, in chapter 2, the operation and repair activities are not performed in abnormal weather. However, in chapter 3, only repair activities are not allowed in abnormal weather. But sometimes there are situations in which operation of the system is not possible when weather is abnormal and thus it becomes necessary to stop the operation of the unit as precautionary measures to avoid excessive damage to the system. And, in such a situation, the repair activities may be done.

In view of the above facts and observations, in this chapter two reliability models for a single-unit system working under two weather conditions – normal and abnormal are developed to evaluate the profit of the system when repair activities including inspection are permitted in abnormal weather. In each model, the unit may fail completely either directly from normal mode or via partial failure. There is a single server who attends the system immediately whenever needed. In model 1, the repair of the unit is done at complete failure while in model 2, the unit is repaired both at its partial and complete failure. The unit becomes degraded after repair at its complete failure; however, it works as new after repair at its partial failure. The server inspects the degraded failed unit to examine the feasibility of repair. If repair of the degraded unit is not feasible, it is replaced by new one in order to avoid the unnecessary expanses on repair. The operation of the system is stopped in abnormal weather. The switch devices
are perfect and switch over is instantaneous. The system is analyzed stochastically by adopting regenerative point technique to obtain some reliability characteristics such as mean time to system failure (MTSF), steady state availability, busy period of the server and expected number of visits by the server. The MTSF and profit of the models are compared graphically for the results obtained for a particular case where all distributions are taken as negative exponential.

2. Notation

\( E \) : Set of regenerative states

\( O \) : Unit is operative and in normal mode

\( \overline{WO} \) : Unit is good but waiting for operation due to abnormal weather

\( r_1/r_2/\lambda \) : Constant failure rate of the unit from normal mode to partial failure mode / partial failure mode to complete failure / normal mode to complete failure

\( \lambda_1 \) : Constant failure rate of the degraded unit

\( \beta/\beta_1 \) : Constant rate of change of weather from normal to abnormal / abnormal to normal weather

\( p/q \) : Probability that repair of the degraded unit is not feasible / feasible

\( PF/PFWO \) : Unit is partially failed and operative in normal weather / waiting for operation due to abnormal weather

\( PUr/PUr \) : Unit is partially failed and operative but under repair / under repair but not working due to abnormal weather

\( FUr/FUr \) : Unit is completely failed under repair in normal weather/ abnormal weather

\( DO/DWO \) : The degraded unit is operative under normal weather / waiting for operation due to abnormal weather

\( DFUi/DFUi \) : The degraded unit is failed and under inspection in normal weather / abnormal weather
\( g(t)/G(t) \): pdf / cdf of repair time of the unit at complete failure and partial failure

\( g_i(t)/G_i(t) \): pdf / cdf of inspection time of the degraded unit

\( h(t)/H(t) \): pdf / cdf of repair time of the unit at complete failure and partial failure

\( q_{ij}(t), Q_{ij}(t) \): pdf / cdf of passage time from regenerative state \( i \) to regenerative state \( j \) or to a failed state \( j \) visiting state \( k \) once in \((0, t]\).

\( M_i(t) \): Probability that the system is up initially in state \( S_i \in E \) is up at time \( t \) without visiting to any other regenerative state.

\( W_i(t) \): Probability that the server is busy in state \( S_i \) upto time \( t \) without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states.

\( m_{ij} \): The unconditional mean time taken by the system to transit from any regenerative state \( S_i \) when it (time) is counted from epoch of entrance in to that state \( S_j \). Mathematically, it can be written as

\[
m_{ij} = \int_0^\infty t d\left[ Q_{ij}(t) \right] = -q_{ij}'(0).
\]

\( \mu_i \): The mean sojourn time in state \( S_i \) which is given by \( \mu_i = E(T) = \int_0^\infty P(T > t) dt = \sum_i m_{ij} \), where \( T \) denotes the time to system failure.

