CHAPTER 3
COHOMOLOGY LOCALIZATION OF KUNNETH GRAPHS

3.1 Introduction

Cohomology which is known in the mathematical world is a concept characterized as the visualization of sequential arrangement of certain groups over a space. Now this study is the refined formation of the homology studies in mathematics under the branch of topology or to be more precise algebraic topology. This concept deals with the ideas in a very abstract formation in the field of mathematics. These theories are developed in the very past 40 to 50 years with the modern research based on abelian groups of structural formation. There have been a series of theories developed in the past many decades to improvise the Ideology.

Some of them are given as follows:

- Simpilical,
- Singular
- de Rham
- Čech.

We shall be initially focusing a specific type of cohomology that works on the topological structures and its characteristics. The scope of this is to mainly understand the structural formation and the further deformation of certain algebraic structures, mainly the ones formed due to the arrangement as formed in the Kunneth theory.

3.2 Cohomology localization of an algebraic structure.

As it is already known cohomology is a sequential formation of associations among Abelian groups, we gather our sight to focus more on the areas governing the association theories among such topological objects. The localization theorem which was used over the ring in adding its multiplicative inverse to a ring of its own type over primes. As it was described the localization
techniques of the ring and its corresponding inverses associated in a sequential way, the current challenge is to categorize the structure of an algebraic object in a space and characterize its map on to the real space to visualize it further. Let us consider an extension from the property of the inverse of the image mapping of a topological object as used in the ring structure to a more generalized manner. Consider a topological object $X$ and its inverse component as $X^{-1}$, which can also be considered as another notation as $Y$. Then the image map of the subset in $X$ given by some $A$ also considered to be homological in nature is a general localization of this kind of a topological object generalized over a space. This concept can also be seen over an affine scheme to be developed in the process to carry out the construction of the variety over a geometrical structure. This was further used by Alexander Grothendick in his attempt to formalize the Weil conjecture.

The idea of the applications of the Weil conjecture helps to link the property of quantize the points of association on the affine or the algebraic varieties over finite fields. We focus to consider the property of cohomology to be localized over an algebraic entity or an idea. The Kunneth theorem which is already a product of the association of topological entities in the homological space is a very interesting property as it gives an extensions to the ideology of the cohomology theories as well which deals with the sequential formation of topological objects in any space.

Consider any such topological objects $X$ and $Y$ as given in the Cohomology space and further to categorize the Kunneth association parameter on it to create a relation of these individual spaces to its product space as created by the product space given by $X \times Y$. The main ideology is to extract the association of these topological objects in cohomology space and further to imagine it in the variety form as a graphical structure to be named as the Kunneth Graphs. There are various ways and phases to carry out this consideration for the identification of the Kunneth graphs in cohomology. This theory will be further used in a developmental study of certain graphical structures in the lieu of its structural justification. The theory of cohomology as studied earlier in the algebraic theory as a sequential formation to categorize the chain structures can be viewed as follows.

Considering the array of topological objects $T_1, T_2, T_3$ in a sequence in any categorized space or a constructed open covering called as the $T$, then the further sequence gives a cohomology space formation among these objects in a array formation which can be further seen as here described in the statement.
We further take into consideration the structure of a sub chain of one such sequence with two subspaces already defined to be in the form of a topological space and contained into this topological space to create a sequence of two such topological objects. These topological objects are further viewed in a sequence as follows.

\[ \ldots \rightarrow T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow \ldots \quad \text{and so on.} \]

The arrows specify the kind of a directed relation that exists in between the topologies under consideration over the ordered arrangement. The main reason to take into consideration of these two topological subspaces is that we are in an attempt to create a condition to be similar to the Kunneth theorem and it’s kind of spaces considered in a formation to be defined in a cohomological space and further to also carry out its independence and dependencies on various other concepts. We see the general statement for the Kunneth theorem as stated below for A and B to be two topological spaces and the association operator over them as a sign provide accordingly.

\[ \frac{\partial}{\partial T_{p+q}} h_p(A,F) \times h_q(B,F) \cong h_k(AXB,F) \]

The special conditions that are above attempted to be defined are also to be treated and made to go through a lot of justification satisfying the conditions followed by the Kunneth’s theory and the formula as well. Some of the justification techniques for the above said event will also include creating and further studying the open as well as generalized sub varieties of these conditions which are for the time being to be categorized as independent structures in the specific space in the form of a bounded sub formation. For the same reason the open sub varieties of these structures are taken into consideration as the scenario given by the Schubert variety cases of an organized Grassmanian having its specially defined sub variety.

