CHAPTER 6

LABELING OF THE GEOGRAPHICAL COORDINATES ON THE KUNNETH GRAPHS IN A REAL PLANE.

6.1 Introduction of the chapter.

This chapter is an extension to the work that attempts to view the investigation of the location coordinates on to the Kunneth graphs considered under a real plane with real geographical coordinates. This investigation of the type of graph in this chapter will induce a location over a geographical plane which traverses an area that boundaries Kunneth kind of arrangement. This kind of arrangement demands a triangular graph over the desired area and the location coordinates over it as that of the Barycentric coordinates parameterizing it.

The main objective of this investigation is to relate the ideology of the Kunneth graph in the form of topological coordinates to the location coordinates over a geographical area. In an attempt to achieve a resemblance to the Kunneth arrangement of the coordinates and the scope of applying this theory over hurricanes, we intend to choose a particular geographical area under study. For the concern over the availability of the peril that is supposed to be discussed preferably the hurricane, we thereby choose the geographical area accordingly. Let us consider an area in the United States of America that lies in the coastal borders of the North Atlantic Ocean traversing the part of the Florida following the areas of Georgia and South Carolina until the areas of North Carolina. This areas are preferably chosen for their importance due to the frequent occurrence of tropical cyclone or the Hurricane. This chapter deals more specifically on the concentration of the coordinate system of the area under consideration and its influence over the Barycentric coordinates relating the Kunneth graphs.

The ideology of location coordinates being treated over the geographical location advances thoroughly in the chapter with the introduction of the basic coordinates over the area and further parameterize it with the Barycentric coordinates. However, the main focus of the chapter lies in obtaining a Kunneth type of arrangement over the area traversed by the coordinates.
6.2 Geographical location of the Area under study.

As discussed in the introduction we consider the geographical location with specific aspects of sharing the coastal line with the North Atlantic Ocean and catering the part of the United States of America. Let us consider the location map of the area under consideration and also as described in the later part of our introduction.

![Map showing the geographical area under investigation for the influence of the Barycentric coordinates.](image)
The above figure is the geographical map that is described earlier which is to be treated with the influence of the Barycentric coordinates and the later procedures to be investigated over it. Let us primarily consider the rough sketch of the area bordering the major states of the USA. (Florida, Georgia, South Carolina and the North Carolina). As we observe the above area is sharing its location with the partial water bodies as in the bordering North Atlantic Ocean, we focus majorly on the land structures and thereby also use the location coordinates over the same and not over the water bodies. The only reason to do this is to isolate the study of the land and the water bodies as the perils that are needed to be studied later behave differently on land and water.

We further see the geographical location coordinates of the area underlying. These location coordinates shall be further used to relate them as the bordering vertices of the specific structure. We consider the whole map bordering the map as a single geographical structure and further relate the Barycentric coordinates over it which shall be later used to fit the Kunneth arrangement of the Kunneth graph in a generalized manner. The below are the coordinates of the specific areas that are taken under reference for study.

<table>
<thead>
<tr>
<th>Area</th>
<th>Latitude</th>
<th>Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Carolina</td>
<td>36</td>
<td>-80</td>
</tr>
<tr>
<td>Georgia</td>
<td>32.5</td>
<td>-85</td>
</tr>
<tr>
<td>Florida</td>
<td>25</td>
<td>-80.7</td>
</tr>
</tbody>
</table>
Fig.6.2 Clock wise directed graph created over the location map.

As we see in the later diagram 6.2 that is a manifested version of the previous figure 6.1. The figure is improved with the labeling of location coordinates for specific location which lie on the borders of the map. The locations observed in the actual map over the ones taken under consideration are rounded off to a certain value to avoid many complications over applying the concept of Kunneth arrangement in the case of triangular graphs. These locations act as vertices when the map is altogether considered as a single graphical structure. We also see that it follows a triangular structure when all the vertices are joined and further also gives scope for creation of inner triangular graphs to be oriented inside the main triangular graph which follow the sub graph pattern of the parent graph so created accordingly. We shall also see the diagrammatical formation of the sub graph or the inner
triangular graphs so formed by further splitting the parent graph accordingly into sub vertices or some locations in between.

