Chapter V

Single Commodity Inventory System with General Disaster Periods

5.1. INTRODUCTION

This chapter deals with a single commodity continuous review \((s,S)\) inventory system in which commodities are damaged due to disaster only. Shortages are not permitted and lead time is assumed to be zero. The demands constitute Poisson process with parameter \(\lambda\). The times between disasters follow general distribution \(G(.)\) which is absolutely continuous with finite mean \(m\). Each unit in the stock, independent of others, either survives a disaster with probability \(p\), or damages completely with probability \((1 - p) = q\). The failed items are disposed off immediately.

The structure of this chapter is similar to chapter 4. As in the previous chapter, the principal aim of the present chapter is to derive the transient and steady state probabilities of the inventory level. A special case in which the disaster affects only the exhibiting item is discussed in Section 5.4. For this special case, the steady state distribution of the inventory levels is shown to be uniform. Illustrations are provided in Section 5.4.1.
Notations

\[ E = \{s+1, s+2, \ldots, S\} \]
\[ M = S - s \]
\[ q = 1 - p \]
\[ N^0 = \{0, 1, 2, \ldots\} \]
\[ \bar{G}(t) = 1 - G(t) \]
\[ \delta(j) = \begin{cases} 1 & \text{if } j \geq 0 \\ 0 & \text{if } j < 0 \end{cases} \]
\[ Q^n(i, j, t) : \text{n-fold convolution of } Q(i, j, t) \text{ with itself.} \]
\[ Q^0(i, j, t) : \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \]

5.2. ANALYSIS OF THE INVENTORY LEVEL

Let \( X(t) \) be the inventory level at time \( t (t \geq 0) \). Then \( X(t) \) takes values on \( E = \{s+1, s+2, \ldots, S\} \). Assume that the disaster epochs are \( 0 = T_0 < T_1 < T_2 < \ldots \).

Define \( X_n = X(T_n+) \), \( n \in N^0 \).

Theorem 5.1

The stochastic process \( (X_n, T_n), n \in N^0 \) is a Markov renewal process with state space \( E \) and the semi-Markov kernel \( \{Q(i, j, t), i, j \in E, t \geq 0\} \) where

\[ Q(i, j, t) = \Pr\{X_{n+1} = j, \ T_{n+1} - T_n \leq t \ | \ X_n = i\} \]
\[ \delta(i-j) \sum_{r=0}^{i-j} \binom{j+r}{j} p^j q^r \int_0^{\lambda u} \left( \frac{\lambda u}{i-j-r} \right)^{i-j-r} e^{-\lambda u} dG(u) \]

\[ + \sum_{k=1}^{\infty} \sum_{r=0}^{S-1} \binom{j+r}{j} p^j q^r \int_0^{\lambda u} \left( \frac{\lambda u}{kM+i-j-r} \right)^{kM+i-j-r} e^{-\lambda u} dG(u) \]

\[ \text{for } j \neq S \]

\[ \delta(i-S) p^S \int_0^{\lambda u} e^{-\lambda u} dG(u) + \sum_{k=0}^{\infty} p^S \int_0^{\lambda u} \left( \frac{\lambda u}{kM+i-s} \right)^{kM+i-s} e^{-\lambda u} dG(u) \]

\[ + \sum_{a=s+1}^{i} \sum_{r=a-s}^{a} \binom{a}{r} p^{a-r} q^r \int_0^{\lambda u} \left( \frac{\lambda u}{i-a} \right)^{i-a} e^{-\lambda u} dG(u) \]

\[ + \sum_{k=1}^{\infty} \sum_{a=s+1}^{S} \sum_{r=a-s}^{a} \binom{a}{r} p^{a-r} q^r \int_0^{\lambda u} \left( \frac{\lambda u}{kM+i-a} \right)^{kM+i-a} e^{-\lambda u} dG(u) \]

\[ \text{for } j = S \]

Proof:

Since the demand process is Poisson, the interarrival times are exponentially distributed. Hence \( X_{n+1} \) depends only on \( X_n \) and \( T_n \). Therefore \( \{(X_n, T_n), n \in \mathbb{N}^0\} \) is a Markov Renewal Process with state space \( E \).