We now again focus on the initial step of considering the two topological sequences in an array that are extracted from the cohomological formation of topological spaces which can also be taken to be discrete or continuous for the time being. As the scenario we are dealing with will be used to manifest the ideology of these kind of structures in many other spaces and subspaces to understand its creditability in its further uses.
Let the consideration of these topological objects in a sequential formation be categorized as $T_1$ and $T_2$ being mutually independent topologies having less interference to deal with the intersection cohomologies. As it will be further exploited to know the depth of this conditions over one another in a special structural form. After the consideration of the topological spaces or objects we now consider the confluence of both the topologies in one single plain and carry out the mapping down of the subspaces in the below considered subspace as follows. Let $t_1$ and $t_2$ be two subspaces derived from the independent topologies $T_1$ and $T_2$. It is believed for these subspaces to have a connected relationship as that of the parent topologies being extracted from one space sharing a sequence of a pattern.

Further to consider this connectedness property between the subspaces their independent Cauchy sequences are taken into consideration. Further this validation is also supported with the investigation of the open varieties derived from these topologies i.e. $T_1$ and $T_2$. These varieties can be further viewed or with the consideration of the affine types of varieties over the space. The main reason to consider the affine variety is that its preapproved result on the complex numbers gives a satisfactory result of a zero set over a system constituting polynomials in $n$ variables over its defined topology which is also contrary to the assumptions of the Zhariski topology. Let us not forget the purpose of our discussion over this part is to find the connected association of the subspaces derived from two independent topologies $T_1$ and $T_2$. For the preliminary case there is a need of the topologies to be associated by the continuity of the maps over them.

**Definition 3.1**

The topologies $T_1$ and $T_2$ which are considered are assumed to be induced by the chain cohomology sequence also known as the **chain complex** as there are a series of topologies in the cohomology group separated by integral coefficients as $n, n-1, n-2$, ....and so on.

We shall demonstrate to prove the connectedness with the following sub topologies in the following proposition.

**Proposition 3.1**

The map of sub topological spaces $t_1 \to t_2$ derived by the combined chain cohomology spaces are continuous.
Proof.

The proof of the continuity is supported with the help of many theoretical assumptions as that in the case of complex numbers of the complex space as seen in one of the proofs by Brian Osserman in his paper on Complex varieties and the works dealing with analytical topology. Let there be a regular function defined on \( t_1 \to T_1 \) as \( f(x) \) for all \( x \) belonging to the parent sub topology \( t_1 \). We define \( f(x) \) to be a series of combinational function as it is defined over the cohomology space for the desired sub topology.

Further the mapping of these functions will be induced with the morphisms that are relating the various points to be considered under this sub topology. We further intend to show \( f(x) \) as a summation of combined structures over the desired topology as follows. As \( f \) is the function taken under consideration it can be viewed as,

\[
f(x) = c_0 + c_1 \times t + \ldots + c_n \times t_n,
\]

As there exists a specific system of roots for the below polynomial function these roots fall under the criteria of being categorized as to fall in the space with a set of elements as that of the circle and further considering all the elements in this circle to be as that of the radius to be separated by the nonzero value \( \epsilon > 0 \) then for each value of \( \epsilon \), there exists a \( \delta > 0 \), depending on \( \epsilon \) such that for any two consecutive values of \( c_i \), which can be considered as \( c_i \) and \( c_{i-1} \) shows a difference in the peculiar way that satisfies the continuity property as

\[
|c_i - c_{i-1}| < \delta \text{this further gives a clear implication as the}
\]

\[
|f(c_i) - f(c_{i-1})| < \epsilon,
\]

which is a good understanding of the continuity function.

Since these two points are in two separate sub topologies mapped by a single function \( f(x) \) which is also expressing the property of an open variety with the sub covering being existed over the two sub spaces and the distance property of these two points being separated by the \( \epsilon \) and \( \delta \), where \( \epsilon > 0 \) and \( \delta > 0 \) gives the scenario of the function being continuous and there also is a need and possibility of every Cauchy sequence in between these points found to be complete over the given
cohomology space. We shall talk about the Existence of the Cauchy sequence among the points in this formation later in our next chapter when the complete set of points are taken into consideration as that of pertaining the idea of defining the Kunneth formula of the association of the points as described in the Kunneth theorem in the cohomology space.

