CHAPTER – 6

FUZZY EOQ MODEL FOR DETERIORATING ITEMS WITH SOME NON-LINEAR MEMBERSHIP FUNCTIONS
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SOME NON-LINEAR MEMBERSHIP FUNCTIONS

6.1 Introduction

Now a classical EOQ is an old-age problem and details of it are available in several books, journals and research papers. Deterioration is one of the important factors in inventory system. Some items like food grains, vegetables, milk, eggs etc. deteriorate during their storage time and retailer suffers loss. A sufficient literature is available for the inventory system for deteriorating items. These models are formulated in crisp environment.

Due to the vagueness and uncertainty of inventory parameters, the fuzzy set theory is more appropriate to formulate inventory model. Normally both linear and non-linear shapes for the membership functions of the fuzzy objective and constraints are proposed. To reflect the decision makers' performances regarding the relative importance of each objective, crisp / fuzzy weights are used.

The fuzzy priorities may be "linguistic variables" such as "very important", "moderately important", "important" and "least important". Membership functions can be defined for these fuzzy priorities in order to develop a combined measure of the degree to which the different objectives are attended.

In this Chapter a multi item EOQ model with stock dependent demand for deteriorating items is considered in fuzzy environment.
Here inventory costs such as holding cost and setup cost have been represented by exponential, hyperbolic and hyperbolic inverse membership functions and profit, deteriorating rate and total investment constraint are represented by linear membership functions. The model has been solved by fuzzy non-linear programming (FNLP) method. Results have been presented along with those of corresponding crisp model and a sensitivity analysis. Till now no literature is available for multi item inventory with above non-linear membership functions.

6.2 Assumptions

1) The scheduling period is constant and no lead-time.

2) Replenishment rate is infinite.

3) Selling price is Known and constant.

4) Demand rate is stock dependent.

5) Shortages are not allowed.

6) Deteriorating rate is age specific failure rate.
6.3 Notations

T_i : Time period for each cycle for the i^{th} item.

R_i : Demand rate per unit time of i^{th} item; [ R_i=a_i+b_iq_i ]

\theta_i : Deterioration rate of i^{th} item

Q_i(t) : Inventory level at time t of i^{th} item.

C_H : Total Holding cost.

C_{1i} : Holding cost per unit of i^{th} item.

C_{3i} : Set up cost for i^{th} item.

S_{di} : Total deteriorating units of i^{th} item.

P_i : Selling price per unit of i^{th} item.

Q_i : Initial stock level of i^{th} item.

PF(Q_i) : Total profit if i^{th} item.

n : Number of items.

( wavy bar (\sim) represents the fuzzification of the parameters )
6.4 Mathematical Formulation

6.4.1 Crisp Model

As \( Q_i(t) \) is the inventory level at time \( t \) of the \( i \)th item, then the differential equation describing the state of inventory is given by

\[
\frac{d}{dt} Q_i(t) + \theta_i Q_i(t) = -(a_i + b_i Q_i(t))
\]

\( 0 \leq t \leq T_i \)

solving the above differential equation using boundary condition \( Q_i(t) = Q_i \) at \( t = 0 \), we get

\[
Q_i(t) = -\frac{a_i}{(\theta_i + b_i)} + \left[ Q_i + \frac{a_i}{(\theta_i + b_i)} \right] e^{-(\theta_i + b_i)t}
\]

---(6.1)

and using boundary condition \( Q_i(t) = 0 \) at \( t = T_i \)

\[
T_i = \frac{1}{(\theta_i + b_i)} \log \left\{ \frac{1 + (\theta_i + b_i)Q_i}{a_i} \right\}
\]

---(6.2)

The holding cost of \( i \)th item in each cycle is

\[
C_H = C_{i i} G_i \left( P_i, Q_i \right)
\]

---(6.3)

where,

\[
G_i(P_i, Q_i) = \int_0^{Q_i} \frac{q_i dq_i}{a_i + (\theta_i + b_i)q_i}
\]

\[
= \frac{a_i}{(\theta_i + b_i)^2} \log \left\{ 1 + \frac{(\theta_i + b_i)Q_i}{a_i} \right\}
\]

By neglecting the higher power terms, we get

\[
G_i(P_i, Q_i) = \frac{Q_i^2}{2a_i} \left\{ 1 - \frac{2(\theta_i + b_i)Q_i}{3a_i} \right\}
\]

The total number of deteriorating units of the \( i \)th item is

\[
S_{di}(Q_i) = \theta_i G_i(Q_i)
\]