\( S/\odot \): Symbol for Laplace-Stieltjes convolution/Laplace convolution

\( \sim / * \): Symbol for Laplace Steiltjes Transform (LST) / Laplace Transform (LT)

Symbol for derivative of the function

The transition states for each model are regenerative. The possible transitions between states along with transition rates for the system models 1 and 2 are shown in figures 1 and 2 respectively. The transition diagram for model 2 is same as that of model 1 except states \( S_1 \) and \( S_5 \). In model 2, \( S_1 = PU_r \) and \( S_5 = PUP_r \) and there are also transitions from state \( S_1 \) to \( S_0 \) and \( S_5 \) to \( S_4 \) with repair time distribution \( g_i(t) \).
State Transition Diagrams

Fig. 1: Model 1

Fig. 2: Model 2
3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

\[ p_{ii} = Q_v(\infty) = \int q_v(t) \, dt \]  

For Model 1

\[
\begin{align*}
p_{01} &= \frac{r_1}{(\lambda + \beta + r_1)} ,
p_{02} &= \frac{\lambda}{(\lambda + \beta + r_1)} ,
p_{04} &= \frac{\beta}{(\lambda + \beta + r_1)} ,
p_{12} &= \frac{r_2}{(r_2 + \beta)} ,
p_{15} &= \frac{\beta}{(r_2 + \beta)} ,
p_{23} &= g*(\beta) ,
p_{26} &= [1 - g*(\beta)] ,
p_{37} &= \frac{\beta}{(\lambda_1 + \beta)} ,
p_{38} &= \frac{\lambda}{(\lambda_1 + \beta)} ,
p_{40} = p_{51} = p_{73} = 1 ,
p_{62} &= g*(\beta_1) ,
p_{67} &= [1 - g*(\beta_1)] ,
p_{80} &= -qh*(\beta) ,
p_{82} &= -ph*(\beta) ,
p_{89} &= 1 - h*(\beta) ,
p_{94} &= -qh*(\beta_1) ,
p_{96} &= -ph*(\beta_1) ,
p_{98} &= 1 - h*(\beta_1)
\end{align*}
\]

It can be verified that

\[
\begin{align*}
p_{01} + p_{02} + p_{04} &= 1 = p_{12} + p_{15} = p_{23} + p_{26} = p_{37} + p_{38} = p_{40} + p_{51} + p_{73} = p_{80} + p_{82} + p_{89} = p_{94} + p_{96} + p_{98}
\end{align*}
\]

The mean sojourn times \( \mu_i \) is the state \( S_i \) are

\[
\begin{align*}
\mu_0 &= \int_0^\infty P(T > t) \, dt = \frac{1}{\lambda + \beta + r_1} ,
\mu_1 &= \frac{1}{r_2 + \beta} ,
\mu_2 &= \frac{1}{\beta}(1 - g*(\beta)) ,
\mu_3 &= \frac{1}{\lambda_1 + \beta} ,
\mu_4 = \mu_5 = \mu_7 = \frac{1}{\lambda_1} ,
\mu_6 &= \frac{1}{\beta_1}(1 - g*(\beta_1)) ,
\mu_8 &= \frac{1}{\beta}(1 - h*(\beta)) ,
\mu_9 &= \frac{1}{\beta_1}(1 - h*(\beta_1))
\end{align*}
\]
For Model 2

The transition probabilities $p_{01}$, $p_{02}$, $p_{04}$, $p_{23}$, $p_{26}$, $p_{37}$, $p_{38}$, $p_{40}$, $p_{62}$, $p_{67}$, $p_{73}$, $p_{80}$, $p_{82}$, $p_{89}$, $p_{94}$, $p_{96}$, $p_{98}$ same as defined in model 1 and remaining are

$$p_{10} = g_1 * (r_2 + \beta),$$
$$p_{12} = \frac{r_2}{r_2 + \beta} \left[ 1 - g_1 * (r_2 + \beta) \right],$$
$$p_{15} = \frac{\beta}{r_2 + \beta} \left[ 1 - g_1 * (r_2 + \beta) \right],$$
$$p_{51} = g_1 * (\beta_1),$$
$$p_{54} = [1 - g_1 * (\beta_1)].$$

(3.4)

The mean sojourn time $\mu_i$ of the state $S_i$ are given by

$$\mu_1 = \frac{1}{r_2 + \beta} \left[ 1 - g_1 * (r_2 + \beta) \right],$$
$$\mu_5 = \frac{1}{\beta_1} \left[ 1 - g_1 * (\beta_1) \right]$$

(3.5)

and the remaining $\mu_i$ for $i = 0, 2, 3, 4, 6, 7, 8, 9$ are same as derived for model 1.

4. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state $i$ to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

For Model 1

$$\phi_0(t) = Q_{01}(t) \phi_1(t) + Q_{04}(t) \phi_4(t) + Q_{02}(t)$$
$$\phi_1(t) = Q_{15}(t) \phi_5(t) + Q_{12}(t),$$
$$\phi_4(t) = Q_{40}(t) \phi_0(t),$$
$$\phi_5(t) = Q_{51} \phi_1(t)$$

(4.1)

For Model 2

The expressions for $\phi_i(t)$ (for $i = 0,4$) are same as defined for model 1 and remaining are

$$\phi_1(t) = Q_{10}(t) \phi_0(t) + Q_{15}(t) \phi_5(t) + Q_{12}$$
\[ \phi_5(t) = Qs_1(t) \lesssim \phi_1(t) + Qs_4(t), \quad (4.2) \]

Taking LST of above relations (4.1) and (4.2) and solving for \( \tilde{\phi}_0(s) \)

Using this, we have

\[ R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s} \quad (4.3) \]

The reliability of the system models can be obtained by taking Laplace inverse transform of (4.3).

The mean time to system failure (MTSF) is given by

**For Model 1**

\[
\text{MTSF} (T_1) = \frac{\left( p_{04} (1 - p_{15} p_{51}) + p_{01} p_{12} \right) \mu_0 + p_{01} \mu_1 + p_{01} p_{15} \mu_5 + (1 - p_{15} p_{51}) p_{04} \mu_4}{(1 - p_{15} p_{51})(1 - p_{04} p_{40})} (4.4)
\]

**For Model 2**

\[
\text{MTSF} (T_2) = \frac{\mu_0 (1 - p_{02}) (1 - p_{15} p_{51}) + p_{01} \mu_1 + \mu_4 \left( p_{01} p_{15} p_{54} + (1 - p_{15} p_{51}) p_{04} \right) + p_{01} p_{15} \mu_5}{(1 - p_{15} p_{51})(1 - p_{04} p_{40}) - p_{01} p_{10} - p_{01} p_{15} p_{54}} (4.5)
\]

**5. Steady State Availability**

Let \( A_i(t) \) be the probability that the system is in up-state at instant ‘t’ given that the system entered regenerative state i at \( t = 0 \). The recursive relations for \( A_i(t) \) are given as

**For Model 1**

\[
A_i(t) = M_i(t) + \sum_{j} q_{ij}(t) A_j(t) \quad (5.1)
\]

where

\[ j = 1, 2, 4; 2, 5; 3, 6; 7, 8; 0; 1, 2, 7; 3, 0, 2, 9; 4, 6, 8; \text{ for } i = \{0, \ldots, 9\}; \]

respectively and

\[ M_i(t) = 0 \text{ for } i = 2, 4, \ldots, 9 \]

while,

\[ M_0(t) = e^{-i\ast\beta\ast\gamma}, \quad M_i(t) = e^{-(\beta\ast\gamma)}, \]

79
\[ M_j(t) = e^{-(\beta_j + \lambda_j)t} \]

**For Model 2**

\( j = 1,2,4;0,2,5;3,6;7,8;0,1,2,7,3;0,2,9;4,6,8; \) for \( i = \{0-9\} \); respectively and while all \( M_j(t) \) (for \( i = 0,2-9 \)) are same as defined in model I and remaining is

\[ M_1(t) = e^{-(\lambda_1 + \gamma_1)G_i(t)} \]

Taking LT of above relations (6.1) and solving for \( \mathcal{A}_0'(s) \).

The steady-state availability is given by

\[ A_0(\infty) = \lim_{s \to 0} s\mathcal{A}_0'(s) \]  

**For Model 1**

\[ A_0 = \frac{N_{11}}{D_{11}} \text{, where} \]

\[ N_{11} = V_2(\mu_0p_{12} + \mu_1p_{01})p_{38}(V_8-V_3) + \mu_3p_{12}V_8(1-p_{04}) \]

\[ D_{11} = V_1V_4[V_2\{p_{37}Z_7V_8+p_{89}(p_{38}p_{98}+zp_{38}p_{96})\} + p_{38}p_{89}p_{96}(p_{67}p_{62}+p_{23}p_{62}Z_8)+p_{38}p_{26}p_{62}Z_8V_8] \]

\[ +p_{38}[p_{04}Z_2V_2R+p_{15}p_{51}V_2\{V_4R_{p02}V_2\} + \{z_7KV_1V_4+V_5L_{p26}\}p_{67}p_{04}p_{38}p_{89}p_{62}V_1H_2] \]