This kind of association will be further viewed in the structural formation of a graph to study its various properties in defining it in real space. Now that we have found the connected aspect of two points in the function mapped within the two sub topologies we will consider the extraction of the map into the localization technique to view the structural formation of the association of the points in cohomology space which satisfy the Kunneth theory. This gives an extension of the theory to support the localization concept of an algebraic structure considered in a cohomology space. The two topologies considered $t_1$ and $t_2$ enjoy a disjoint connected set and further a sequence is taken into consideration which is forming a linear structure whose Cauchy sequences are continuous. It can be viewed as the following:

For any $x_i \in t_1$ for any sequential formation on the sub topology $t_1$ considering any $y_i \in t_2$ an element on any other topology. These two elements are mapped by the function $f$. It is seen that the difference of the function among the two separable elements as seen above is continuous with the help of the $\varepsilon, \delta$ separating the two functions. Which was defined over the sub topologies was separated accordingly thereby giving the continuity which shows the set to be locally connected in the sub topology. Thereby the algebraic structure is created over it. The very first sight of a sub space being locally connected is that it pertains to have a singular element $x_i$ in $t_1$ being surrounded by a set of neighborhood $p$ in a connected set form.

The individual spaces showing a local overlapping is to be considered while the functional mapping takes place in between the two sub topologies as that in the case of continuous functions.
Figure 3.1. The mapping of sub topologies over the cohomologies.

The sub topologies and the continuous function $f$ over them following a sequential formation. The above diagram gives a clear view of the function acting on the sequential formation of the elements in each sub topology. We shall further talk of the structural formation of the elements which are in a sequence and also follow the Kunneth’s theoretical property of the association of the elements in between them in a sequence and its product space. For the same reason let us consider the two sub topologies which are being derived from the parent topologies $T_1$ and $T_2$ to be in a partial or a sub open covering as continuing with the property of association of their sub topologies being connected sequentially. This gives us the existence of the open covering which categorizes the two topologies. The two points or topologies who are induced by the sub topologies which are connected locally are further tested for the kunneth’s condition. As known the kunneth’s theorem is a property that shows an association in between topological objects in a cohomology space with the product space of these topologies. We shall be investigating these kind of association in between the topologies and their product space to come up with an idea of a structural formation to be further tested in some other space for its realistic applications, for an instance the real space. Let $T_1, T_2$, be the two topological structures having their own spaces and also the evidence of they being connected with the property of their sub spaces sequentially connected. Hence we deduce the two topologies to be connected on a cohomological space.

\[ \ldots T1 \rightarrow T2 \ldots \]

We give the sequence associated as above. We now consider the product topology of the two individual sub topologies with the following arrangement as
The association in between the sub topologies $t_1$ and $t_2$ derived from the parent topologies $T_1$ and $T_2$. This further gives a triangular structure that is similar to the inner triangular graphs showing a triangular association between the individual sub spaces. This triangular structure is an interesting concept to be studied in both the algebraic topologies as well as the graph theory.

3.3 The structure of the Kunneth theorem in the cohomology space.

We now concentrate on the sub topologies which is one of the kinds of a Kunneth property as seen in the above diagram. The investigation of this arrangement is needed to study the structural formation of the algebra of the topologies that are formed due to their individual sub topologies in the given cohomology space. As we have already seen the existence of the connected sequence in between the sub-topologies which is also visible due to the Cauchy connectedness property. This phase of the research takes into consideration the two sub topologies in an indication of the parent topologies of who’s the sub topologies area a part of to be connected in a nontrivial formation of sets including the connected sequence. As $t_1$ and $t_2$ are subspaces, we now look into the continuity of the entire association of the functional mapping among the three entities as to be visualized to be the independent topologies or independent sub topologies.

A minimum requirement for a smooth set of curves to form a triangular structure is the discontinuity of it on at most three positions or in a more precise scenario the three continuous
smooth curves which may or may not be a straight line visualization will interpret to a triangular structure with the required varieties in it. We are more interested in a closed triangular structure that is formed due to this formation as seen in the mapping sets joined at the ends due to the Kunneth property in Cohomology spaces. These mapping sets that form the triangular structure can also be treated to the basis of the Cauchy Schwartz inequality to find the closeness of the curve that is obtained due to the connection.

The Cauchy Schwartz inequality can be further idealized as a relation among vectors so as to express the following result. For \( p \) and \( q \) to be vectors in form of an inner product space as follows.

\[
|\langle p, q \rangle|^2 \leq \langle p, p \rangle \cdot \langle q, q \rangle
\]

where \( \langle \cdot, \cdot \rangle \) represents the dot product for the given set of vectors in the desired space.

