CHAPTER 6

APPLICATION OF DIVERGENCE MEASURES TO WORLD UNIVERSITIES RANKING – A CASE STUDY

6.1 INTRODUCTION

In the present chapter, we have taken a case study of ranking problem for world universities. First, we have taken the data from TOI for 2008, 2009 and 2010 for first 15 universities in Rank. For ranking the methodology for five / six indicators have considered by the institution.

Section 2 discusses the application of non-symmetric divergence measures. How do they diverge from their ranks due to the variation of indicators? Section 3 provides some results from numerification point of view while section 4 presents the application of Theil’s [103] measure of Information Improvement. This measure witnesses the improvement in rank.

Now, we present the case study here with Pie diagrams.

World University Ranking 2008

The Times Higher Education-QS World University Ranking aim to capture the essence of a world-class university. This involves quality teaching and research, both of which we attempt to measure. Because a global institution will also be a magnet for academic talent, we analyze institution’s abilities to attract staff and students form beyond their own shores, but we also consider the informed opinion of two groups-university academics and major recruiters of graduates-that have a unique insight into institution' strengths.
The means that our ranking of the world's top 200 universities is a compound index that uses qualitative data derived from surveys alongside quantitative data on topic-such as staff and student numbers and scholarly citations. The 2008 rankings use much the same method as in 2007 but with more and better data.

For us to consider a university, it must undertake work in at least two of the major academic fields: the natural science; biomedicine; engineering and information technology; the social science; and the arts and humanities. It must also teach undergraduates. This means that these rankings do not include many excellent specialist institutions, mainly medical colleges and business schools. But many universities that do not teach a full range of subjects, principally specialist science and technology college. A fundamental tenet of this ranking is that academic knows which the best universities are. Setting their opinion has been the most time-consuming part of the exercise and is also its most distinctive feature. We have shown 13 indicators in this pie diagram which is given below:

![Pie chart showing various academic positions](image)

**Fig 6.1**
As the chart shows, the academics we consult are spread around the world a range from lecturers to university presidents. They have to enter our survey from an academic site (edu or an equivalent), and they have a simple task: to name up to 30 institutions they regard as being the best in the world in field in which they work. In practice, the academic peer reviewers—who cannot vote their own institution choose about 20 universities each. To increase the statistical power of the survey, we aggregate data up to three years old, although we use only the most recent reply form anyone who has responded more than once. The result for 2008 is a survey of 6,354 people. With each person nominating an average of 20 universities, this means we have more than 120,000 data points.

This academic review, the most significant element of the rankings, accounts for 40 percent of an institution's score. The other qualitative element, accounting for 10 percent, is a survey of major employers of graduates across the world and across a range of businesses. The people contacted are active recruiters. They are simply asked from which universities they like to recruit, and again this survey amalgamates up to three years of data. This year's recruiter survey includes the opinion of 2,339 recruiters in every field, from mining to the media.

The other half of a university's possible score in this ranking uses quantitative measures intended to capture quality in key university activities.

The first of these, worth 20 per cent of a university's score, is a measure of staff-to-student ratio, which we believe tells us sometimes about whether a university has enough people to teach the students it admits. QS gather the data on staff a student numbers from a range of sources.
Some come from national statistical bodies such as the Higher Education Statistics Agency in the UK and the National Center for Education Statistics in the US, but many of the data collected from university directly, under a controlled definition of who may be counted as a staff member of student. Both figures are full-time equivalents.

Our understanding of the information that makes up this column has been growing. One of the most spectacular climbs in our top 200, in which the University of Michigan rose from 38 in 2007 to 18 now, has been caused mainly by our use of more accurate staff and student numbers.

A further 20 percent of an institution's score comes from a measure intended to assess the international esteem of its research. Here, we look at the number of citations to published paper from a university and divide it by the number of full-time equivalent staff it employs. We use this approach rather than the more conventional measure of citations per paper because it reveals the density of creative brainpower on campus.

The citations data we use cover a five-year period and come from Scopus. We accept that all such data have some inherent biases, especially towards publishing in English.