Let the number of replenishments in \((0, t)\) be \( N(t) \) and define

\[ \Omega_k(i, j, t) = \Pr\{X_1 = j, N(T_1) = k, T_1 \leq t \mid X(0+) = i\}; k = 0, 1, 2, \ldots \]

then semi-Markov kernel \( Q(i, j, t) \) is given by

\[ Q(i, j, t) = \sum_{k=0}^{\infty} \Omega_k(i, j, t) \]

To derive the expression for \( \Omega_k(i, j, t) \) \((k = 0, 1, 2, \ldots)\) assume that the next disaster after the initial one occurs in \((u, u+\delta u)\) where \( u < t \). There are five cases.

1. \( k = 0 \) and \( j \neq S \).

In this case there is no replenishment in \((0, u]\). Assume that the disaster that happened at time \( u \) destroys \( r \) items \((r = 0, 1, 2, \ldots, i-j \text{ if } i \geq j)\). In order that the inventory level is \( j \) just after this disaster, the inventory level must
have reduced to \( j+r \) (\( j+r \leq i \)) due to demands in (0, u) from the initial inventory level \( i \). Therefore

\[
\Omega_0(i, j, t) = \delta(i-j) \sum_{r=0}^{j-1} \binom{j+r}{r} p^j q^r \int_0^u \frac{(\lambda u)^{i-j-r} e^{-\lambda u}}{(i-j-r)!} \, dG(u)
\]

(5.5)

(2) \( k \neq 0, j \neq S \).

Here there are \( k \) replenishments in (0, u) due to depletion of inventory by demand and the stock level is instantaneously brought to \( S \) each time. If the disaster at \( u \) destroys \( r \) items (\( r = 0, 1, 2, \ldots, S - j \)), in order to have the inventory level \( j \) just after the disaster at \( u \), the arrivals in (0, u) must have demanded \( (kM + i - j - r) \) units in (0, u). Hence

\[
\Omega_k(i, j, t) = \sum_{r=0}^{S-j} \binom{j+r}{r} p^j q^r \int_0^u \frac{(\lambda u)^{kM+i-j-r} e^{-\lambda u}}{(kM+i-j-r)!} \, dG(u)
\]

(5.6)

(3) \( k = 1, j = S \).

There are two possibilities. (i) There is exactly one replenishment due to demand in (0, u) and the S units in the inventory survives the disaster at \( u \). (ii) There is no replenishment in (0, u) and a replenishment is triggered by the disaster at \( u \). The former case will happen when the demands in (0, u) are exactly for \( i - s \) units and the disaster at \( u \) affects none of the items in the stock. The latter case happens when the inventory level is \( a \) just before the disaster and at least \( (a - s) \) units (\( s+1 \leq a \leq i \)) are destroyed by the disaster at \( u \). So we have

\[
\Omega_1(i, S, t) = p^S \int_0^u \frac{(\lambda u)^{i-s} e^{-\lambda u}}{(i-s)!} \, dG(u)
\]

\[
+ \sum_{a=s+1}^i \sum_{r=a-s}^a \binom{a}{r} p^a q^r \int_0^u \frac{(\lambda u)^{i-a} e^{-\lambda u}}{(i-a)!} \, dG(u)
\]

(5.7)

(5.3) TIME DEPENDENT AND LIMITING DISTRIBUTIONS.
In this case also there are two possibilities. (i) There are exactly \( k \) replenishments due to demand in \((0, u)\) and the \( S \) units in the inventory survive the disaster at \( u \). (ii) There are \((k - 1)\) replenishments due to demand in \((0, u)\) and a replenishment is triggered by the disaster at \( u \). In the former case exactly \([(k - 1)M + i - s]\) units are demanded in \((0, u)\) and the disaster at \( u \) affects none of the items in the stock. The latter case happens when the inventory level is brought to \( a \) by \([(k - 1)M + i - a]\) demands in \((0, u)\) and at least \( a - s \) units \((s + 1 \leq a \leq S)\) are destroyed by the disaster at \( u \). So we have

\[
\Omega_k (i, S, t) = p^S \int_{0}^{(k - 1)M + i - s} e^{-\lambda u} (\lambda u)^{(k - 1)M + i - s - 1} dG(u) + \sum_{a=s+1}^{S} \sum_{r=a-s}^{a} \binom{a}{r} p^{a-r} q^r \int_{0}^{(k - 1)M + i - a} e^{-\lambda u} (\lambda u)^{(k - 1)M + i - a - 1} dG(u)
\]

(5) \( k = 0, \ j = S \).