The application of the Cauchy Schwartz inequality will merely give the separation or the distance in between the position of each separated topological entity in that triangular formation. For the same reason we restrict the application of the Cauchy Schwartz inequality of the triangle to merely consider the structural formation of the association gained due to the Kunneth formulization. The following diagram shows a difference in between the Kunneth kind of formation which is not restricted to the triangular structure that is connecting straight line graphs to the smooth curves that connect to form the triangular structures which are straight lines in nature.

3.4 The Connected arrangement of the Kunneth property.

We now move the focus on the investigation of the connected property of the Kunneth association found in between the sub topologies of the parent topologies \( T_1 \) and \( T_2 \). For the same reason let us consider the sequences in between the product space of the sub topologies \( \times T_1 \times T_2 \) and the individual topologies as well. We shall see the entire triangular space as a separated
independent space which is formed due to these three topological entities. These three topologies also give rise to a manifold kind of association which further associates the structure in a closed ring that provides an algebraic association.

These triangular structural association over the cohomology space is also well expressed using the Schubert varieties over the concerned space. For the same reason there is a need to consider a grassmannian over the sequence of topologies. The objective of studying these kinds of varieties over the space is to get a sequential pattern to gain the clear visibility of the Kunneth formation. We consider a sequence of points in the triangular formation where there are topological spaces to be governed by the vector spaces of some dimension $k$. we need to show the Schubert variety over the structure to gain the consistency of the triangular formation which can be further investigated over the real space to attain a interference of patterns found in the structures in $R^3$.

After the Schubert varieties are estimated in the Kunneth formation of this triangular structure we can further name it as an identical system of topologies in the graphical formation to be further studied in the graph theory to attain its physiological aspects. This analysis will further be carried out with the help of various tools in the graph theory to define a graph of the specific arrangement of the Kunneth theory to be named as the Kunneth Graph. This graph will also satisfy the minimum requirements of Cohomology localization of the Kunneth theory in the form of a graph using this arrangement of the topologies and their association in Cohomology spaces to be visualized in the real space later or to be more specific to carry out the parametrization of the graphs following the Kunneth arrangement in the Cohomology space to the real space.

### 3.5 Schubert Variety of the Kunneth arrangement.

The Kunneth arrangement in the cohomology space with the association of the two topologies and their product space is taken in to consideration in this discussion. The Kunneth arrangement in the previous part is seen as below. However, we now consider their parent topologies $T_1, T_2$ and their product space as $T_1 \times T_2$. 
Fig. 3.3 The formal association among the Topologies on the Kunneth scenario.

The association in between the parent topologies $T_1$ & $T_2$, and their product space $T_1 \times T_2$ giving a triangular formation among them. As we see the Kunneth arrangement is in a similar form as that of the arrangement of the sub topologies been associated. The only difference is the absence of the directed association in between them. This was shown in the form of arrows directed from one sub topology towards the other and also the product spaces. The only reason this association is to view the association in almost all directions from the topologies connected in the arrangement.

We now consider a set of points in the space underlying the above arrangement. For the same reason to imbibe the set of a well-defined Grassmanian among it there is a need to imagine singularities of these points to projected in the real space. These points of singularities which need to be also treated under the coordinate system. For the same reason we need to further consider the concept of coordinate singularity as we are checking for any discontinuity of this arrangement in the real plane or the real space as its further visualization needs to be done in the real space.

We further consider a set of points in the Kunneth arrangement ($Ka$) to be formed in a sequential arrangement governed by a set. Let the set of points in that set which is a part of the Kunneth arrangement be defined as $p_1, p_2, p_3, \ldots, p_n$.

As considering the set $Si$ is well ordered for any $i^{th}$ Interval in the $Ka$. As $Si$ is a subset of $Ka$ and $Si$ is a construction of points $p_1, p_2, p_3, \ldots, p_n \in Si$ and $Si$ is well ordered gives us the assumption of the subset inside the $Ka$ which is connected among the individual topologies as seen in the most of the previous part of the discussion we declare the existence of the subset to be sharing an intersection to a
set of a grassmannian type. Further this intersection of the $\text{Gr}$ with the $Si$ may be considered for any such kind of grassmannian intersection.