\[ +p_{23}k_1V_4H_3+V_5k_2L+p_{82}p_{26}Z_8p_{67}V_1V_4 + \{p_{89}p_{94}Z+p_{80}H_4\}V_2L +p_{26}p_{67}Z_8(p_{82}V_1V_4+L) \]

\[ V_1 = (1-p_{15}p_{51}), \quad V_2 = (1-p_{26}p_{02}), \quad V_3 = (p_{89}p_{96}+p_{82}), \]

\[ V_8 = (1-p_{89}p_{98}), \quad V_4 = (1-p_{04}), \quad V_5 = (p_{89}p_{94}+p_{80}), \]

\[ K = (p_{89}p_{96}+p_{26}p_{82}), \quad K_1 = (p_{89}p_{96}p_{62}+p_{82}), \quad R = (V_8-V_3), \]

\[ L = (p_{01}p_{12}+p_{02}V_4) \]

**For Model 2**

\[ A_0 = \frac{N_{21}}{D_{21}} \text{, where} \]

\[ N_{21} = V_2\{(\mu_0V_1+\mu_1p_{01})p_{38}(V_8-V_3)+\mu_3V_8\{p_{01}p_{12}+p_{02}V_1\}\} \]

\[ D_{21} = D_{11}+p_{38}\{p_{01}Z_4V_6+L_1Z_6-p_{01}p_{06}\}V_6Z_7-p_{89}p_{96}\{Z(L_1+p_{01}p_{10}V_2)+(p_{01}V_6p_{67}+L_1p_{26})\} \]
6. Busy Period Analysis

Let $B_i(t)$ be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered regenerative state $i$ at $t = 0$. The recursive relations for $B_i(t)$ are as follows:

$$B_i(t) = W_i(t) + \sum_{c=1}^{9} q_{ij}(t)(c)B_j(t). \quad (6.1)$$

**For Model 1**

where

$$j = 1, 2, 4; 2, 5; 3, 6; 7, 8; 0; 1, 2, 7, 3; 0.2, 9; 4, 6, 8; \text{ for } i = \{0 - 9\}; \text{ respectively and}$$

and $W_i(t) = 0$ for $i = \{0, 1, 3, 5, 7, 9\}$ while

$$W_2(t) = e^{-\beta_1 t} \tilde{G}(t), \quad W_6(t) = e^{-\beta_1 t} \tilde{G}(t), \quad W_8(t) = e^{-\beta_1 t} \tilde{H}(t),$$

$$W_9(t) = e^{-\beta_1 t} \tilde{H}(t).$$

**For Model 2**

$$j = 1, 2, 4; 2, 5; 3, 6; 7, 8; 0; 4, 1, 2, 7, 3; 0, 2, 9; 4, 6, 8; \text{ for } i = \{0 - 9\}; \text{ respectively and}$$

$W_i(t) = 0$ for $i = \{0, 3, 4, 7\}$ however

$W_2(t), W_6(t), W_8(t), W_9(t)$ are same as defined in model I and remaining are

$$W_1(t) = e^{-\gamma_1 t} \tilde{G}_1(t), \quad W_5(t) = e^{-\gamma_1 t} \tilde{G}_1(t)$$

Taking LT of above relations (7.1) and determine $B'_i(s)$ for each model. Using this, we get in the long run, the time for which server is busy as
\[ B_0(\infty) = \lim_{s \to 0} s B_0^r(s) \quad (6.2) \]

For Model 1

\[ B_0 = \frac{N_{12}}{D_{11}}, \text{ where} \quad (6.3) \]

\[ N_{12} = p_{12} p_{38} (1 - p_{44}) \left[ \mu_2 \{ V_{88} p_{89} p_{96} p_{67} \} + \mu_5 \{ V_{88} p_{26} + p_{90} p_{89} p_{96} \} + (\mu_8 + \mu_9 p_{89}) V_2 \right] \]

and \( D_{11} \) is already defined.

For Model 2

\[ B_0 = \frac{N_{22}}{D_{21}}, \text{ where} \quad (6.4) \]

\[ N_{22} = p_{38} (L) (\{ \mu_2 + \mu_6 p_{26} \} V_{88} + p_{89} p_{96} (p_{23} \mu_6 - \mu_2 p_{67}) + V_2 (\mu_8 + \mu_9 p_{89}) + p_{01} p_{15} \mu_5 (V_{88} - V_3) \}) \]

and \( D_{21} \) is already defined.