This happens only when \( i = S \), when there is no demand in \((0, u)\) and the disaster at time \( u \) affects none of the units in the stock. So we get

\[
\Omega_0 (S, S, t) = p^S \int_{0}^{(k - 1)M + i - a} e^{-\lambda u} dG(u)
\]

Substituting (5.5) - (5.9) in (5.4) we get (5.2). Hence the theorem.

5.3 TIME DEPENDENT AND LIMITING DISTRIBUTIONS

Let \( p(i, j, t) = \Pr \{ X(t) = j \mid X(0+) = i \} \), \( i, j \in E \). Then we have
Theorem 5.2

The time dependent probabilities of the inventory levels are given by

\[ p(i,j,t) = \sum_{r=s+1}^{S} \int_{0}^{\infty} R(i,r,dr) k(r,j,t-u); i, j \in E \] (5.10)

where

\[ R(i,j,t) = \sum_{n=0}^{\infty} Q^n(i,j,t); i, j \in E \]

and

\[ k(i,j,t) = G(t) \sum_{n=\delta(j-i-1)}^{\infty} \frac{(\lambda t)^{nM+i-j} e^{-\lambda t}}{(nM+i-j)!} \] (5.11)

Proof:

The stochastic process \( \{X(t), t \geq 0\} \) is a semi-regenerative process with the embedded MRP \( \{(X_n, T_n), n \in \mathbb{N}^0\} \). Conditioning on the first disaster epoch \( T_1 \) we see that \( p(i,j,t) \)'s satisfy the following Markov renewal equations,

\[ p(i,j,t) = k(i,j,t) + \sum_{r=s+1}^{S} \int_{0}^{\infty} Q(i,r,dr) p(r,j,t-u); i, j \in E \] (5.12)

where \( k(i,j,t) = \Pr\{X(t) = j, T_1 > t | X(0+) = i\}; i, j \in E \).

To derive the expressions of \( k(i,j,t) \) in (5.11) note that, since \( T_1 > t \), the depletion of inventory is only due to demand and there may be \( n \) replenishments in \( (0, t) \). If \( i < j \) then there should be at least one replenishment and \( n \) varies from 1 to \( \infty \) in (5.11), otherwise \( n \) varies from zero to \( \infty \). The solution of (5.12) is given by (5.10). Hence the theorem.

Consider the underlying Markov chain \( \{X_n, n \in \mathbb{N}^0\} \) associated with the MRP \( \{(X_n, T_n), n \in \mathbb{N}^0\} \). Its transition probability matrix \( Q = (q_{ij}) \); \( i, j \in E \), is given by

\[ \rho_j = \frac{\sum_{i \in E} \int_0^t k(i,j,t) dt}{m}; j \in E \] (5.15)
Since the transition from any state $i$ to any state $j$ ($i, j \in E$) is possible with positive probability, the finite Markov chain $\{X_n, n \in \mathbb{N}\}$ is irreducible and hence it is recurrent. Since the chain is irreducible, it possesses a unique stationary distribution,

$$\Pi = (\pi_{s+1}, \pi_{s+2}, \ldots, \pi_S)$$

which satisfies $\Pi Q = \Pi$ and $\sum \pi_j = 1. \quad (5.14)$

Let $P = (p_{s+1}, p_{s+2}, \ldots, p_S)$ denote the steady state probability vector of the inventory level where $p_j = \lim_{t \to \infty} p(i, j, t)$. Then we have

**Theorem 5.3**

If $G(t)$ is absolutely continuous with finite expectation, $m$, then the steady state probabilities of the inventory levels are given by

$$p_j = \frac{\sum \pi_i \int_0^\infty k(i, j, t) dt}{m} \quad (5.15)$$
Since $G(t)$ is absolutely continuous with finite expectation, it follows from (5.10) - (5.12) that

$$\sum_{i \in E} \pi_i \int_0^\infty k(i, j, t) dt = \sum_{i \in E} \pi_i m_i,$$

where $m_i = \text{mean sojourn time in state } i = \int_0^\infty t dG(t) = m$. Substitution yields (5.15).