The set of points over $\text{Gr} \subseteq Si$ which shows an intersection of the set of the grassmannian and the sequential set formation. As the set of sequential formation is a local embedding of a well ordered set of points $p_1, p_2, p_3, \ldots, \; p_n \in Si$. There should not be an ignorance to the fact that it’s an embedding in the parent Kunneth Arrangement $Ka$. The grassmannian intersection of the set $Si$ and the element on the $\text{Gr}$ will be forming a character subset as $\text{Gr} (k, \pi)$. The points on the set $Si$ will be further considered as the vector entity over the well ordered set. The explanation of the set $\text{Gr} (k, \pi)$ gives a liberal understanding of the dimension of a set under the grassmannian property to be of ‘k’ for the individual topological spaces intersected with the sets $pi$. The concept of a projective variety as previously used in the explanation of the grassmannians which are visualized under its formation of the algebraic structure are taken into consideration. We recall the consideration of the sequential arrangement of points $p_1, p_2, p_3, \ldots, \; p_n \in Si$ where $Si$ is a subspace of the given topological spaces or individual objects as $T1$ and $T2$. Let the concept recall the similar Grassmannians in the complex space as that to be true for any $k=1$ in the Grassmannian $\text{Gr} (k, \pi)$.

We now intend to find a linear character of the selected subspaces to achieve the exclusive property of identifying the property of $\text{Gr}$. As known in the earlier part of our assumption the individual topologies are a part of the Kunneth arrangement given as $Ka$, whereas the category of the selected sub topologies in them are sequentially connected. This in turn gives an extension to the theoretical assumption that there exists a sequence of points as well in the topologies. As $p_1, p_2, p_3, \ldots, \; p_n \in Si$ and $Si$ is subspace of $T_i$ for $i = 1$ and $2$. We further give the projection of the space which is extracted from the individual topologies as considered in the arrangement $Ka$. This kind of projection of the sequential arrangement in between the topologies is considered to give a visualization of the Grassmanian type of formation and also its kind of an algebraic structure. This structure thus formed can also be formed as the algebraic structure formed by the identification of the grassmannian type of varieties. We shall see the statements in the later part after considering the initial statements for the visualization for the Grassmanian type of visualization in this arrangement $Ka$. 
Fig. 3.4 Relative association of topologies following the Kunneth Schemes.

The above diagram which shows the Kunneth arrangement of the topologies also shows the relative properties in between the independent topologies and their product spaces as well. Let us not forget the triangular formation obtained in the due process of this kind of arrangement.

Now further we shall visualize the formation of the individual topologies and the inclusion of the grassmannians embedded in each of them with the sequential formation of subspaces thus giving the specific varieties altogether. Now considering the individual topologies as follows.
Fig. 3.5 Individual topologies on a grassmannian formation of topologies.

This figure is an attempt to visualize each set of the individual topologies with the identification of each set of a grassmannian included in it. The grassmannian are formed in the provision of the sequential formation of the set of points in each $S_i$ where each subset is a provision of points as $p_1, p_2, p_3, \ldots, p_n \in S_i$. These kind of arrangement is further to be considered as in the line of giving a Schubert variety which can be attempted to be viewed as follows. For the same reason the set of sequential formation of the points embedded in the subspaces $S_i$ of each topological space is taken into a consideration.

As obtained the grassmannian over the arrangement $K_a$, the sets are further being well ordered to give a local embedding that shows as follows. These well ordered property can be further considered as the $S_1 \in S_2 \in S_3$ and so on. This gives the grassmannians to follow a sub variety of a certain kind. This sub variety in the extended form further gives an extension to the Schubert type of variety after having an intersection of a set with that of an embedded intersection. This gives the clear idea of the singularized algebraic structure embedded into an arrangement known as the $K_a$ which is also named as the Kunneth graphs $K_g$. The visualization of the Schubert variety gives a channel to our satisfactory conclusion of the desired algebraic formation which was considered in the cohomology space to have a structure which can be localized and also projected into the real space. This gives the understanding of the localization of the desired algebraic structure in the cohomology space. We further will see the projection of this structure in the real planes which is similar to the concept of
cohomology localization of the algebraic structure in the real spaces. Further these results will also focus on the part of investigating the previously identified Kunneth graph $Kg$ which is a structural observation of the Kunneth arrangement $Ka$.

### 3.6 Projection of the Algebraic structure in the Real plane.

We recall the basic projection of the vectors from one plane to another. This kind of projection is used to understand the ideology of visualizing the topological objects already described in the cohomology space in the form of the algebraic structure onto the real plane and thereby the real space in a more generalized form.