Our final two measures, each worth 5 percent of a university's are designed to tell us how serious and institution is about globalization and how successful it is at attracting staff and students from around the world. This lets us see which institutions and nations are most committed to bringing in the best talent and which institutions people want to be. The results obtained by each institution on all these measures are graded using a Z-score, with the top mark set at 100 to make the table readable.

We are always keen for ideas to improve these rankings, but we have considerable confidence in the strength of the methods we use today.
Ranking 2009

THE SCORECARD

We have shown 6 indicators in this pie diagram which is given below:

Fig 6.2

Methodology 2010

The inaugural 2010-2011 methodology is 13 separate indicators grouped under five categories: Teaching (30 percent of final score), research (30 percent), citations (research impact) (worth 32.5 percent), international mix (5 percent), industry income (2.5 percent). The number of indicators is up from the Times-QS rankings published between 2004 and 2009, which used six indicators.

A draft of the methodology was released on 3 June 2010. The draft stated that 13 indicators would first be used and that this could rise to 16 in future rankings, and laid out the categories of indicators as "research indicators" (55 percent), "institutional indicators" (25 percent), "economic activity/innovation" (10 percent), and "international diversity" (10 percent).
also included the weighting signed to each of the 13 indicators, shown below:

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<th>Individual indicators</th>
<th>Percentage weighting</th>
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<td>International diversity</td>
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<td>Teaching-the learning environment</td>
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<td>• PhD’s awards per academic</td>
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<td>• Undergrad, admitted per academic</td>
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<td>• Income per academic</td>
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<td></td>
<td>• PhDs/ undergraduate degrees awarded</td>
<td>• 2.25%</td>
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<td>Research-volume, Income and reputation</td>
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<td>• Research income (scaled)</td>
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<td></td>
<td>• Papers per research and academic staff</td>
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<td>• Public research income/ total research income</td>
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<td>Citations-research influence</td>
<td>• Citation impact (normalized average citation per paper)</td>
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SECTION 2

6.2 APPLICATION OF INFORMATION THEORETIC WEIGHTED NON-SYMMETRIC DIVERGENCES

WORLD UNIVERSITY RANKINGS

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<th>( q_i )</th>
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NUMERIFICATION AND GRAPHICAL REPRESENTATION OF $\chi^2(P//Q;W)$:

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$\chi^2(P//Q;W) = 0.00102072$

Fig 6.3
NUMERIFICATION AND GRAPHICAL REPRESENTATION
OF $\chi^2(Q//P;W)$:

Table 6.4

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Fig 6.4
NUMERIFICATION AND GRAPHICAL REPRESENTATION OF K(P//Q;W):

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\[ K(P//Q;W) = -0.03496 \]
Interpretation: It is notable as when \( p \) is less than \( q \), the self information turns to be negative as a result, the Weighted Relative Divergence is negative. Again, when \( p \) is greater than \( q \), the self information turns to be positive as a result, the Weighted Relative Divergence is positive. Hence due to this behaviour, the totality of the curve is asymmetric.

**NUMERIFICATION AND GRAPHICAL REPRESENTATION OF**

\( K(Q//P;W) \):

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K\( (Q//P;W) = 0.035984 \)
NUMERIFICATION AND GRAPHICAL REPRESENTATION OF \( F(P//Q;W) \):

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\( F(P//Q;W) = -0.01761 \)
NUMERIFICATION AND GRAPHICAL REPRESENTATION OF
F(Q//P;W):

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\[ F(Q//P;W) = 0.017863 \]
NUMERIFICATION AND GRAPHICAL REPRESENTATION OF G(P//Q;W):

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\[ G(P//Q;W) = 0.017864 \]
Interpretation: In this divergence measure, the numerical results show a strange behaviour, for first five values of p. When p is less than q, the self-information turns positive; the divergence also turns positive as a result. When p is greater than q, the self-information turns negative then divergence is immediately negative. Hence due to this behaviour, the totality of the curve is asymmetric.

