Chapter VII

Multi-Commodity Perishable Inventory

Problem with Shortages

7.1 INTRODUCTION

A continuous review \((s, S)\) multi-commodity inventory system perishable due to decay and disaster allowing shortages is studied in this chapter. The \(n\) commodities are denoted by \(C_1, C_2, \ldots, C_n\). The maximum inventory level and the re-ordering point of commodity \(C_k\) are \(S_k\) and \(s_k\) respectively, \((k = 1, 2, \ldots, n)\). Lead time is assumed to be zero and the sales are considered as lost during stock out period. Fresh orders are placed whenever the inventory levels of all the commodities fall to or below their re-ordering points after the previous replenishment. Demands for commodity \(C_k\) are assumed to follow Poisson process with rate \(\lambda_k\) and the life times of commodity \(C_k\) follow exponential distribution with parameter \(\omega_k\). The distribution of the times between the disasters is exponential with mean \(1/\mu\). Each unit of commodity \(C_k\), independent of others, survives a disaster with probability \(p_k\) or is destroyed completely with probability \(1-p_k\). The damaged items are disposed off from the inventory immediately.

The objectives of this chapter are to find transient and stationary probabilities of the inventory states and the optimal value of the \(2n\)-tuple,
(s_1, s_2, ..., s_n, S_1, S_2, ..., S_n) at steady state. The scheme of presentation is similar to chapter VI. The time dependent solution is arrived at in section 7.3. Section 7.4 deals with the stationary probabilities and the replenishment periods whereas section 7.5 discusses optimization problem. Some numerical examples are provided in the last section. The present chapter also generalizes the results of chapter II to multi-commodity case.

7.2 NOTATIONS

\( S_k \): Maximum inventory level of commodity \( C_k \) (k = 1, 2, ..., n)

\( s_k \): Re-ordering level of commodity \( C_k \) (k = 1, 2, ..., n)

\( M_k \): \( S_k - s_k \)

\( M \): \( M_1 \times M_2 \times \ldots \times M_n \)

\( q_k \): \( 1 - p_k \)

\( N^0 \): \{0, 1, 2, ..., \}

\( R^+ \): The set of non-negative real numbers

\( E_k \): \{0, 1, 2, ..., \( S_k \) \}

\( E_k s \): \{0, 1, 2, ..., \( s_k \) \}

\( E \): \((E_1 \times E_2 \times \ldots \times E_n) - (E_{1s} \times E_{2s} \times \ldots \times E_{ns})\)

\( \Delta_k \): \( \{(i_1, i_2, ..., i_n) \in E \mid i_k = 0\} \)

\( s^* \): \( f((s_1, s_2, ..., s_n)); f \) is defined in (7.12)

\( S^* \): \((S_1 + 1) \times (S_2 + 1) \times \ldots \times (S_n + 1) - (s_1 + 1) \times (s_2 + 1) \times \ldots \times (s_n + 1)\)

\( E^* \): \{1, 2, ..., \( S^* \) \}

\( \alpha \): \((0, 0, ..., 1); \) \( S^* \) components

\( e \): \((1, 1, ..., 1)^T; \) \( S^* \) components

\( A \): \( (a_{i_1i_2...i_n})_{S^* \times S^*}; a_{i_1i_2...i_n}'s \) are given by (7.4)

\( \delta(i, j) \): 1 if \( i = j \); 0 otherwise
\[
A_{j_k} = \begin{cases} 
\sum_{r=j_k-s_k}^{j_k} \binom{j_k}{r} p_k^{j_k-r} q_k^r & \text{if } j_k > s_k \\
1 & \text{if } j_k \leq s_k 
\end{cases}
\]

\(D_{s^*} = \text{The determinant of the submatrix obtained from } A \text{ by deleting the first } i^* \text{ rows, the last and first } i^* - 1 \text{ columns, } i^* \in E^* - \{S^*\}\)

7.3. TRANSIENT PROBABILITIES

Let \(X_k(t)\) denote the inventory level of commodity \(C_k\) (\(k=1,2,\ldots,n\)) at any time \(t \geq 0\). If \(X(t) = \{X_1(t), X_2(t), \ldots, X_n(t)\}\), then \(\{X(t), t \in \mathbb{R}\}\) is a continuous time Markov chain with state space \(E\). We assume that the initial probability vector of this chain is \(\alpha\).