Hence the theorem.

### 5.4 A SPECIAL CASE

In this sub-section we discuss a special case in which the disaster affects only an exhibiting item.

**Theorem 5.4**

If the disaster affects only an exhibiting item and it is replaced instantaneously by another one upon failure, then the steady state probabilities of the inventory level are uniformly distributed.

**Proof:**

In this case the semi-Markov kernel \(\{Q(i, j, t), i, j \in E, t \geq 0\}\) is given by

$$Q(i, j, t) = p \sum_{k=\delta}^{\infty} \int_{(j-i+1)} e^{-\lambda u} \frac{(i-j+kM)(i-j+kM)!}{(i-j+kM-1)!} dG(u)$$

$$+ q \sum_{k=\delta}^{\infty} \int_{(j-i)} e^{-\lambda u} \frac{(i-j+kM)(i-j+kM-1)!}{(i-j+kM-1)!} dG(u)$$

for \(i, j \in E\).
and the transition probabilities,

\[ q_{ij} = Q(i, j, \infty) = p \sum_{k=\delta}^{\infty} \int_{0}^{\infty} e^{-\lambda u} \frac{(\lambda u)^{(i-j+kM)}}{(i-j+kM)!} dG(u) + \int_{0}^{\infty} e^{-\lambda u} \frac{(\lambda u)^{(i-j+kM-1)}}{(i-j+kM-1)!} dG(u) \]

for \( i, j \in E \) (5.18)

Since \( q_{ij} \) is a function of \((i-j)\),

\[ \sum_{i=s+1}^{n} q_{ij} = p \sum_{n=0}^{\infty} \int_{0}^{\infty} e^{-\lambda u} \frac{(\lambda u)^n}{n!} dG(u) + q \sum_{n=0}^{\infty} \int_{0}^{\infty} e^{-\lambda u} \frac{(\lambda u)^n}{n!} dG(u) \]

\[ = (p+q) \int_{0}^{\infty} dG(u) = 1 \]

for \( j \in E \) (5.19)

Therefore the transition probability matrix \( Q \) is doubly stochastic and from the uniqueness of solution it follows that the invariant measure \( \pi_j = 1/M \) for \( j \in E \). Also note that

\[ \sum_{i=s+1}^{\infty} \int_{0}^{\infty} k(i, j, du) = \int_{0}^{\infty} \sum_{n=0}^{\infty} (\lambda u)^n e^{-\lambda u} \frac{[1-G(u)]}{n!} du = \int_{0}^{\infty} [1-G(u)] du = m \]

Therefore from (5.15) we get \( p_j = 1/M \) for \( j \in E \). Hence the theorem.

### 5.4.1 Illustrations

As in chapter 4 we shall illustrate the above results by taking the general distribution of disaster periods as gamma distribution with parameters \((\nu, \mu)\). Then for \( i, j \in E \),
\[ Q(i, j, t) = p \sum_{k=0}^{\infty} \int_{0}^{\infty} e^{-(\lambda+\mu)u} A(i-j+kM) \mu^\nu u(i-j+kM+v-1) \frac{du}{(i-j+kM)!(v-1)!} \]

\[ + q \sum_{k=0}^{\infty} \int_{0}^{\infty} e^{-(\lambda+\mu)u} A(i-j+kM) \mu^\nu u(i-j+kM+v-2) \frac{du}{(i-j+kM-1)!(v-1)!} \]