NUMERIFICATION AND GRAPHICAL REPRESENTATION OF $G(Q//P;W)$:

**Table-6.10**

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$G(Q//P;W) = -0.017607$
NUMERIFICATION AND GRAPHICAL REPRESENTATION OF
\( D(P//Q;W) \):

Table-6.11

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<tr>
<th>i</th>
<th>( p_i )</th>
<th>( q_i )</th>
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\( D(P//Q;W) = 0.000512 \)
Interpretation: Here self-information \( \ln \left( \frac{p_i + q_i}{2q_i} \right) \) is again negative when \( p \) is less than \( q \), but the divergence is positive due to the factor \( (p_i - q_i) \) as it is negative in this case but total divergence is again positive. Obviously it is asymmetric.

**NUMERIFICATION AND GRAPHICAL REPRESENTATION OF**

**D(Q//P;W):**

**Table-6.12**

<table>
<thead>
<tr>
<th>i</th>
<th>( p_i )</th>
<th>( q_i )</th>
<th>( w_i )</th>
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\( D(Q//P;W) = 0.000514 \)
6.3 NUMERICAL VERIFICATION OF INEQUALITIES AMONG NON-SYMMETRIC DIVERGENCE MEASURES

After calculating, we get the following values:

\[ \chi^2(P || Q; W) = 0.001963, \chi^2(Q || P; W) = 0.0019619, \]
\[ K(P || Q; W) = 0.003681, K(Q || P; W) = -0.00172, \]
\[ F(P || Q; W) = 0.001595, F(Q || P; W) = -0.001104, \]
\[ G(P || Q; W) = -0.0011047, G(Q || P; W) = 0.0015953, \]
\[ D(P || Q; W) = 0.000981, D(Q || P; W) = 0.000973, \]

Let \( R = 1.10, r = 0.97 \), we have:
(i) \[
\frac{1}{R^3} \chi^2(P/Q;W) \leq \chi^2(Q/P;W) \leq \frac{1}{r^3} \chi^2(P/Q;W)
\]
\[
\Rightarrow \frac{1}{(1.10)^3} \leq 0.001963 \leq \frac{1}{(0.97)^3} \leq 0.001963
\]
\[
\Rightarrow 0.001475 \leq 0.0019619 \leq 0.002152
\]

(ii) \[
\frac{1}{2R} \chi^2(P/Q;W) \leq K(P/Q;W)
\]
\[
\Rightarrow \frac{1}{2(1.10)} \leq 0.001963 \leq 0.003681
\]
\[
\Rightarrow 0.000893 \leq 0.003681
\]

(iii) \[
\frac{r^2}{2} \chi^2(Q/P;W) \leq K(P/Q;W)
\]
\[
\Rightarrow \frac{(0.97)^2}{2} \leq 0.0019619 \leq 0.003681
\]
\[
\Rightarrow 0.000922 \leq 0.003681
\]

(iv) \[
\frac{1}{2R(R+1)^2} \chi^2(P/Q;W) \leq F(P/Q;W)
\]
\[
\Rightarrow \frac{1}{2 \times 1.10(1.10+1)^2} \leq 0.001963 \leq 0.001595
\]
\[
\Rightarrow 0.000202 \leq 0.001595
\]

(v) \[
\frac{1}{(R+1)^2} K(P/Q;W) \leq F(P/Q;W)
\]
\[
\Rightarrow \frac{1}{(1.10 + 1)^2} \cdot 0.003681 \leq 0.001595
\]

\[
\Rightarrow 0.000835 \leq 0.001595
\]

(vi) \[
\frac{1}{2} \left( \frac{r}{r + 1} \right)^2 \chi^2(Q \parallel P; W) \leq F(P \parallel Q; W)
\]

\[
\Rightarrow \frac{1}{2} \left( \frac{0.97}{0.97 + 1} \right)^2 \cdot 0.0019619 \leq 0.001595
\]

\[
\Rightarrow 0.000238 \leq 0.001595
\]