Let the transition probability matrix of the Markov chain \(\{X(t)\}\) be

\[
P(t) = [P_{i_1j_1, \ldots, i_nj_n}(t)]_{S^* \times S^*}
\]

where

\[
P_{i_1j_1, \ldots, i_nj_n}(t) = \Pr\{X_1(t) = j_1, \ldots, X_n(t) = j_n \mid X_1(0) = i_1, \ldots, X_n(0) = i_n\}
\]

\((i_1, i_2, \ldots, i_n), (j_1, j_2, \ldots, j_n) \in E\) (7.1)

**Theorem 7.1**

The transition probability matrix \(P(t)\) is uniquely determined by

\[
P(t) = \exp(Bt) = I + \sum_{m=1}^{\infty} \frac{B^m t^m}{m!}
\]

where the matrix \(B = A + G\), in which \(A\) and \(G\) are defined as follows:

\[
A = \begin{bmatrix} a_{i_1j_1, \ldots, i_nj_n} \end{bmatrix}_{S^* \times S^*} \text{ and } G = \begin{bmatrix} g_{i_1j_1, \ldots, i_nj_n} \end{bmatrix}_{S^* \times S^*}
\]

\((i_1, i_2, \ldots, i_n), (j_1, j_2, \ldots, j_n) \in E\) (7.3)

with
\[ a_{i_1 j_1 \ldots i_n j_n} = \left\{ \begin{array}{ll}
\left[ \mu + \sum_{k=1}^{n} \left[ 1 - \delta(0, i_k) \right] (\lambda_k + i_k \omega_k) \right] + \mu \prod_{l=1}^{i_1} p_{i_1}^{l_1} p_{i_2}^{l_2} \ldots p_{i_k}^{l_k} & \text{if } i_k = j_k = k = 1, 2, \ldots, n \\
(\lambda_k + i_k \omega_k) + \mu \left( \prod_{k=1}^{n} \frac{i_k}{j_k} \right) \prod_{l=1}^{i_1} p_{i_1}^{l_1} q_{i_1}^{j_1} & \text{if } i_k = j_k + 1; i_l = j_l (l = 1, 2, \ldots, n; l \neq k)
\end{array} \right. \]

\[ = \mu \left( \prod_{k=1}^{n} \left( \frac{i_k}{j_k} \right) \right) p_{i_1}^{j_1} q_{j_1}^{i_1} - 1 \text{ if } \sum_{k=1}^{n} (i_k - j_k) > 1 \]

\[ \left( i_k - j_k \right) \geq 0 \]

otherwise

and

\[ g_{i_1 j_1 \ldots i_n j_n} = \left\{ \begin{array}{ll}
[\lambda_k + (s_k + 1) \omega_k] + \mu (A_{i_1} A_{i_2} \ldots A_{i_n}) & \text{if } i_k = s_k + 1, \text{ and } i_j \leq s_j \text{ for } j \neq k; \\
\mu (A_{i_1} A_{i_2} \ldots A_{i_n}) & \text{if } \sum_{k=1}^{n} (i_k - j_k) > 1; (j_1, j_2, \ldots, j_n) = (S_1, S_2, \ldots, S_n) \]

\[ = 0 \text{ if } (j_1, j_2, \ldots, j_n) \neq (S_1, S_2, \ldots, S_n) \]

Proof:

