CHAPTER – 3

FUZZY EOQ MODEL FOR DETERIORATING ITEMS WHERE DEMAND DEPENDS UPON SELLING PRICE AND FREQUENCY OF ADVERTISEMENTS
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3.1 Introduction

Deterioration of an item plays an important role in inventory management. The commodities like food grains vegetables, fruits, chemicals etc. deteriorate (dryness, damage, spoilage vaporization etc.) during their storage period. Therefore the loss due to deterioration cannot be ignored while determining the optimal inventory policy.

Certain models have been developed in the area of deteriorating inventories considering the demand rate to be constant, stock-dependent, time-dependent, ramp type or selling price dependent. A significant research has been done with constant demand. Cohen M.A.[29] used constant demand rate to obtain an ordering policy. Giri B.C., Pal S., Goswami A. and Chudhari K.S.[46] and Datta T.K. and Pal A.K.[34] are developed an EOQ models with stock-dependent demand. Hariga M.[55], Chang H.J., Dye C.Y.[19], and Teng J.T., Yang H.L. and Ouyang L.Y.[96] taken time-varying demand where Wu K.S., Ouyang L.Y.[103] and Wee H.M.[99] used ramp type and selling price dependent demand respectively. However in the present competitive market, the frequency of advertisement of an item changes its demand. The advertisement of an item by well-known media such as Newspapers, Magazines, Radio, T.V.,
Cinema and through the sales representatives motivate the people to buy more and more.

Also the selling price is one of the important factors in purchasing of an item. Naturally lesser selling price increases the demand whereas higher price has the reverse effect. Hence it can be realistically assumed that the demand rate is a function of frequency of advertisements and selling price of an item simultaneously.

Recently, Bhunia A.K. and Maiti M.[13] are used the demand depend upon selling price, frequency of advertisement and linear trend in time to develop inventory model for deteriorating items.

For developing the EOQ model, it is always considered that the deterioration rate, various costs in inventory control are constant in the crisp model. But in reality, it is not so certain. Hence this uncertainty should be treated as fuzzy number.

As discussed in section 2.1, Yao et al [105,107] used the Centroid and Signed distance methods to defuzzify the total cost.

Currently Chen S.H. and Wang C.C. [22] consider the backorder inventory model with the fuzzy yearly demand, fuzzy order cost, fuzzy inventory cost and fuzzy backorder cost. Then Function principle method is used to defuzzify the total cost. Similarly in [23,24], Chen S.H. used the same Function principle method but these models are also for non-deteriorating items.

Here in this chapter the total cost and optimum order quantity are obtained in fuzzy sense for deteriorating items and specially considering demand being dependent on selling price and
frequency of advertisement by using function principal method for defuzzification to obtain total fuzzy inventory cost. Also the median rule is apply to find the optimum Economic Order Quantity [EOQ] and shortage quantity. Solution procedure is illustrated by a numerical example. The sensitivity analysis of the optimum solution with respect to the changes in the different parameter values is also discussed.

3.2 Assumptions

1) The scheduling period is constant and no lead-time.

2) Demand rate \( R \) is dependent linearly on the unit selling price and non-linearly on frequency of advertisement i.e.,

\[
R = (a-bp)N^\alpha
\]

where \( a, b \) and \( \alpha \) are non-negative constants.

3) Shortages are allowed and totally backlogged.

4) Deteriorating rate is age specific failure rate.

5) The advertisement cost is fraction of the total selling price per cycle.

3.3 Notations

\( T \) : Scheduling time of one cycle.

\( R \) : Demand rate per unit time; \( R = (a-bp)N^\alpha \).

\( \theta \) : Deterioration rate.
Q(t) : Inventory level at time t.

C_H : Total Holding cost per cycle.

C_1 : Holding cost per unit.

C_S : Total Shortage cost per cycle.

C_2 : Shortage cost per unit.

S_d : Total deteriorating units.

C_D : Total deteriorating cost per cycle.

C_d : Deteriorating cost per unit.

C_A : Advertisement cost per cycle.

P : Selling price per unit.

N : Number of advertisements.

μ : Advertisement cost \( (0 < μ < 1) \)

S : Initial stock level.

S_1 : Maximum shortage level.

TC : Total inventory cost per cycle.

(wavy bar (~) represents the fuzzification of the parameters)
3.5 Mathematical Analysis

3.5.1 Crisp Model

The initial stock level is $S$ at time $t = 0$, then inventory level decreases due to demand mainly and partially by deterioration. The stock reaches to zero level at $t = t_1$. Then shortages occur and accumulate to the level $S_1$ at $t = T$.