(vii) \[
\frac{R + 3}{2(R + 1)^2} \chi^2(P \parallel Q; W) \leq D(P \parallel Q; W)
\]

\[
\Rightarrow \frac{1.10 + 3}{2(1.10 + 1)^2} \cdot 0.001963 \leq 0.000981
\]

\[
\Rightarrow 0.000912 \leq 0.000981
\]

(viii) \[
\frac{r^3(r + 3)}{2(r + 1)^2} \chi^2(Q \parallel P; W) \leq D(P \parallel Q; W)
\]

\[
\Rightarrow \frac{(0.97)^3(0.97 + 3)}{2(0.97 + 1)^2} \cdot 0.0019619 \leq 0.000981
\]

\[
\Rightarrow 0.000917 \leq 0.000981
\]

(ix) \[
D(P \parallel Q; W) \leq R(R + 3)F(P \parallel Q; W)
\]

\[
\Rightarrow 0.000981 \leq (1.10)(1.10 + 3) \cdot 0.001595
\]

\[
\Rightarrow 0.000981 \leq 0.007193
\]
(x) \[ \chi^2(Q//P;W) \leq \chi^2(P//Q;W) \]

\[ \Rightarrow 0.0019619 \leq 0.001963 \]

(xi) \[ \frac{1}{2} \chi^2(P//Q;W) \leq rK(P//Q;W) \]

\[ \Rightarrow \frac{1}{2} * 0.001963 \leq 0.97 * 0.003681 \]

\[ \Rightarrow 0.000982 \leq 0.003571 \]

(xii) \[ D(P//Q;W) \leq \frac{r(r+3)}{(r+1)^2} K(P//Q;W) \]

\[ \Rightarrow 0.000981 \leq \frac{0.97(0.97+3)}{(0.97+1)^2} * 0.003681 \]

\[ \Rightarrow 0.000981 \leq 0.003652 \]

(xiii) \[ \frac{R^2(R+3)}{(R+1)^2} K(Q//P;W) \leq D(P//Q;W) \]

\[ \Rightarrow \frac{(1.10)^2(1.10+3)}{(1.10+1)^2} * (-0.00172) \leq 0.000981 \]

\[ \Rightarrow -0.001935 \leq 0.000981 \]

(xiv) \[ (r+3)F(Q//P;W) \leq D(P//Q;W) \]

\[ \Rightarrow (0.97 + 3)(-0.001104) \leq 0.000981 \]

\[ \Rightarrow -0.004383 \leq 0.000981 \]

(xv) \[ (r+1)^2 F(Q//P;W) \leq \frac{1}{2} \chi^2(P//Q;W) \]
\[ (0.97 + 1)^2 (-0.001104) \leq \frac{1}{2} * 0.001963 \]

\[ -0.004285 \leq 0.000982 \]

\[ D(P // Q; W) \leq \frac{2(R + 3)}{(R + 1)} G(Q // P; W) \]  
(xvi)  
\[ 0.000981 \leq \frac{2(1.10 + 3)}{(1.10 + 1)} * 0.0015953 \]

\[ 0.000981 \leq 0.006229 \]

\[ \frac{1}{2} \chi^2(P // Q; W) \leq 2(R + 1) G(Q // P; W) \]  
(xvii)  
\[ \frac{1}{2} * 0.001963 \leq 2(1.10 + 1) * 0.001595 \]

\[ 0.000982 \leq 0.0067 \]

\[ K(Q // P; W) \leq \frac{1}{2r^2} \chi^2(P // Q; W) \]  
(xviii)  
\[ -0.00172 \leq \frac{1}{2(0.97)^2} * 0.001963 \]

\[ -0.00172 \leq 0.001043 \]

\[ K(Q // P; W) \leq \frac{R}{2} \chi^2(Q // P; W) \]  
(xix)  
\[ -0.00172 \leq \frac{1.10}{2} * 0.0019619 \]

\[ -0.00172 \leq 0.001079 \]
\[(xx)\] \[F(Q /\ P; W) \leq \frac{R}{2} \left( \frac{R}{R+1} \right)^2 \chi^2(Q /\ P; W)\]

\[\Rightarrow -0.001104 \leq \frac{1.10}{2} \left( \frac{1.10}{1.10+1} \right)^2 \times 0.0019619\]