Therefore

\[ q_{ij} = Q(i, j, \infty) = p \sum_{k=0}^{\infty} \left( \frac{n+v-1}{n} \right) \left( \frac{\mu}{\mu+\lambda} \right)^{\nu} \left( \frac{\lambda}{\mu+\lambda} \right)^{n} \]

\[ + q \sum_{k=0}^{\infty} \left( \frac{n+v-1}{n} \right) \left( \frac{\mu}{\mu+\lambda} \right)^{\nu} \left( \frac{\lambda}{\mu+\lambda} \right)^{n} \]

for \( i, j \in E \)

\[ \sum_{i=0}^{\infty} q_{ij} = (p+q) \left( \frac{\mu}{\mu+\lambda} \right)^{\nu} \sum_{n=0}^{\infty} \left( \frac{n+v-1}{n} \right) \left( \frac{\lambda}{\mu+\lambda} \right)^{n} \]

\[ = \left( \frac{\mu}{\mu+\lambda} \right)^{\nu} \left( 1 - \frac{\lambda}{\mu+\lambda} \right) = 1 \]

for \( j \in E \)

Since \( \hat{G}(t) = e^{-\mu t} \sum_{a=0}^{\nu} \frac{(\mu t)^a}{a!} \)

\[ \int_{0}^{\infty} k(i, j, t) dt = \sum_{n=0}^{\infty} \frac{1}{\mu+\lambda} \sum_{a=0}^{\nu-1} \left( \frac{\mu}{\mu+\lambda} \right)^{a} \left( \frac{\lambda}{\mu+\lambda} \right)^{b} \]

for \( i, j \in E \)

Therefore

\[ \sum_{i=0}^{S} \int_{0}^{\infty} k(i, j, t) dt = \frac{1}{\mu+\lambda} \sum_{a=0}^{\nu-1} \left( \frac{\mu}{\mu+\lambda} \right)^{a} \sum_{b=0}^{\lambda} \left( \frac{\lambda}{\mu+\lambda} \right)^{b} \]

\[ = \frac{1}{\mu} \sum_{a=0}^{\nu-1} \left( \frac{\mu}{\mu+\lambda} \right)^{a+1} \left( 1 - \frac{\lambda}{\mu+\lambda} \right)^{-(a+1)} \]

for \( j \in E \)
In this case, the expected replenishment cycle time is \( M/(\lambda + \mu q/v) \).
Therefore, if the fixed ordering cost is \( K \), unit purchase cost of the item is \( c \),
the holding cost per unit time is \( h \), and the unit damage cost is \( d \), then the cost
function to be minimized is

\[
C(s,S) = \frac{(K + cM)}{M} + \frac{h}{M} \sum_{i=s+1}^{S} i + \frac{d \mu q}{v}
\]

\[
= \left[ \left( \frac{K}{M} \right) + c \left( \frac{\lambda + \mu q}{v} \right) \right] + h s + \left( \frac{h}{2} \right) (M + 1) + \frac{d \mu q}{v}
\]

which is minimum for \( s = 0 \). Therefore, the optimum cost function reduces to

\[
C(0,S) = \left[ \left( \frac{K}{S} \right) + c \left( \frac{\lambda + \mu q}{v} \right) \right] + h s + \left( \frac{h}{2} \right) (S + 1) + \frac{d \mu q}{v}
\]

which is minimum for \( s = 0 \). Therefore, the optimum cost function reduces to

\[
C(0,S) = \left[ \left( \frac{K}{S} \right) + c \left( \frac{\lambda + \mu q}{v} \right) \right] + h s + \left( \frac{h}{2} \right) (S + 1) + \frac{d \mu q}{v}
\]

(5.28)

Since

\[
\Delta^2 C(0,S) = \frac{2K[\lambda + \mu q]}{S(S+1)(S+2)} > 0,
\]

(5.29)

the cost function in (5.28) is convex. If \( S^* \) denotes the optimum value of \( S \),
then it is given by

\[
S^* (S^* - 1) < \frac{2K[\lambda + \mu q]}{h} < S^* (S^* + 1)
\]

(5.30)

The following three tables show that there is increase in the optimum
values of \( S \) with the increase of the values of \( \lambda, \mu \) and \( q \). In all the tables \( K = 200 \), \( h = 2.5 \) and \( v = 3 \). Figure 5.1 depicts the effect of disaster on the cost
function.

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(Optimum values of $S$ for $\lambda = 6$)

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Table 5.3
(Optimum values of $S$ for $\lambda = 11$)

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Figure 5.1
(The effect of disaster on the cost function)
\[ K = 200, c = 20, h = 2.5, d = 5, \lambda = 1, \nu = 3, q = 0.5. \]