7. Expected Number of Visits by the Server

Let \( N_i(t) \) be the expected number of visits by the server in \((0, t]\) given that the system entered the regenerative state \( i \) at \( t = 0 \). The recursive relations for \( N_i(t) \) are given as

\[ N_i(t) = \sum_{i,j} Q_{ij}(t) (K + N_j(t)) \quad (7.1) \]

For Model 1

\[ j = 1, 2, 4; 2, 5; 3, 6; 7, 8; 0; 1; 2, 7; 3; 0, 2, 9; 4, 6, 8; \text{ for } i = \{0-9\}; \text{ respectively and} \]

\[ K = \begin{cases} 0 & \text{for } i = \{2, 4-9\} \\ 1 & \text{for } i = 0, j = 2; i = 1, j = 2 \text{ and } i = 3, j = 8 \end{cases} \]

For Model 2

\[ j = 1, 2, 4; 0, 2, 5; 3, 6; 7, 8; 0, 4, 1; 2, 7; 0, 2, 9; 4, 6, 8; \text{ for } i = \{0-9\}; \text{ respectively} \]

and \[ K = \begin{cases} 0 & \text{for } i = \{1, 2, 4-9\} \\ 1 & \text{for } i = 0, j = 1 \text{ and } j = 2; i = 3, j = 8 \end{cases} \]

Taking LST of above relations (7.1) and solving for \( \tilde{N}_i(s) \). The expected number of visits per unit time are given by

82
\[ N_0(\infty) = \lim_{s \to 0} s \tilde{N}_0(s) \quad (7.2) \]

**For Model 1**

\[ N_0 = \frac{N_{13}}{D_{11}}, \text{ where} \quad N_{13} = p_{38}(1-p_{04})V_2p_{12}(V_8-V_3), \text{ and } D_{11} \text{ is already defined.} \quad (7.3) \]

**For Model 2**

\[ N_0 = \frac{N_{23}}{D_{23}}, \text{ where} \quad N_{23} = p_{38}V_2(V_8-V_3)(p_{01}p_{12}+p_{02}V_1), \text{ and } D_{11} \text{ is already defined.} \quad (7.4) \]

**8. Profit Analysis**

Profit incurred to the system model in steady state can be evaluated as

\[ P_i = K_0A_0 - K_1B_0 - K_2N_0 \quad (i = 1, 2), \text{ where} \]

- \( K_0 \) = Revenue per unit up-time of the system
- \( K_1 \) = Cost per unit time for which server is busy
- \( K_2 \) = Cost per unit visit by the server

and \( A_0, B_0, N_0 \) are already defined.

**9. Particular Case**

Suppose \( g(t) = \alpha e^{-\alpha t} \), \( h(t) = \theta e^{-\theta t} \),

\[ g_1(t) = \alpha_1 e^{-\lambda_1 t}, \quad h_1(t) = \theta_1 e^{-\theta_1 t}. \]

The following results are obtained

\[ \text{MTSF} (T1) = \frac{r_1X_1(\beta + \beta_1) + \beta_1r_2(\beta + \beta_1)}{\beta_1X_1X_8} \quad (9.1) \]
MTSF (T2) = \frac{[\beta_1 (r_1 + \beta) Y_4 + r_1 \beta_4 X_1 (X_4 + \beta) + X_4 \beta (r_1 \alpha_1 + Y_4)]}{\beta_1 X_1 \{Y_4 (\lambda + r_1) - r_1 \alpha_1 (X_4 + \beta)\}} \quad (9.2)

Also,

Availability (A_0) = \frac{N_{11}}{D_{11}} \text{ (for model 1)}.

(A_0) = \frac{N_{21}}{D_{21}} \text{ (for model 2)} \quad (9.3)

Busy Period (B_0) = \frac{N_{12}}{D_{11}} \text{ (for model 1)}.