The net revenue for the \( i \)th item is

\[
N(Q_i) = (P_i - C_i)Q_i - P_i \cdot S_{di}(Q_i)
\]
\[ N(Q_i) = (P_i - C_i)Q_i - P_i\theta_iG_i(Q_i) \quad \text{---(6.4)} \]

The profit of \( i \)-th item is

\[ PF(Q_i) = N(Q_i) - C_{1i}G_i(P_i, Q_i) - C_{3i} \quad , \quad i = 1, 2, \ldots, n. \]

\[ PF(Q_i) = (P_i - C_i)Q_i - P_i\theta_iG_i(Q_i) - C_{1i}G_i(P_i, Q_i) - C_{3i} \quad , \quad i = 1, 2, \ldots, n. \]

\[ PF(Q_i) = (P_i - C_i)Q_i - (C_{1i} + P_i\theta_i)G_i(P_i, Q_i) - C_{3i} \quad , \quad i = 1, 2, \ldots, n \quad \text{---(6.5)} \]

Hence the problem is

\[ \text{Max } PF = \sum_{i=1}^{n} \left[ (P_i - C_i)Q_i - (C_{1i} + P_i\theta_i)G_i(P_i, Q_i) - C_{3i} \right] \]

subject to

\[ \sum_{i=1}^{n} C_iQ_i \leq B \quad \text{---(6.6)} \]

\[ Q_i \geq 0, \quad i = 1, 2, 3, \ldots, n \]

### 6.4.2 Fuzzy Model

When above profit, costs, deteriorating rate and total budget become fuzzy, the said crisp model is transform to

\[ \tilde{\text{Max }} PF = \sum_{i=1}^{n} \left[ (P_i - C_i)Q_i - (\tilde{C}_{1i} + P_i\tilde{\theta}_i)G_i(P_i, Q_i) - \tilde{C}_{3i} \right] \]

subject to

\[ \sum_{i=1}^{n} C_iQ_i \leq \tilde{B} \]

\[ Q_i \geq 0, \quad i = 1, 2, 3, \ldots, n \]

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6.5 Mathematical Analysis

A crisp non-linear programming problem may be defined as follows:

Max $g_0(x,c_0)$
subject to

$g_r(x,c_r) \leq b_r$, \quad r = 1,2,\ldots,m.

$x \geq 0$

Where $x=(x_1,x_2,\ldots,x_n)^T$ is a variable vector $g_0,g_r$'s are algebraic expressions in $x$ with coefficients $C_0=(C_{01},C_{02},\ldots,C_{0n})^T$ and $C_r=(C_{r1},C_{r2},\ldots,C_{rl})^T$ respectively.

Introducing fuzziness in the crisp parameters, the above problem in fuzzy environment becomes

Max $g_0(x,\tilde{c}_0)$
subject to

$g_r(x,\tilde{c}_r) \leq \tilde{b}_r$, \quad r=1,2,\ldots,m.

$x \geq 0$

In fuzzy set theory the fuzzy objective goal, coefficients and resources are defined by their membership functions, which may be linear and/or non-linear. According to Bellman and Zadeh [8] and following Carlsson and Korhonen [16], above problem is transform to
Max $\alpha$

subject to

$g_o(x, \mu c_o^{-1}(\alpha)) \leq \mu r^{-1}(\alpha)$

$g_r(x, \mu c_r^{-1}(\alpha)) \geq \mu r^{-1}(\alpha), r=1, 2, \ldots, m$

$x \geq 0, \alpha \in [0, 1]$

where

$\mu c^{-1}=\{\mu c_i^{-1}, \mu c_2^{-1}, \ldots, \mu c_n^{-1}\}$

Fuzzy goal, costs, deteriorating rate and total budget are representing by combinations of linear and non-linear membership functions:

Here we consider three different types of membership functions such as i) Exponential ii) Hyperbolic and iii) hyperbolic Inverse membership functions to represent the fuzzy costs and fuzzy goal, deteriorating rate and total budget are represented by linear membership function.

### 6.5.1 Exponential Membership Function

In this case

$$\mu_{Ci}(u) = \begin{cases} 
1 & ; u > C_i \\
t(C_i-u)/PC_i & ; C_i-PC_i \leq u \leq C_i \\
0 & ; u > C_i-PC_i 
\end{cases}$$

where $0 < q < 1$, $t>0$, $i = 1, 3$. 
It's pictorial representation is

Fig: Exponential membership function of $\tilde{C_i}$

Let $q=0.5$ and $t=1$ then

$$\mu_{ci^{-1}}(\alpha) = c_i - P_{ci} \log_{0.5} (0.5 + 0.5\alpha)$$

### 6.5.2 Hyperbolic Membership Function

For each fuzzy parameter $\tilde{A}$, the corresponding hyperbolic membership function is defined by

$$\mu_A(u) = \frac{1}{2} \tanh((A(u) - b)c) + \frac{1}{2} \quad \text{where} \quad c > 0.$$  

The hyperbolic membership function can be determined by asking the decision maker to specify the two points $A(0.25)$ and $A(0.5)$ within $A_{max}$ and $A_{min}$.