For a fixed \( i_0 = (i_1, i_2, \ldots, i_n) \), the difference-differential equations satisfied by the transition probabilities are the following:

\[ P_{i_0}^{j_1 j_2 \ldots j_n} (t) = -[\mu + \sum_{k=1}^{n} \left( 1 - \delta(0, j_k) \right) (\lambda_k + j_k \omega_k)] P_{i_0}^{j_1 j_2 \ldots j_n} (t) \]

\[ + \sum_{k=1}^{n} [\lambda_k + (j_k + 1) \omega_k] [1 - \delta(S_k, j_k)] P_{i_0}^{j_1 j_2 \ldots (j_k+1) \ldots j_n} (t) \]

\[ + \mu \left( \sum_{l_1=0}^{s_1-j_1} \sum_{l_2=0}^{s_2-j_2} \ldots \sum_{l_n=0}^{s_n-j_n} \prod_{k=1}^{n} \left( \frac{f_k + l_k}{j_k} \right) p_{i_1}^{l_1} q_{j_1}^{l_1} \right) P_{i_0}^{j_1+l_1 j_2+l_2 \ldots (j_n+l_n)} (t) \]

\[ (j_1, j_2, \ldots, j_n) \in E - \{(S_1, S_2, \ldots, S_n)\} \]
\[ P'_{t_0} S_1 S_2 \ldots S_n(t) = -[\mu + \sum_{k=1}^{n} (\lambda_k + S_k \omega_k)] P'_{t_0} S_1 S_2 \ldots S_n(t) + \sum_{k=1}^{n} [\lambda_k + (s_k + 1) \omega_k] \]

Defining a relation \( \sigma \) on \( E \) as follows: \( \sigma (i) = j \) if \( i < k \) or \( i = k \) or \( i > k \) if \( i \neq k \).

From equations (7.3) - (7.7) we can easily see that the Kolmogorov equations,

\[ P'(t) = P(t)B \quad \text{and} \quad P'(t) = BP(t) \]

with the condition,

\[ P(0) = I \]

are satisfied by \( P(t) \). The solution of (7.8) with (7.9) is (7.2). The finiteness of \( B \) guarantees the convergence of the series in (6.2) and the solution obtained is unique. Hence the theorem.

### 7.4. STEADY STATE PROBABILITIES AND REPLENISHMENT CYCLES

Since the transition from any state \((i_1, i_2, \ldots, i_n)\) to any state \((j_1, j_2, \ldots, j_n)\) in \( E \) is possible with positive probability the Markov chain \( \{X(t), t \geq 0\} \) is irreducible. Therefore

\[ \lim_{t \to \infty} P_{i_1 i_2 \ldots i_n, j_1 j_2 \ldots j_n}(t) = \pi_{j_1 j_2 \ldots j_n} \quad (j_1, j_2, \ldots, j_n) \in E \]
exist and are independent of the initial state. \( \pi_{j_1,j_2,...,j_n} \)'s are obtained by solving

\[
\Pi B = 0 \quad \text{and} \quad \Pi e = 1
\]  

(7.11)
simultaneously.

Define a relation \( \leq \) in \( E \) as follows:

For \((i_1, i_2, ..., i_n), (j_1, j_2, ..., j_n) \in E\), \((i_1, i_2, ..., i_n) \leq (j_1, j_2, ..., j_n)\) if

1. \( i_1 < j_1 \)
2. \( i_1 = j_1; \quad i_2 < j_2 \)
3. \( i_1 = j_1; \quad i_2 = j_2; \quad i_3 < j_3 \)
4. \( i_1 = j_1; \quad i_2 = j_2; \quad i_3 = j_3; \quad i_4 < j_4 \)
5. \( i_1 = j_1; \quad i_2 = j_2; \quad ..., i_n = j_n; \quad i_n < j_n \)

Then clearly \( \leq \) is a partial order relation in \( E \). Arrange the elements of \( E \) in ascending order. In this arrangement \((0, 0, ..., s_n+1)\) will be the first element and \((S_1, S_2, ..., S_n)\) will be the \( S^* \)th element. Now define a function \( f \) from \( E \) to \( E^* \) as

\[
f((i_1, i_2, ..., i_n)) = i^* \quad \text{if} \quad (i_1, i_2, ..., i_n) \text{ is the } i^* \text{th element in the arrangement, } i^* \in E^*.
\]  

(7.12)

Since \( f \) is a bijection, henceforth \((i_1, i_2, ..., i_n)\) will be represented by \( i^* \).