![Projection of the Algebraic structure in the Real plane](image)

**Fig. 3.6** Projection of the Algebraic structure in the Real plane.

The adjoining fig. 3.6 shows an attempt to relate the parameterization of the Kunneth graph $Kg$ in the cohomology space to be projected in the real plane and thereby to achieve a homological association in between the given set of topologies thus described in the $Ka$. The pattern observed in the real plane is any arbitrary pattern thus obtained not to scale for the projection of the Kunneth graphs from the cohomology space.
Now to consider the Kunneth graph $Kg$ in the real plane as that of the structural formation achieved in the cohomology space. In order to consider $Kg$ in the real plane we need to primarily focus on the projection of the topologies from the cohomology space to the real plane. For this similar reason we take into understanding the various projection techniques used in most of the mathematical scenarios. The most recently used and discussed is the one with the idea of cartography that widely carries out the map projection of the earth’s surface to be mapped or traced on to a plane surface. This again is the example of a real life scenario being applied to the abstract idea of projection from one space in mathematics to another where in our case one being the cohomology space and the other being the real plane.

We will be needed to also focus on the initial part of the development of this projection theory on this kind of projection with the help of retract as our projection deals with topologies and their individual association. Let us consider the topology $T_i$ in the Kunneth graph $Kg$ embedded with elements $x$ and each embedding is structured with a with a separation of $x_i$ for all the values of $i = 1, 2, 3, \ldots n$. The set of axioms required to understand the projection of this topology is related to the generalization of the projection map with the existence of the certain retract existed in between the topologies.

The main focus should be to use the concept of retract in order to prove the axiom of the projection existing in between the topological entities. As previously considering the topologies we now take their subspaces to be bounded and mutually exclusive. These subspaces are further to be brought up to the level of consideration of the existence of sequential points embedded in it. As each separation of points is separated with a local distance of $x_i$ and $x_{i-1}$ we take its finite distance to be exact over the sequence as

$$|x_i - x_{i-1}| < \epsilon,$$

Where $\epsilon > 0$ is any abstract value in $R^3$.

This sequential property thus gives the property of the mapping governing the two points in the desired subspace to be continuous for the given set formation in the array form.
Let us not forget each topology has a subspace embedded in the form of sequential formation of points and they further are well ordered as defined in the previously stated cohomology group. We take a map from the points of a subspace on to the exterior points for that subspace which are nothing but condensed into another subspaces. We consider a mapping set to traverse from one subspace to another subspace as given with the following statement. The separation property observed in the above statement gives a relation to the map being continuous which satisfies the basic need to exist a retraction in between any two subspaces of the underlying topology $T_1$. The property of retract is the basic need to achieve a projection of the desired topological arrangement in the form of a Kunneth arrangement also to be known as the Kunneth graph $K_\text{gin}$ the real plane from the cohomology space. Further this retract will need the support of the retraction to be needed to prove the absolute retract for the same reason. We now consider the subspace of the topology $T_1$ which is given by the continuous map over the points in the subspace.

Let this mapping be described ahead with a function given as $\omega$. We give the mapping statement of this relation as to be relating the points achieved in the subspace $S_1$. Let a map between the points $x_i$ and $x_{i-1}$ be given as follows.

$$\omega: x_i \rightarrow x_{i-1}$$

be continuous as the distance between the points is less than the arbitrary

$$|x_i - x_{i-1}| < \epsilon$$

as they are in a sequential arrangement as assumed earlier in the definition of the subspace $S_1$ of $T_1$. As we take the projection from a cohomology space to the real plane the elements retain the properties of the desired domain. For any $S_1$ in $\mathbb{R}^3$ which is projected from the cohomology space embedded in a topology $T_1$ will showcase a sequential formation of a set of points which are in a well ordered formation which was also verified with a Schubert type of variety due to the continuity parameter observed in the previous few topics above and the well description of the structures defined as the graphical formation more preferably named as Kunneth graphs given with the abbreviation $K_\text{g}$. This structure thereby formed in the cohomology space and later projected in the real space can be further treated or studied under various graphical parameters to understand its efficiency over the structure used in real space. We shall also be observing the correlation of the $K_\text{g}$ in the real planes and thereby the real space with the property of the moment graphs which shall also be investigated in the real space. This graphs thus studied in this method will be modeled to be applied on further studies of
various patterns in the real life scenarios as we advance to the later chapters.