(B_0) = \frac{N_{22}}{D_{21}} \text{ (for model 2)} \quad (9.4)

Expected Number of Visits (N_0) = \frac{N_{11}}{D_{11}} \text{ (for model 1)},

(N_0) = \frac{N_{22}}{D_{21}} \text{ (for model 2)} \quad (9.5)

where,

X = (\beta + r_2 + \alpha_1), \quad X_1 = (\lambda + \beta + r_1),

X_4 = (\alpha_1 + \beta_1), \quad X_8 = (r_2 + \beta),

Y_4 = (\lambda X_4 - \beta \beta_1)

N_{11} = A_{40} X_8 \left[ (r_1 + r_2) \lambda Y_2 \left\{ Y_3 - p\theta (X_6 + \beta_1) \right\} + r_2 (\lambda + r_1) Y_2 X_3 \right],

N_{12} = A_{40} X_8 \lambda_1 (\lambda + r_1) \left\{ (Y_3 X_4 - p\theta \alpha \beta) + (Y_3 + \alpha p\theta) \beta + Y_2 (X_7 + \beta) \right\},

N_{13} = A_{40} X_8 Y_2 (\lambda + r_1) \left\{ Y_3 - p\theta (X_6 + \beta_1) \right\},

N_{21} = A_{40} X_8 Y_2 \left[ \lambda_1 (Y_4 + r_1 X_4) \left\{ Y_3 - p\theta (X_6 + \beta_1) \right\} + Y_3 (r_1 X_4 + \lambda Y_4) \right],

N_{22} = A_{40} X_8 \lambda_1 \left\{ (r_2 X_4 + \lambda Y_4) \left\{ Y_3 (X_3 + \beta) + Y_2 (X_6 + \beta_1) \right\} + r_1 Y_2 \beta \left\{ Y_3 - p\theta (X_6 + \beta_1) \right\} \right\}.
\[ N_{23} = A_{40} \lambda X X_4 Y_2 \left[ \{ r_5 X_4 + \lambda Y_4 \}\{ Y_4 - p\vartheta (X_6 + \beta) \} \right], \]

\[ D_{11} = X_8 r_2 (\lambda + r_1) \left[ 3 \beta^2 \lambda \left\{ (X_6 + X_7) R Y_2 + (X_2 + X_5) Y_3 R_3 \right\} + \beta (X_1 + \beta_1) Y_2 Y_3 R_2 \right] +
\]

\[ r_2 \lambda_4 \left\{ (X_1 + \beta_1) R_4 X_8 \left\{ Y_1 Y_2 - \alpha \beta_1 p\vartheta (X_2 + X_7 + Y_5) \right\} + \lambda_4 \beta^2 \lambda_4 R_2 X_3 \left\{ (\lambda + r_1) \right\} \right\} \]

\[ Y_1 Y_2 - \alpha p\vartheta (X_7 + X_8) \right\} - \alpha \lambda q \lambda_2 \left( X_3 + \beta_1 \right) \left( X_7 + \beta_1 \right) + R_4 \alpha q \theta \beta_1 \lambda_4 \left[ r_2 \right. \]

\[ (X_8 + X_1) \left( X_4 + \beta_1 \right) + (X_2 + \beta_1) \beta X_8^2 \right\} + \lambda_3 \left( X_1 + \beta_1 \right) r_2 R_2 X_8 \alpha \lambda_2 \beta \left( \lambda + r_1 \right) \left[ p (X_2 + X_7) + q \left( X_6 + \beta_1 \right) \right] + \beta \lambda_4 \lambda_5 \theta \lambda_3 \left[ p \left\{ R_2 Y_3 \left( \lambda + r_1 \right) - R_4 \right\} \right.

\[ X_5 \beta^2 \beta_1 \left\{ X_8 \right\} \right\} + q \left( X_6 + \beta_1 \right) X_2 \left( \lambda + r_1 \right) \left( R_2 - R_4 X_5 \right) \right] + \alpha \beta \beta_1 r_2 R_8 X_8 q \theta \left[ (X_2 + X_5) \right.

\[ (2 X_1 + \beta_1) \left( \lambda + r_1 \right) + \lambda X_8^2 \left( \lambda + r_1 \right) \right\} + q \theta \lambda_1 \left[ \beta \left\{ \beta_1 \left( X_6 + X_7 \right) + X_6 X_7 \right\} + \beta, X_7 \right] \right\}

\[ (\lambda Y_4 + r r_2) \alpha \left( X_4 + \beta_1 \right) \]

\[ Z = (\mu_8 + \mu_9), \quad Z_1 = (\mu_1 + \mu_3), \]

\[ Z_2 = (\mu_6 + \mu_4), \quad Z_4 = (\mu_0 + \mu_1), \]