![Diagram](image)

**Fig.6.3**

*Formation of the inner triangulated graphs over the main graph traced over the demography.*

This diagram is the improvisation of the previous map which borders the outer part of the geographical area under consideration. The main feature of this figure is the formation of the inner triangulated graphs for the main graph. The vertices of the bordering locations are labeled with the latitude and longitudinal coordinates to uniquely define them. This labeling caters a unique way to identify each vertex over the main graph, even if there is a formation of the inner triangulated graphs which in turn are acting as sub graphs for the main triangulated graph. This labeling will further cater the influence of the Barycentric coordinates over the triangulated graph and its sub graphs. This induces the Barycentric functional in place of the location coordinates. The desired functional of the coordinates that is needed to form in the given graph is related to the centralized triangular formation projected in a planar field. The projection of the planar graph which needs to be investigated over the
Barycentric coordinates demands a coordinate system that possesses three location coordinates in lieu of two as observed in the place of two which are the latitude and the longitude.

We recall the previous work expressed above on the triangular graphs for the formation of inner triangulated graphs where the vertex acts as a topological entity and needs to be acted upon the properties of the topological axioms of the Kunneth arrangement in the case of analyzing the graphs as Kunneth graphs. There has been many such works on the analysis of triangular smooth structures with the help of Barycentric coordinates. As this system demands three coordinates, the planar structure in the form of a smooth polygon gives limited scope of getting a third coordinate over the previously observed two coordinates. This part of the study bifurcates into further sections depending on the availability of the triangular graphs and its structural analysis under the given triangular subgraphs so formed and the coordinate to be used as the third part in defining the Barycentric coordinate. We shall primarily talk upon the formation of the inner triangulated graphs before advancing the coordinates for the desired Barycentric coordinates.

6.3 Formation of the Inner triangulated graphs.

We refer to the diagram developed in the fig. 6.3 which shows the geographical area under consideration for the triangular graph under a real plane as it possesses coordinates from latitudinal and longitudinal observations. This coordinates are acting as the bordering vertices for the triangulated graph under consideration. In planar topological aspect of observing a perfect smooth manifold which is in the form of an Earth’s surface, the piecewise continuous part of it or the sub part of the surface itself behaves as a collection of topological entities that are behaving the basic axioms as that act upon the generalized topological bodies. In the case of analyzing the topological entities as vertices of a triangulated graph, we consider the triangular graph as an entire collection of topological entities being associated with the topological aspects. These aspects will further cater the association in between the various vertices of the triangular graph. However, the triangular graph which is considered under the smooth triangular plane is observed to have boundaries with certain locations forming the vertices that also are considered as the topological entities in the specific points of the graph.
As the graph is at a whole a collection of various topological entities, there always exists a scope of existence of a topological entity in between any two topologies. This statement needs less justification under the ideology of the previously known Archimedean property or the very popular Sandwich property of any two objects following a specific ordered formation. The property thus states that when any two quantities are places in the ordered form of their values then there exists a value which lies in between the quantitative appreciation of both the previously considered value. Let us attempt in describing it in the mathematical language. For any two quantities \( \square > 0 \) and \( \square > 0 \) belonging to a mathematical domain of a set of values. Further also to consider the two given quantities are in a well ordered form. As in this case we can elaborate as \( \square > \square \), then there exists an entity \( \$ \) which is not equal to ‘0’ and also not equal to \( \square \) and \( \square \). Then it is seen that \( \square > \$ > \square \). This property can be used in almost all the cases to generalize discrete entities where there exists an ordered array.

Applying the similar concept of ordered topological entities in the case of the map under consideration, there demands a need to exist an entity which lies well in between the two extreme positioned vertices when considered the ordered pattern among them. This kind of arrangement demands the continuity property of the graph taken under consideration over the given map. We further coincide the property over the map as seen in the diagram of figure 6.3 of the current chapter. Thereby the application of the Archimedean property of such a type bestows the graph to split it further into sub triangulated graphs which are also directed as the parent graph. The case of the parent graph being directed with three different cases as seen in the previous chapters shall follow in the same manner to direct the sub graphs so created. This part hereafter focuses more on the sub-graph created and its further properties to the main ideology.