The differential equation describing the state of inventory in the interval $(0, t_1)$ is given by

$$\frac{d}{dt}Q(t)+\theta Q(t)=-\left(a-bp\right)N^\alpha; \quad 0 \leq t \leq t_1 \quad --- (3.1)$$
Solving above differential equation using boundary condition at 
\( t = 0, Q(t) = S \), we get,
\[
Q(t) = -\frac{(a-b)pN^q}{\theta} + \left(\frac{S\theta + (a-b)pN^q}{\theta}\right)e^{-\theta t} \quad 0 \leq t \leq t_1 \quad --- (3.2)
\]
using boundary condition at \( t = t_1, Q(t_1) = 0 \), we get
\[
t_1 = \frac{1}{\theta} \log \left\{ 1 + \frac{S\theta}{(a-b)pN^q} \right\} \quad --- (3.3)
\]
The differential equation describing the state of inventory in the
interval \( (t_1, T) \) is given by,
\[
\frac{d}{dt} Q(t) = -(a-b)pN^q \quad ; \quad t_1 \leq t \leq T \quad --- (3.4)
\]
integrating both sides and solving using condition at \( t = t_1, Q(t_1) = 0 \)
, we get,
\[
Q(t) = -(a-b)pN^q t + (a-b)pN^q t_1 \quad ; \quad t_1 \leq t \leq T \quad --- (3.5)
\]
using condition at \( t = T, Q(t) = -S_1 \), we get,
\[
S_1 = (a-b)pN^q T - (a-b)pN^q \frac{1}{\theta} \log \left( 1 + \frac{S\theta}{(a-b)pN^q} \right) \quad --- (3.6)
\]
Total deteriorating units during the time interval \((0, T)\) are
\[
S_d = \int_0^{t_1} \theta Q(t) \, dt \quad ; \quad 0 \leq t \leq t_1
\]
\[
S_d = \theta \int_0^{t_1} \left[ -\frac{(a-b)pN^q}{\theta} + \left(\frac{S\theta + (a-b)pN^q}{\theta}\right)e^{-\theta t} \right] \, dt
\]
Solving above integral, we get,
\[
S_d = -(a-b)pN^q t_1 - \frac{S\theta + (a-b)pN^q}{\theta} (e^{-\theta t_1} - 1)
\]
62
Therefore the deteriorating cost is given by,

\[ C_D = C_d S_d \]

\[ \therefore C_D = C_d \theta \left[ \frac{(a-b)pN^\alpha t_1}{\theta} - \left( \frac{S\theta + (a-b)pN^\alpha}{\theta^2} \right)(e^{-\theta t_1 - 1}) \right] \]

--- (3.7)

Holding cost over the time period \((0, T)\) is given by,

\[ C_H = C_1 \int_0^t Q(t)dt \]

Solving above integral using equation (3.2), we get

\[ C_H = C_1 \left[ \frac{(a-b)pN^\alpha t_1}{\theta} - \left( \frac{S\theta + (a-b)pN^\alpha}{\theta^2} \right)(e^{-\theta t_1 - 1}) \right] \]

--- (3.8)

Shortage cost is given by

\[ C_S = C_2 \left[ \int_{t_1}^T Q(t)dt \right] \]

Solving above integral by using equation (3.5), we get

\[ C_S = C_2 \left[ \frac{(a-b)pN^\alpha}{2}(T-t_1)^2 \right] \]

--- (3.9)

Advertisement cost per cycle is

\[ C_A = \mu (S - S_d) P \cdot N \]

\[ \therefore C_A = \mu S - \frac{S^2 \theta}{(a-b)pN^\alpha} \]

Then the total inventory cost is given by,

\[ TC = C_H + C_D + C_S + C_A \]
TC=(C_1+C_dθ) \int_0^{t_1} Q(t)dt+C_S+C_A

TC=(C_1+C_dθ)\left[\frac{(a-bp)N^q}{θ}t_1 - \frac{(Sθ+(a-bp)N^q)}{θ^2}(e^{-θt_1}-1)\right] + C_2\left[\frac{(a-bp)N^q}{2}(T-t_1)^2\right] + μ\left[\frac{S - \frac{S^2θ}{(a-bp)N^q}}{}\right]P.N

--- (3.10)

The above equation can be simplified using series form of logarithmic term and ignoring second and higher terms as follows

1) \log(1+x)=x+\frac{x^2}{2}+\frac{x^3}{3}+\ldots\ldots\text{for }|x|<1

2) e^{-x}=1-x+\frac{x^2}{2}-\frac{x^3}{3}+\ldots\ldots

3) Second and higher terms are negligible with this approximation. Assuming their validity, for all relevant expressions, we get