Let \( D_{i^*} \) be the determinant of the submatrix obtained from \( A \) by deleting the first \( i^* \) rows, the last and first \( i^* - 1 \) columns, \( i^* \in E^* - \{S^*\} \), and \( D_{S^*} = 1 \).

With these notations we have

**Theorem 7.2**

The steady state probabilities of the inventory states are given by

\[
\pi_{i_1i_2....i_n} = \frac{D_{i^*}}{\sum_{S^*} D_{i^*}} \quad ; \quad i^* \in E^*
\]  

(7.13)

\[
F(s^*,S^*) \prod_{k' = i^*} (-a_{k',k^*})
\]
where

\[ F(s^*, S^*) = \sum_{i=1}^{S^*} \frac{D_i^*}{\prod_{k=i}^{S^*} (-a_{k^*, k^*})} \]  

(7.14)

**Proof:**

As in the previous chapter, we can see that the solution of (7.11) is

\[ \pi_{i_1, i_2, ..., i_n} = \pi_{i_1} = \frac{D_{i_1}^* \pi_{S^*}}{S^* - 1} \prod_{k=i_1}^{S^*} (-a_{k^*, k^*}) \]

(7.15)

and

\[ \pi_{S_1, S_2, ..., S_n} = \pi_{S^*} = \frac{1}{-a_{S^*, S^*} F(s^*, S^*)} \]

(7.16)

Substituting (7.16) in (7.15) we get (7.13). Hence the theorem.

Let \( T_0 = 0 < T_1 < T_2 < \ldots \ldots \) be the epochs when the orders are placed. This occurs whenever the inventory levels of all the commodities \( C_k \) fall to their reordering levels or below those for the first time after the previous replenishment (\( k = 1, 2, \ldots, n \)). Since lead time is assumed to be zero, the stock level is immediately brought to \( (S_1, S_2, \ldots, S_n) \). Thus clearly \( \{T_m, m \in \mathbb{N}^0\} \) is a renewal process.

Arguing in the similar lines of Theorem 6.3, the expected replenishment cycle time is obtained by

**Theorem 7.3**

If \( E(T) \) represents the expected time between two successive re-orders, then

\[ E(T) = F(s^*, S^*) = \frac{1}{-a_{S^*, S^*} \pi_{S^*}} \]

(7.17)
7.5 COST ANALYSIS

Let $M_k^*$ represent the random re-ordering quantity of commodity $C_k$. Then

$$E(M_k^*) = E(T) \left[ \sum_{(i_1, i_2, \ldots, i_n) \in E - \Delta_k} \pi_{i_1i_2 \ldots i_n} \left\{ \lambda_k + i_k \omega_k + \mu \sum_{j_k=0}^{i_k} j_k \left( p_{j_k} - q_{j_k} \right) \right\} \right]$$

$$= E(T) \left[ \sum_{(i_1, i_2, \ldots, i_n) \in E - \Delta_k} \pi_{i_1i_2 \ldots i_n} \right]$$

(7.18)

Let $h_k$ be the unit holding cost per unit time, $c_k$ the unit procurement cost and $d_k$ the unit damage cost, $b_k$ be the unit shortage cost of commodity $C_k$ ($k=1, 2, \ldots, n$), $K$ be the fixed ordering cost per order. Then the cost function is

$$C(s_1, s_2, \ldots, s_n, S_1, S_2, \ldots, S_n) = \frac{K + \sum_{k=1}^{n} c_k E(M_k^*)}{E(T)}$$

$$+ \sum_{k=1}^{n} \left[ h_k + d_k (\omega_k + \mu q_k) \right] H_k(s^*, S^*) + \sum_{k=1}^{n} \sum_{(i_1, i_2, \ldots, i_n) \in E - \Delta_k} \lambda_k b_k \pi_{i_1i_2 \ldots i_n}$$