\[\Rightarrow -0.001104 \leq 0.000296\]

\[(xxi)\] \[F(Q /\ P; W) \leq RF(P /\ Q; W)\]

\[\Rightarrow -0.001104 \leq 1.10 \times 0.001595\]

\[\Rightarrow -0.001104 \leq 0.001754\]

\[(xxii)\] \[F(Q /\ P; W) \leq \left( \frac{R}{R+1} \right)^2 K(Q /\ P; W)\]

\[\Rightarrow -0.001104 \leq \left( \frac{1.10}{1.10+1} \right)^2 \times (-0.00172)\]

\[\Rightarrow -0.001104 \leq -0.000472\]

\[(xxiii)\] \[\frac{r^3}{4(r+1)} \chi^2(Q /\ P; W) \leq G(Q /\ P; W)\]

\[\Rightarrow \frac{(0.97)^3}{4(0.97+1)} \times 0.0019619 \leq 0.0015953\]

\[\Rightarrow 0.000227 \leq 0.0015953\]

\[(xxiv)\] \[\frac{r}{2(r+1)} K(P /\ Q; W) \leq G(Q /\ P; W)\]
\[
\Rightarrow \frac{0.97}{2(0.97 + 1)} \leq 0.0015953
\]

\[
\Rightarrow 0.000907 \leq 0.0015953
\]

\[(xxv)\]
\[
\frac{r^2}{2(r+1)} K(Q / P; W) \leq G(Q / P; W)
\]

\[
\Rightarrow \frac{(0.97)^2}{2(0.97 + 1)} \leq 0.0015953
\]

\[
\Rightarrow -0.000411 \leq 0.0015953
\]

\[(xxvi)\]
\[
\frac{r}{2} \frac{(r+1)}{2} F(P / Q; W) \leq G(Q / P; W)
\]

\[
\leq \frac{R}{2} (R + 1) F(P / Q; W)
\]

\[
\Rightarrow \frac{(0.97)}{2} (0.97 + 1) \leq 0.0015953
\]

\[
\leq \frac{(1.10)}{2} (1.10 + 1) \leq 0.001595
\]

\[
\Rightarrow 0.001524 \leq 0.0015953 \leq 0.001842
\]

\[(xxvii)\]
\[
\frac{r^2 (r + 3)}{2(r+1)^2} K(Q / P; W) \leq D(P / Q; W)
\]

\[
\Rightarrow \frac{(0.97)^2 (0.97 + 3)}{2(0.97 + 1)^2} \leq 0.000981
\]

\[
\Rightarrow -0.000827 \leq 0.000981
\]
(xxviii) \[ D(P//Q;W) \leq \frac{2(r+3)}{(r+1)} G(Q//P;W) \]

\[ \Rightarrow 0.000981 \leq \frac{2(0.97+3)}{(0.97+1)} * 0.0015953 \]

\[ \Rightarrow 0.000981 \leq 0.00643 \]

(xxix) \[ G(P//Q;W) \leq \frac{1}{4r^2(r+1)} \chi^2(P//Q;W) \]

\[ \Rightarrow -0.0011047 \leq \frac{1}{4(0.97)^2(0.97+1)} * 0.001963 \]

\[ \Rightarrow -0.0011047 \leq 0.000265 \]

(xxx) \[ G(P//Q;W) \leq \frac{R}{4(R+1)} \chi^2(Q//P;W) \]

\[ \Rightarrow -0.0011047 \leq \frac{1.10}{4(1.10+1)} * 0.0019619 \]

\[ \Rightarrow -0.0011047 \leq 0.000257 \]

(xxxi) \[ G(P//Q;W) \leq \frac{1}{2r(r+1)} K(P//Q;W) \]

\[ \Rightarrow -0.0011047 \leq \frac{1}{2(0.97)(0.97+1)} * 0.003681 \]

\[ \Rightarrow -0.0011047 \leq 0.000963 \]