\left|\frac{Sθ}{(a-bp)N^q}\right| < 1 \Rightarrow S < \frac{(a-bp)N^q}{θ}

for minimizations second derivative should be greater than zero, we get the following condition,

\frac{(a-bp)N^q(C_1+C_dθ+C_2)}{C_2} \geq \log\left(1+\frac{Sθ}{(a-bp)N^q}\right)

Therefore total inventory cost becomes,
TC = \left(C_1 + C_d \theta\right) \frac{S^2}{(a-bp)N^\alpha} + \left[C_2 \left(\frac{(a-bp)N^\alpha}{2}\right)^2\right]

\begin{align*}
&+ \mu \left[ S - \frac{S^2 \theta}{(a-bp)N^\alpha} \right] P.N
\end{align*}

\begin{equation}
\text{----(3.11)}
\end{equation}

To obtain optimum order quantity differentiating TIC partially w.r.t. \( S \) and equate to zero

\begin{equation}
\frac{dT C}{dS} = \frac{2\left(C_1 + C_d \theta\right)}{(a-bp)N^\alpha} S + \frac{C_2}{(a-bp)N^\alpha} S - \frac{2\mu P N}{(a-bp)N^\alpha} S + \mu P N - C_2 T = 0
\end{equation}

\begin{equation}
\text{----(3.12)}
\end{equation}

The optimum order level is given by,

\begin{equation}
S^0 = \frac{(a-bp)N^\alpha \left(C_2 T - \mu P N\right)}{2\left(C_1 + C_d \theta - \mu P N \theta\right) + C_2}
\end{equation}

\begin{equation}
\text{----(3.13)}
\end{equation}

### 3.6 Methodology

#### 3.6.1 Function Principle method

Defining the economic order quantity (EOQ) under fuzzy inventory model requires arithmetic operations on fuzzy quantities. It appears that the method known as the function principle is more useful for the fuzzy numbers with trapezoidal membership function as shown in fig -3.1

\[ \mu_A(x) = \begin{cases} 
    \frac{w(x-c)}{(a-c)} & c \leq x \leq a \\
    w & a \leq x \leq b \\
    \frac{w(x-d)}{(b-d)} & b \leq x \leq d \\
    0 & \text{otherwise}
\end{cases} \]

Where \( 0 \leq w \leq 1 \)
Such a number will be given a brief notation \((c,a,b,d;w)\) or even \((c,a,b,d)\) if the maximum \(w\) is understood. In the remainder of the paper, we deal only with non-negative \(c \leq a \leq b \leq d\).

By using median rule to obtain find the minimization of \(C\). The median of \(C_m\) of \(C\) can be derived from

\[
\frac{[(C_{m-C_1})+(C_{m-C_2})]/2}={[(C_{4-C_m})+(C_{3-C_m})]} / 2.
\]

Then \(C_m=(C_1+C_2+C_3+C_4) / 4\).

In all practical applications, \(C_m\) falls in between \(C_2\) and \(C_3\). In some unlikely cases as shown in figure- it could happen that \(C_m>C_3\) (or \(C_m<C_2\)). Here we do not consider such cases.
3.7 Fuzzy Model

In the above developed crisp model, it was assumed that all the parameters were fixed or could be predicted with certainty, but in real life situations, they will fluctuate little from the actual value. Therefore these parameters of model could not be assumed constant. Usually rate of deterioration is vague in nature, thus instead of considering rate of deterioration as constant the EOQ model is developed with the assumption that deterioration rate is a fuzzy number. Similarly holding cost and shortage cost are also considered as a fuzzy number.

In this model deterioration rate, holding cost and shortage cost are represented by trapezoidal fuzzy numbers. By using function principle method fuzzy total cost and fuzzy optimum ordered quantity is obtained. The equation of fuzzy total inventory cost is

\[
T\tilde{C} = \left(\tilde{C}_1 \oplus C_d \otimes \tilde{\theta}\right) \frac{\tilde{S}^2}{(a-bp)N^\alpha} \oplus \tilde{C}_2 \left[\frac{(a-bp)N^\alpha}{2}\left(\frac{T - \tilde{S}}{(a-bp)N^\alpha}\right)^2\right] \\
\oplus \tilde{\mu} \otimes \left[\tilde{S} - \frac{\tilde{S}^2 \otimes \tilde{\theta}}{(a-bp)N^\alpha}\right] P.N
\]

where

\[
\tilde{S} = \frac{(a-bp)N^\alpha \otimes \left(\tilde{C}_2 \otimes T - \tilde{\mu} \otimes P.N \right)}{2(\tilde{C}_1 + C_d \otimes \tilde{\theta} - \tilde{\mu} \otimes \tilde{\theta} \otimes P.N)} \oplus \tilde{C}_2
\]