(7.19)
7.6 NUMERICAL ILLUSTRATIONS

In this section we provide some numerical examples for two commodity inventory problems. Numerical examples show that optimal values of \( s_k = 0 \), \((k = 1, 2, ... , n)\) when shortage cost is zero. This can be seen from tables 7.3 and 7.4. Figure 7.1 illustrates the effect of disaster on the optimum values of \((s_1, S_1), (s_2, S_2)\). Tables 7.1 and 7.2 compare the optimum values \((s_1, S_1), (s_2, S_2)\) of a two commodity problem for disaster rates \( \mu = 1 \) and 0 respectively when the shortage costs are \( b_1 = 200 \) and \( b_2 = 100 \). The third and fourth tables compare the same when shortage costs are set at zero. The effect of shortage on the optimum inventory level can be seen by comparing tables 7.1 and 7.3, and tables 7.2 and 7.4. The optimum values are found out with the aid of a computer giving upper bounds to \( S_i = 9 \) and assigning \( s_i = 0, 1 ; i = 1,2 \).

Table 7.1
Optimum values \((s_1, S_1), (s_2, S_2)\) for \( \mu = 1 \) and \( b_1=200, b_2=100, p_1=.3, p_2 =.1, K=100, c_1=20, c_2=10, h_1 = 4, h_2 = 2, d_1= 4/3, d_2 = 2/3 \).

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Table 7.2
Optimum values \((s_1, S_1), (s_2, S_2)\) for \(\mu = 0\) and \(b_1=200, b_2=100, p_1=.3, p_2 =.1, K=100, c_1=20, c_2=10, h_1 = 4, h_2 = 2, d_1 = 4/3, d_2 = 2/3\).

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Table 7.3
Optimum values \((s_1, S_1), (s_2, S_2)\) for \(\mu = 1\) and \(b_1=0, b_2=0, p_1=.3, p_2 =.1, K=100, c_1=20, c_2=10, h_1 = 4, h_2 = 2, d_1 = 4/3, d_2 = 2/3\).

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Table 7.4
Optimum values \((s_1, S_1), (s_2, S_2)\) for \(\mu = 0\) and \(b_1=0, b_2=0, p_1=.3, p_2 =.1, K=100, c_1=20, c_2=10, h_1 = 4, h_2 = 2, d_1 = 4/3, d_2 = 2/3\).

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Figure 7.1

(Optimum values of \((s_1, S_1), (s_2, S_2)\)

\[ p_1 = .3, \quad p_2 = .1, \quad K = 100, \quad c_1 = 20, \quad c_2 = 10, \quad h_1 = 4, \quad h_2 = 2, \quad d_1 = 4/3, \quad d_2 = 2/3, \]
\[ b_1 = 200, \quad b_2 = 100, \quad \lambda_1 = 2, \quad \lambda_2 = 1, \quad \omega_1 = 1, \quad \omega_2 = 1. \]

Two Commodity Inventory Problem with Markov Shift in Demand

S. INTRODUCTION

Three models of a two commodity system are discussed in this chapter. The type of commodity inventory problem we consider in this chapter is that in which the demand epochs constitute a Markov chain. Each demand or demand one unit of demand, or one unit of demand is not permitted and the lead time is assumed to be zero. The interarrival times of demands are i.i.d. random variables with absolutely continuous distribution function \(G(.)\) having finite mean \(\mu\).

In the first model, the replenishment policy is to order for \(C_i\) alone so as to bring the inventory level to \(S_i\) whenever the inventory level of \(C_i\) falls to the reordering point \(s_i\) after the previous replenishment \((i = 1, 2)\). In the second model, the replenishment policy is to order for both \(C_1\) and \(C_2\), so as to make the inventory levels maximum \((S_1\) and \(S_2))\) whenever the inventory level of at least one of the commodities reaches its reordering point \((s_1\) or \(s_2\)) after the previous replenishment.

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