\[ Z_6 = (\mu_4 + \mu_3), \quad Z_7 = (\mu_1 + \mu_2), \]

\[ Z_8 = (\mu_2 + \mu_6), \quad R_1 = X_4 X_5 X_6 X_1, \]

\[ R_2 = X_4 X_5 X_6 X_7, \quad R_3 = X_1 X_5 X_6 X_7, \]

\[ R_4 = X_4 X_5 X_6 X_7, \quad K_2 = (\mu_2 + \mu_5), \]

\[ Y_5 = X_5 X_7 + \beta \beta_1, \quad Y_6 = X_5 X_7 - \beta \beta_1 \]

\[ X_2 = (\alpha + \beta), \quad X_5 = (\alpha_1 + \beta_1), \]

\[ X_6 = (\theta + \beta), \quad X_7 = (\theta + \beta_1), \]

\[ Y_2 = (X_2 X_5 - \beta \beta_1), \quad Y_3 = (X_6 X_7 - \beta \beta_1), \]

\[ X_3 = (\lambda - \beta), \quad A_{40} = \beta X_2 X_3 X_4 X_5 X_6 X_7, \]

\[ D_{21} = \left[ 3 \beta^2 \lambda_4 \left\{ (X_6 + X_7) R_2 Y_2 + (X_2 + X_5) Y_3 R_3 \right\} + \beta (X_1 + \beta_1) Y_2 Y_3 R_2 \right\} \right\{ Y_4 \left( \lambda + r_1 \right) - \eta \alpha_1 \right], \]

85
10. Conclusion

From figures 3 and 4, it is observed that the mean time to system failure (MTSF) of the system models keeps on increasing with increase of abnormal weather rate ($\beta$) for fixed values of other parameters. But there is a decline trends for MTSF when normal weather rate ($\beta_1$) and direct failure rate ($\lambda$) increases. And, MTSF of model 2 is more than that of model 1. Figures 5 and 6 indicate that profit of the system models goes on decreasing with the increase of abnormal weather rate ($\beta$) and direct failure rate ($\lambda$). However, profit of model 2 decreases more rapidly as compared to profit of model 1. It is also interesting to note that there is an increase in the profit of the system models as normal weather rate ($\beta_1$) increases.

Thus, on the basis of results and graphs drawn for a particular case, it is analyzed that

(i) The idea of repairing the unit both at its partial and complete failure subject to weather conditions can be beneficial economically to the system only if rate of change of weather from abnormal to normal ($\beta$) is high and direct failure rate ($\lambda$) is low.
(ii) The idea of stopping the operation of the unit in abnormal weather is helpful in increasing the reliability of the system.
Graph between MTSF v/s Abnormal Weather Rate

\[ p=0.4, q=0.6, \alpha_1=1, \alpha_2=1.2, \lambda_1=0.003, \theta=20, r_1=0.3, r_2=0.5 \]

\[ \lambda=0.001, \beta_1=0.1 \]

\[ \lambda=0.001, \beta_1=0.3 \]

\[ \lambda=0.005, \beta_1=0.1 \]

Fig. 3: Abnormal Weather Rate (\( \beta \))

Graph between MTSF v/s Abnormal Weather Rate

\[ p=0.4, q=0.6, \alpha_1=1, \alpha_2=1.2, \lambda_1=0.003, \theta=20, r_1=0.3, r_2=0.5 \]

\[ \lambda=0.001, \beta_1=0.1 \]

\[ \lambda=0.001, \beta_1=0.3 \]

\[ \lambda=0.005, \beta_1=0.1 \]

\[ \lambda=0.001, \beta_1=0.3 \]

Fig. 4: Abnormal Weather Rate (\( \beta \))
Graph between Profit v/s Abnormal Weather Rate

\[
p=0.4, q=0.6, \alpha=1.2, \lambda_1=0.003, \theta=20, r_1=0.3, r_2=0.5, k_0=5000, k_1=450, k_2=150
\]

Fig. 5: Abnormal Weather Rate (\(\beta\))

Graph between Profit v/s Abnormal Weather Rate

\[
p=0.4, q=0.6, \alpha=1.2, \lambda_1=0.003, \theta=20, r_1=0.3, r_2=0.5, k_0=5000, k_1=450, k_2=150
\]

Fig. 6: Abnormal Weather Rate (\(\beta\))