and

\[
\frac{\tilde{S}^2}{(a-bp)N^\alpha}, \quad \frac{\tilde{S}^2 \otimes \tilde{\theta}}{(a-bp)N^\alpha}, \quad \tilde{C}_2 \otimes T, \quad \tilde{\mu} \otimes P.N, \quad C_d \otimes \tilde{\theta} \quad \text{and} \quad \tilde{\mu} \otimes \tilde{\theta} \otimes P.N \quad \text{are fuzzy points.}
\]
3.7.1 Defuzzification by Function Principle

In equation (3.12), Suppose $\tilde{C}_1$, $\tilde{C}_2$ and $\tilde{\theta}$ are fuzzy numbers with the trapezoidal membership function defined in (3.6), such as

$$\tilde{C}_1 = (C_{11}, C_{12}, C_{13}, C_{14})$$
$$\tilde{C}_2 = (C_{21}, C_{22}, C_{23}, C_{24})$$
$$\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$$
$$\tilde{\mu} = (\mu_1, \mu_2, \mu_3, \mu_4)$$

By using function principle, the membership function of $\text{TIC}$ can be defined as $\text{TIC} = (\text{TC}_1, \text{TC}_2, \text{TC}_3, \text{TC}_4)$, where

$$\text{TC}_i = \left(\frac{S^2}{(a-b)pN^\alpha} + C_{2i} \right) \left[ \frac{(a-b)pN^\alpha}{2} \right]^{2} \text{P.N} + \left[ \frac{S^2 \tilde{\theta}}{(a-b)pN^\alpha} \right] \text{P.N}$$

By using median rule, above equation can be revised as

$$\text{TC}_m = \frac{1}{4} \left( \sum_{i=1}^{4} C_{1i} + \sum_{i=1}^{4} C_{2i} \tilde{\theta} \right) \frac{S^2}{(a-b)pN^\alpha} \left( \sum_{i=1}^{4} C_{2i} \right)$$

$$\left[ \frac{(a-b)pN^\alpha}{2} \right]^{2} \left[ \frac{S}{(a-b)pN^\alpha} \right]^{2} \text{P.N} + \left( \sum_{i=1}^{4} \mu_i \right) \left[ \frac{S^2 \tilde{\theta}}{(a-b)pN^\alpha} \right] \text{P.N}$$

To obtain optimum order quantity differentiating $\text{TIC}_m$ partially w.r.t. $S$ and equate to zero

$$\frac{d\text{TC}_m}{dS} = 0$$
\[
\Rightarrow \frac{1}{4} \left( 2^{\alpha} \left( \sum_{i=1}^{4} C_{1i} + \sum_{i=1}^{4} C_d \theta_i \right) \frac{S}{(a-bp)N^\alpha} + \frac{\left( \sum_{i=1}^{4} C_{2i} \right)}{2^{\alpha}(a-bp)N^\alpha} \right)
\]

\[
\left[ \left( (a-bp)N^\alpha T - S \right)^{(-1)} \right] + \left( \sum_{i=1}^{4} \mu_i \right) \left[ 1 - \frac{2^\alpha S^i \theta_i}{(a-bp)N^\alpha} \right] P \cdot N = 0
\]

Solving the above equation, the optimum order level is given by,

\[
S^O = \frac{(a-bp)N^\alpha \left( \sum_{i=1}^{4} C_{2i} T - \sum_{i=1}^{4} \mu_i P \cdot N \right)}{2 \left( \sum_{i=1}^{4} C_{1i} + \sum_{i=1}^{4} C_d \theta_i - \sum_{i=1}^{4} \mu_i \theta_i P \cdot N \right)} + \sum_{i=1}^{4} C_{2i}
\]

### 3.8 Numerical Example

1) Crisp model :

**Input** :

\[ a=100, \ b=0.5, \ P=4, \ N=2, \ \alpha=0.3, \ C_1=0.5, \ C_2=5, \ C_d=4, \ T=1, \theta =0.05, \mu =0.05, \]

**Output** :

\[ S=86.18, \ t_1=0.70, \ S_1=35.97, \ TC=81.10 \]

In table 3.1 sensitivity analysis of fuzzy model with advertisement. The optimum values are presented along with the combination of two values of number of orders (N) and two intervals of values of fuzzy parameters. Decision maker can select the optimum results of any one suitable case.
It is expected that if expenditure on advertisement increases, then total inventory cost increases. Also it is seen that, due to increase in the values of various parameters the total inventory cost in fuzzy model increases. So the control on the values of these parameters is necessary.
2) Fuzzy Model