The graph of the hyperbolic membership function $\mu_A(u)$ is
Fig: Hyperbolic membership function

\[ \mu_{ci}^{-1}(\alpha) = b_{ci} + \frac{1}{c_{ci}} \tanh^{-1}(2\alpha - 1) \]

### 6.5.3 Hyperbolic Inverse Membership Function

For each fuzzy parameter $\tilde{A}$, the corresponding hyperbolic membership function is defined by

\[ \mu_A(u) = a \tanh^{-1}((A(u) - b)c) + \frac{1}{2} \quad \text{where } a > 0, \ c > 0. \]

The hyperbolic inverse membership function can be determined by asking the decision maker to specify the three points $A(0)$, $A(0.25)$ and $A(0.5)$ within $A_{\text{max}}$ and $A_{\text{min}}$.

Following figure gives the pictorial representation of the hyperbolic membership function.
Fig: Hyperbolic inverse membership function

\[ \mu_{ci}^{-1}(x) = b_{ci} + \frac{1}{C_{ci}} \tanh \left( \frac{x - 1}{a_{ci}} \right) \]

**Model 6.a):** Fuzzy costs represented by exponential membership function and fuzzy goal, deteriorating rate and total budget are represented by linear membership function

Let

\[ \mu_{ci}(x) = c_{i} - P_{ci} \log_{0.5}(0.5 + 0.5x) \]

\[ \mu_{ci}(x) = c_{i} - P_{ci} \log_{0.5}(0.5 + 0.5x) \]

\[ \mu_{bi}(x) = b_{ci} + P_{bi}(1-x) \]

\[ \mu_{PF}(x) = PF_{0} - PF_{1}(1-x) \]

\[ \mu_{B}(x) = B + P_{B}(1-x) \]

Here \( P_{PF}, P_{bi} \) are the maximum acceptable violation of the aspiration levels \( PF_{0} \) and \( B \).
Then the fuzzy model reduces to crisp model as:

\[
\text{Max } \alpha
\]

subject to

\[
PF(\theta, \alpha) \geq PF_0 - P_{PF}(1-\alpha)
\]  \hspace{1cm} (6.8)

\[
\sum_{i=1}^{n} C_i Q_i \leq B + P_B (1-\alpha)
\]

\[
Q_i \geq 0, \alpha \in [0,1]
\]

Where \( Q_i > 0 \) \((i=1,2,\ldots,n)\) are decision variables and

\[
PF(Q_i, \alpha) = \sum_{i=1}^{n} \left[ (P_i - C_i)Q_i - [(C_{i1} - P_{i1})\log_{0.5}(0.5+0.5\alpha) + P_i(\theta_i + P_{yi}(1-\alpha))\left(\frac{Q_i^2}{2a_i} - \frac{2(\theta_i + P_{yi}(1-\alpha)) + b_i}{3a_i}\right) - (C_{3i} - P_{ci}\log_{0.5}(0.5+0.5\alpha)] \right)
\]

**Model 6.b:** Fuzzy costs represented by hyperbolic membership function and fuzzy goal, deteriorating rate and total budget are represented by linear membership function:

Let

\[
\mu_{C_{ii}}^{-1}(\alpha) = b_{C_{ii}} + \frac{1}{C_{C_{ii}}} \tanh^{-1}(2\alpha - 1)
\]

\[
\mu_{C_{3i}}^{-1}(\alpha) = b_{C_{3i}} + \frac{1}{C_{C_{3i}}} \tanh^{-1}(2\alpha - 1)
\]

\[
\mu_{\theta_i}^{-1}(\alpha) = \theta_{ci} + P_{yi}(1-\alpha)
\]

\[
\mu_{PF}^{-1}(\alpha) = PF_0 - P_{PF}(1-\alpha)
\]

\[
\mu_{B}^{-1}(\alpha) = B + P_B (1-\alpha)
\]

Here \( P_{PF}, P_{yi} \) are the maximum acceptable violation of the aspiration levels \( PF_0 \) and \( B \).
Then the fuzzy model reduces to crisp model as:

Max $\alpha$
subject to
$PF(\theta_j, \alpha) \geq PF_0 - PF_P (1-\alpha)$ \hspace{1cm} \text{(6.9)}
$\sum_{i=1}^{n} C_i Q_i \leq B + P_B (1-\alpha)$
$Q_i \geq 0, \alpha \in [0,1]$

Where $Q_i > 0$ ($i=1,2,\ldots,n$) are decision variables and

$PF(Q_i, \alpha) = \sum_{i=1}^{n} [(P_i-C_i)Q_i - ((b_{ci} + \frac{1}{C_{ci}} \tanh^{-1}(2\alpha-1)))$
$+ P_i(\theta_j + P_{bi}(1-\alpha)) \left\{ \frac{Q_i^2}{2a_i} \left[ 1 - \frac{2[(\theta_j + P_{bi}(1-\alpha)) + b_i]}{3a_i} \right] \right\}$
$-(b_{ci} + \frac{1}{C_{cs}} \tanh^{-1}(2\alpha-1))]$

Model 6.c): Fuzzy costs represented by hyperbolic inverse membership function and fuzzy goal, deteriorating rate and total budget are represented by linear membership function:

Let $\mu_{c_{ni}}^{-1}(\alpha) = b_{c_{ni}} + \frac{1}{C_{c_{ni}}} \tanh \left[ \frac{\alpha-1}{a_{c_{ni}}} \right]$

$\mu_{c_{ni}}^{-1}(\alpha) = b_{c_{ni}} + \frac{1}{C_{c_{ni}}} \tanh \left[ \frac{\alpha-1}{a_{c_{ni}}} \right]$

$\mu_{\theta_i}^{-1}(\alpha) = \theta_i + P_{bi}(1-\alpha)$

$\mu_{PF}^{-1}(\alpha) = PF_0 - PF_P (1-\alpha)$

$\mu_B^{-1}(\alpha) = B + P_B (1-\alpha)$

Here $PF_P$, $P_{bi}$ are the maximum acceptable violation of the aspiration levels $PF_0$ and $B$. 
Then the fuzzy model reduces to crisp model as

Max $\alpha$

subject to

$$\text{PF}(\theta, \alpha) \geq \text{PF}_0 - \text{PF}(1-\alpha)$$  

$$\sum_{i=1}^{n} C_i Q_i \leq B + P_B(1-\alpha)$$  

$$Q_i \geq 0, \alpha \in [0,1]$$

Where $Q_i > 0$ (i=1,2,---n) are decision variables and

$$\text{PF}(Q_i, \alpha) = \sum_{i=1}^{n} [(P_i - C_i)Q_i - [(b_{ci} + \frac{1}{C_{ci}} \tanh \left( \frac{\alpha-1}{a_{ci}} \right) ] + P_i(\theta_i + P_{bi}(1-\alpha)) \left\{ Q_i^2 \left[ \frac{1}{2a_i} - \frac{2[(\theta_i + P_{bi}(1-\alpha)) + b_i]}{3a_i} \right] \right\}$$

-$$[(b_{ci} + \frac{1}{C_{ci}} \tanh \left( \frac{\alpha-1}{a_{ci}} \right) ]$$

6.6 Numerical Example

\boxed{n = 2}

$P_1 = 10, C_1 = 7, a_1 = 110, b_1 = 0.5, \theta_1 = 0.025,$

$P_2 = 10, C_2 = 6.75 a_2 = 100, b_2 = 0.5, \theta_2 = 0.03,$

$PF_0 = 500, P_{PF} = 50, B = 1800, P_B = 100, P_{\theta_1} = 0.005, P_{\theta_2} = 0.005,$

Model 6.a): $C_{11} = 2, C_{31} = 65, C_{12} = 2.2, C_{32} = 50,$

$P_{C_{11}} = 0.05, P_{C_{31}} = 10, P_{C_{12}} = 0.05, P_{C_{32}} = 10$

The optimum results are:

$\alpha = 0.7256, Q_1 = 71.8234, Q_2 = 77.7292$

$PF = 257.6075, B = 1027.4379$
Model 6.b): \(b_{c11}=1.8, \ b_{c12}=2, \ b_{c31}=64, \ b_{c32}=49,\)
\(c_{c11}=1.2, \ c_{c12}=1.5, \ c_{c31}=53, \ c_{c32}=42\)
The optimum results are:
\[\alpha = 0.6663, \quad Q_1=72.28094, \quad Q_2=78.13349\]
\[PF=259.0751, \quad B=1030.500\]