(xxxii) \[ G(P//Q;W) \leq \frac{1}{2(r+1)} K(Q//P;W) \]
\[ -0.0011047 \leq \frac{1}{2(0.97 + 1)} \times (-0.00172) \]

\[ -0.0011047 \leq -0.000437 \]

\[ \frac{2r^2(r+3)}{r+1} G(P \parallel Q;W) \leq D(P \parallel Q;W) \]

\[ \Rightarrow \frac{2(0.97)^2(0.97 + 3)}{0.97 + 1} \times (-0.0011047) \leq 0.000981 \]

\[ -0.004189 \leq 0.000981 \]

\[ \frac{3R+1}{2R^2(R+1)} \chi^2(P \parallel Q;W) \leq D(Q \parallel P;W) \]

\[ \leq \frac{3r+1}{2r^2(r+1)^2} \chi^2(P \parallel Q;W) \]

\[ \Rightarrow \frac{3(1.10) + 1}{2(1.10)^2(1.10 + 1)^2} \times 0.001963 \leq 0.000973 \]

\[ \leq \frac{3(0.97) + 1}{2(0.97)^2(0.97 + 1)^2} \times 0.001963 \]

\[ 0.000791 \leq 0.000973 \leq 0.001051 \]

\[ \frac{r(3r + 1)}{2(r + 1)^2} \chi^2(Q \parallel P;W) \leq D(Q \parallel P;W) \]

\[ \leq \frac{R(3R + 1)}{2(R + 1)^2} \chi^2(Q \parallel P;W) \]
\[
\Rightarrow \frac{(0.97)(3*0.97+1)}{2(0.97+1)^2} * 0.0019619 \leq 0.000973
\]

\[
\leq \frac{(1.10)(3*1.10+1)}{2(1.10+1)^2} * 0.0019619
\]

\[
\Rightarrow 0.000959 \leq 0.000973 \leq 0.001052
\]

(36) \quad \frac{D(Q//P;W)}{D(Q//P;W)} \leq \frac{(3r+1)}{r(r+1)^2} \leq \frac{(3*0.97+1)}{0.97(0.97+1)^2} * 0.003681

\[
\Rightarrow 0.000973 \leq 0.003823
\]

(37) \quad \frac{D(Q//P;W)}{D(Q//P;W)} \leq \frac{(3r+1)}{r} \leq \frac{9.07}{9.7} * 0.001959

\[
\Rightarrow 0.000973 \leq 0.006429
\]

(38) \quad \frac{D(Q//P;W)}{D(Q//P;W)} \leq \frac{(3r+1)}{r} \leq \frac{9.07}{9.7} * 0.001959

\[
\Rightarrow 0.000973 \leq 0.006429
\]

(39) \quad \frac{D(Q//P;W)}{D(Q//P;W)} \leq \frac{(3r+1)}{r} \leq \frac{9.07}{9.7} * 0.001959

\[
\Rightarrow 0.000973 \leq 0.006429
\]
\[ 2(3 \cdot 0.97 + 1) \cdot (-0.0011047) \leq 0.000973 \]

\[ \Rightarrow -0.004385 \leq 0.000973 \]

SECTION 4

6.4 APPLICATION OF WEIGHTED THEIL'S INFORMATION IMPROVEMENT

WORLD UNIVERSITY RANKINGS

<table>
<thead>
<tr>
<th>2008 Rank</th>
<th>2009 Rank</th>
<th>2010 Rank</th>
<th>Institute</th>
<th>Country</th>
<th>Overall Weightage 2008</th>
<th>( p_i )</th>
<th>Overall Weightage 2009</th>
<th>( q_i )</th>
<th>Overall Weightage 2010</th>
<th>( r_i )</th>
<th>( w_i )</th>
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<td>100</td>
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Application of Information Improvement Through Thiel’s Measure

Table-6.14

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<td>$r_i$</td>
<td>$w_i$</td>
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*Note: Analysis:* Negative improvement shows that the rank has come down. Harvard University rank has come down from 1st to 11nd positive improvement shows that the rank has been improved from 31rd to 11nd and then 1st in 2010 as the case of University of Cambridge.