Let us take into consideration the above figure 6.3 again as in the part of creation of the inner triangulated graph from the parent graph over the desired area which caters the geographical locations of the US coastal line more specifically the Florida until the North Carolina. As we also observe the vertices of the graph are labeled with the location coordinates that gives a unique location identity to each location. We manifest the discussion on the properties of the inner triangular graphs before relating it with the previously spoken Barycentric coordinates over this triangulated graphs. We recall a research work recently given by U. Dougrusoz on his collaborated work on the theories of graph study involving the triangle graph and it being influenced by the Hamiltonian cycles over a specific
pattern. It mainly portrays the path or a cycle formed which is Hamiltonian in nature on the triangulated graph as formed and its need to fit the path in the main triangulated graph. This triangulated graph fits the condition of the path that is observed to be directed in a certain way to hold the nature to be satisfying the Hamiltonian property of a cycle.

This cycle is further assumed to be catering the inner triangulated graphs so derived under the main triangle graph or as in our discussion the graph which was addressed as the parent triangulated graph. The derivation and the formation of the inner triangulated graphs from the main triangulated graph can be understood from the relation of the sub paths so created in between the main path forming and the unidirectional cycle that attains the Hamiltonian type of arrangement. The main concept of including the study of this work given by U. Dougruso is to understand the formation of the inner triangulated graphs from the main graph under the axioms of following a directional path or a cycle and it also traverses through each edge of the main graph and the edges so formed in between two main edges of the parent graph with the similar Archimedean property of well-ordered association of the magnitude of each edge. The addressing of the Archimedean property refers to the one influencing the beginning of this topic where the existence of a sub vertex appears in between the two ordered vertices depending on their magnitude of quantity. We shall also see the splitting of the graph to form sub triangulated pattern as given earlier and thereby the inner triangulated graph as desired by the topic.
Fig. 6.4
Inner triangulated graph (case 1).

Fig. 6.5 Inner triangulated graph (case 2).

Fig. 6.6 Inner triangulated graph (case 3)
Before we begin with the description of the above three figures let us relate back to the triangulated graphs as seen in the earlier topics where the assumptions also funded to be traversing a smooth curve bordering the vertices of the parent graph where it need not be following a straight line or a geometrically linear formation of the path. However, there is a strict condition to be followed for the graph to be continuous over a certain domain. The Cauchy sequences observed over the Kunneth arrangement of similar kind of triangular graphs takes care of the certain graph to be continuous over the desired domain. We now focus on the three diagrams as given above. All the three diagrams are reasoning to the certain category of graphs with three different cases.

However, the different cases which are discussed over the similar kind of triangular graphical formation refer to the direction of each graph which being clock wise, anti-clock wise and the non-directed graphs. Whereas in the case the three cases refer to the creation of the sub triangulated graphs thus to follow a path or a cycle further associating the Hamiltonian type of the conditions where each cycle traverses a vertex just once in its path along the graph and thus splitting the graph. This again falls in to a different way of observing the main graph. The primary method is to join small triangulated graphs and thereby associate each vertex to form a single graph out of two or more graphs as seen in the first two cases. However, the third case shows the triangulated graph creating sub triangulated graphs or the inner triangulated graphs with the existence of the vertices in between the two vertices of the parent graph. The main reason to introduce or recall the previous work on such kind of inner triangulate graphs is to relate the main triangulated geographical graph so formed under the labeling of the location coordinates and the property of creating the inner triangulated graphs or the sub triangulated graphs following certain axioms of the theories in defining the directed graphs.

The inner triangulated graphs so formed in the map (refer figure 6.4) is directed clockwise with the labeled edges as coordinates of the geographical locations hold the axioms of the directed graph. Consider the various bordering edges of the graph as following a path given by,

\[(32.5, -85) \rightarrow (36, -80) \rightarrow (25, -80.7) \rightarrow (32.5, -85)\]

This path thus observed to form a cyclic graph as it starts from the edge labeled as \((32.5, -85)\) and traverses to the other two vertices and comes back to the original vertex where it started from. There can be also identified an Eulerian type of a path that follows this kind of property of the cyclic graph. Therefore there exists an Eulerian path in the below considered graph that is cyclic in nature.
and clockwise directed. Now as there exists a single path under consideration for the graph considered, the identification of the path which is also directed towards a vertex arrangement in an ordered form thereby gives scope of that path to be Hamiltonian in nature. The main aspect of the Hamiltonian path to exist in a directed graph is that it should travel each vertex just once around the path traversed around it and should not repeat any further vertex if the path demands to be closed under a cyclic formation. Thus the property of a cyclic graph to achieve Hamiltonian confirms indirectly the ordered or the clockwise directed aspect of the graph. We are yet to investigate the magnitude values of each vertex that is the quantitative value due to the numerical value of the location coordinates in the study of the association in between the Kunneth arrangement that is discussed to be the main aspect of the study.