Table 3.1

SENSITIVITY ANALYSIS (With Advertisement)

<table>
<thead>
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<th>N</th>
<th>α</th>
<th>C₁</th>
<th>C₂</th>
<th>θ</th>
<th>μ</th>
<th>S</th>
<th>t₁</th>
<th>S₁</th>
<th>TC</th>
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<td>(3,.4,.5,.6)</td>
<td>(.01,.03,.05,.07)</td>
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<td>24.90</td>
<td>0.74</td>
<td>60.23</td>
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<tr>
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<td>0.3</td>
<td>(.4,.5,.6,.7)</td>
<td>(4,.5,.6,.7)</td>
<td>(.03,.05,.07,.09)</td>
<td>(.04,.05,.06,.07)</td>
<td>73.36</td>
<td>26.24</td>
<td>0.73</td>
<td>75.83</td>
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<td>(.03,.04,.05,.06)</td>
<td>74.20</td>
<td>24.90</td>
<td>0.75</td>
<td>60.22</td>
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<td></td>
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<td>(.4,.5,.6,.7)</td>
<td>(4,.5,.6,.7)</td>
<td>(.03,.05,.07,.09)</td>
<td>(.04,.05,.06,.07)</td>
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<td>26.24</td>
<td>0.73</td>
<td>75.83</td>
</tr>
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<td>34.14</td>
<td>0.72</td>
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<td>(.04,.05,.06,.07)</td>
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<td>0.72</td>
<td>96.23</td>
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<td>93.11</td>
<td>38.15</td>
<td>0.70</td>
<td>120.03</td>
</tr>
</tbody>
</table>
3.9 Without Advertisement Model (as a Particular case)

As \( \alpha = 0 \), the proposed model is transform to without advertisement model.

In this case Demand rate is \( a-bp \) and the optimum values of total inventory cost as well as optimum ordered quantity are as follows

3.9.1 Crisp Model

\[
TC = (C_1 + C_d \theta) \frac{S^2}{(a-bp)} + C_2 \left[ \frac{(a-bp)}{2} \left( \frac{T}{(a-bp)} - S \right) \right]^2
\]

\[
S^o = \frac{(a-bp)C_2 T}{2(C_1 + C_d \theta) + C_2}
\]

3.9.2 Fuzzy Model

\[
TC_m = \frac{1}{4} \left\{ \left( \sum_{i=1}^{4} C_{1i} + \sum_{i=1}^{4} C_d \theta_i \right) \frac{S^2}{(a-bp)} + \left( \sum_{i=1}^{4} C_{2i} \right)^* \right\}

\[
\left[ \frac{(a-bp)}{2} \left( \frac{T}{(a-bp)} - S \right) \right]^2
\]

\[
S^o = \frac{(a-bp)^* \left( \sum_{i=1}^{4} C_{2i} \right) * T}{2 \left( \sum_{i=1}^{4} C_{1i} + \sum_{i=1}^{4} C_d \theta_i \right) + \sum_{i=1}^{4} C_{2i}}
\]
3.10 Numerical Example
   i) Crisp Model
   Input :
       \[ a = 100, \quad b = 0.5, \quad P = 4, \quad C_1 = 0.5, \quad C_2 = 5, \quad C_d = 4, \]
       \[ T = 1, \quad \theta = 0.05, \]
   Output :
       \[ S = 76.56, \quad t_1 = 0.76, \quad S_1 = 22.89 \quad TC = 53.59, \]

   ii) Fuzzy Model
   For the earlier values of \( a, b, p, c_d, \) and \( T \)

   \[
   \begin{array}{|c|c|c|c|c|c|}
   \hline
   C_1 & C_2 & \theta & S & t_1 & S_1 & TC \\
   \hline
   (3.4,5.6) & (3.4,5.6) & (0.01,0.03,0.05,0.07) & 77.09 & 0.77 & 22.09 & 47.03 \\
   (4.5,6.7) & (4.5,6.7) & (0.03,0.05,0.07,0.09) & 76.13 & 0.76 & 23.59 & 60.14 \\
   \hline
   \end{array}
   \]

3.11 Concluding Remark
   As discussed earlier the function principle method is not considered in many fuzzy inventory models for defuzzification. The models are lack of deterioration of an items, so it is necessary to develop the inventory models for deteriorating items with the above method. Here we have used the function principle method for with-shortages inventory model of deteriorating items. This model can be extended with finite replenishment. The proposed model can also be developed for multi-item, multi-objective with or without chance constraints.