Model 6.c):
\(a_{c11}=1.4, \ a_{c12}=2.1, \ a_{c31}=34, \ a_{c32}=42,\)
\(b_{c11}=1.6, \ b_{c12}=2.7, \ b_{c31}=36, \ b_{c32}=47,\)
\(c_{c11}=1.8, \ c_{c12}=2.8, \ c_{c31}=38, \ c_{c32}=48.\)
The optimum results are:
\[\alpha = 0.7785115, \quad Q_1=114.3249, \quad Q_2=170.3124\]
\[PF=259.0751, \quad B=1030.500\]
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<tr>
<th>Model</th>
<th>$C_{11}$</th>
<th>$C_{12}$</th>
<th>$C_{31}$</th>
<th>$C_{32}$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$B$</th>
<th>$Q_1$</th>
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* Not feasible with respect to the range considered for the objective goal (210 – 260)
### Table 6.2
Effect Of Variations in $P_{PF}$ (Model 6.a)

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<th>PF</th>
<th>B</th>
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### Table 6.3
Effect Of Variations in $P_{PF}$ (Model 6.b)

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<td>0.8253</td>
<td>70.7334</td>
<td>77.3836</td>
<td>242.6244</td>
<td>1017.4730</td>
</tr>
<tr>
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<td>0.9039</td>
<td>69.8989</td>
<td>77.0898</td>
<td>230.5614</td>
<td>1009.6489</td>
</tr>
<tr>
<td>1000</td>
<td>0.9421</td>
<td>69.9888</td>
<td>76.4247</td>
<td>221.6158</td>
<td>1005.7886</td>
</tr>
</tbody>
</table>
Table 6.4
Effect Of Variations in $P_{PF}$ (Model 6.c)

<table>
<thead>
<tr>
<th>$P_{PF}$</th>
<th>$\alpha$</th>
<th>$Q_1$</th>
<th>$Q_2$</th>
<th>PF</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3390</td>
<td>73.6523</td>
<td>78.5596</td>
<td>260.0000</td>
<td>1066.0935</td>
</tr>
<tr>
<td>50</td>
<td>0.67082</td>
<td>71.1128</td>
<td>77.8235</td>
<td>256.6153</td>
<td>1027.1686</td>
</tr>
<tr>
<td>100</td>
<td>0.7853</td>
<td>70.27195</td>
<td>77.5255</td>
<td>250.7505</td>
<td>1021.9510</td>
</tr>
<tr>
<td>200</td>
<td>0.8682</td>
<td>69.6061</td>
<td>76.9163</td>
<td>246.7964</td>
<td>1013.1783</td>
</tr>
<tr>
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<td>0.9400</td>
<td>69.06414</td>
<td>76.4141</td>
<td>243.5254</td>
<td>1005.9944</td>
</tr>
<tr>
<td>1000</td>
<td>0.9683</td>
<td>68.7076</td>
<td>76.3612</td>
<td>242.2800</td>
<td>1003.1409</td>
</tr>
</tbody>
</table>

From the above tables, the following observations are made:

i) In Table 6.1, the optimum values are given for these fuzzy models with different combination of membership functions along with the crisp model. Here amount of profit varies from Rs. 193 to Rs. 299. As our permissible profit range is (210-160), only three crisp models profits fall within this range. Actually, if we make parametric studies on the crisp model with the different values of different parameters, results of some of those studies will coincide with the optimum values of fuzzy model. This laborious and time consuming parametric study can avoid by using fuzzy analysis. Depending on the experience or from past observed data, the exact form of membership functions for the
variations of different inventory costs, objective and budget can be defined and then the optimum values are obtained for the appropriate fuzzy model.

ii) In table 6.2, 6.3 and 6.4, it is seen that $\alpha$ increases but it never becomes one as it is expected. The values of decision variables (Qi's) become invariant but profit and budget decreases with the increase in tolerance of objective.

6.7 Concluding Remark

In this Chapter, a real life inventory problem in fuzzy environment has been proposed and presented its situations along with a sensitivity analysis approach. Here three types of membership functions are considered to represent the nature of variations in inventory costs. Objective goal and budget are represented by linear membership functions. Depending on the earlier experience or past values of these parameters, the other types of membership functions such as piecewise linear, cubical parabolic, L-R fuzzy number, etc. can be considered to construct the membership functions for the above inventory parameters and then the model can be easily solved by the present method. The proposed methodology can be extended to other inventory problems including the ones with two storage, multi-objective, etc.