Further moving the focus of study on the triangular structures as seen in the three triangular diagrams given in figure 6.4, 6.5, 6.6 and also taking the figure of the map given in 6.3 for the understanding of the inner triangulated graphs relating them with the labeling of each vertex in an ordered form. The case 1 and case 2 as seen in the figure 6.4 and 6.5 provide the structures of inner triangulated graph formed due to the joining of the internal vertices and in some cases to identify a vertex which falls in between the two vertices with the linear ordered property and considering that vertex as a breakeven point to separate the two initial vertices of the main parent graph and to structure a vertex to separate them. However, the path that needs to be also in a cyclic formation is expected to traverse through each vertex just once while travelling through the entire graph. This also generally separates the entire main graph or to refer the parent graph into sub graphs with the inclusion of the vertex in between the two end vertices bordering the parent graph. This gives us the scope of separating the discussion on the triangulated graphs and its further aspect on inscribing the inner triangulated graph formed within it over the real plane.

6.4 Triangulated graphs and the inner triangulated graphs in the real plane.

The main portion of investigation in this part of the study is the features of the triangulated graphs to inscribe the inner triangulated graph inside it over the real plane. This thus induces to use the graphs as seen in the previous part as the triangulated graphs observed in the previous topic. *(Please refer 6.3)* Before we move on to the next concept of graphical studies in the main graph so created let us primarily focus on defining some concepts as follows.
**Definition 6.1 Walk in a graph.**

For any graphical structure, where a number of vertices are taken into consideration with either a continuous linear arrangement or nonlinear arrangement is known as a path observed in a graph that mostly is also directed. This linear combination involves the edges and the vertices creating the walk by joining them.

**Definition 6.2 Self-avoiding path.**

In a graphical structure that follows the basic axioms to be projected in a plane, a certain type of a walk that traverses through all the vertices of the graph in such a manner that each vertex is just traversed once and then complete the cycle over the graph. This case also includes the graph to be closed and having a finite number of vertices. The walk observed following such conditions can be addressed as a self-avoiding.

Let us observe the graph which is directed in the diagram 6.2. As it is a simple graph with a closed walk in terms of a path that traverses each edge just once in the entire graph. We identify one such a walk which is self-avoiding. A precise investigation on the main graph that has the three vertices as seen in figure 6.2 which is bordering needs to be have certain properties that cater the needs of a cyclic graph to the fundamental theory of studying the further aspects of whether an orientation of a sub-graph exists or not. Let us consider the pair of vertices of the linearly formed sub-graph of the main graph that has a single linear edge sharing it.

Let \( G_1 = (V_1, E_1) \) be a graph that is built on the collection of the three vertices of the triangulated graph formed and further the vertex \( V_1 \) be the collection of the three vertices of the main graph. As in the figure the location coordinates labeling the main vertices will be taken for consideration. Therefore, \( V_1 = \{(25, -80.7), (32.5, -85), (36, -80)\} \) follows. The coordinates of the graph use latitude and longitudinal as the real example of the location labeling that gives a unique identity for each location. As we see this chapter induces a few varied observations over the graphical map of the figure 6.2 like the identification of the closed cyclic graph in the associated vertices of the map.

The influence of the self-avoiding walk over the same graphical structure, the labeling of each vertex with a unique identity of by using the location coordinates thereby giving a quantitative magnitude for each vertex in the graph and finally not to forget the triangulated structures and the
scope of inscribing the inner triangulated graphs in the parent graph. This observations not only give scope of observing the graph microscopically but also rendering a specified study of graphical aspects over the further sought inner triangular graphs over it. The fourth coming chapter discusses these all features specifically and also provides a major link to its next upcoming chapter on the major applications of the theory onto the real life problems like the hurricanes.