Preface

Chaos is a ubiquitous and robust nonlinear phenomenon often encountered in nature in diverse physical, chemical and biological systems. A large body of the studies which have appeared in the literature relating to chaotic dynamics of damped, periodically and quasiperiodically driven nonlinear oscillators both in experimental and in theoretical works are often fragmentary in nature in the parameter space. A systematic understanding of the dynamics covering whole ranges of parameters will be more valuable. Consequently, much more informations concerning the physical features through the study of bifurcations and chaos still remain to be extracted. Thus, detailed investigations on the inter-relationship between the system parameters and the system behaviours including bifurcations and their mechanisms, chaos,its routes, and so on are of considerable interest. Such analyses allow one to deduce a certain global picture about the dynamical transitions occuring in the systems. They also act as guides in uncovering the dynamical behaviours in real experiments.

In this thesis we wish to make a systematic study of bifurcations and chaos phenomena in certain generalised nonlinear oscillators driven by external periodic and quasiperiodic forces, which can throw much insight into the behaviour of the underlying nonlinear dynamical systems. In particular, we wish to con-
centrate on two classes of dynamical systems—namely, (1) nonlinear oscillators subjected to external periodic forcing, and (2) nonlinear oscillators subjected to two-frequency quasiperiodic forcing, and investigate the dynamics in a one or two-parameter space. It has been shown that these systems exhibit a rich variety of bifurcations, regular and complex structures including strange chaotic and nonchaotic behaviours. Specifically, we consider the following class of nonlinear dynamical systems—namely periodically forced Duffing-van der Pol oscillator, velocity-dependent nonpolynomial potential systems, and quasiperiodically driven velocity-dependent potential systems, Murali - Lakshmanan - Chua (MLC) circuit, Duffing oscillator, and a cubic map for our investigations. Detailed analysis on each of the cases is carried out and the results are presented with reference to a one or two-parameter plane. The investigations were carried out during the period 1994-1999, under the supervision of Prof. M. Lakshmanan, at the Centre for Nonlinear Dynamics, Department of Physics, Bharathidasan University, Tiruchirappalli - 620 024.

The thesis consists of eight chapters, including the introducing Chapter I. Chapter II contains a brief discussion of simple attractors, strange attractors, their classifications, and characterizations. We bring out in Chapter III a detailed study of the bifurcations and chaos phenomena in the double-well Duffing-van der Pol oscillator. The existence of different attractors in the system parameters \( f - \omega \) domain is examined and a detailed account of the various steady states for fixed damping is presented. Various routes to chaos namely, period doubling, quasiperiodic, intermittent and crises routes are discussed in detail. Further, phase-locking, mode-locking, periodic windows, and co-existence of multiple attractors are reported. Moreover, local and global bifurcations have been elucidated in detail.

In Chapter IV, we consider the velocity dependent-potential systems—namely,
(1) rotating parabola system and (2) nonlinear harmonic oscillator. We have also shown that the damped and forced versions of these systems exhibit a rich variety of bifurcations phenomena. It includes the familiar period doubling bifurcations, preceded by a symmetry breaking bifurcation. Besides, the period doubling route to chaos, intermittency, antimonotonicity and crisis routes are observed. Another interesting physical situation is the case in which there is an additional parametric modulation in the angular velocity of the rotating parabola system. We investigate the dynamical features associated with the system. In Chapter V, we focus our attention on the combined effect of both the external and parametric forcing in the parabola system and in particular when the frequency ratio of the forcing is kept irrational. In this case, besides the standard attractors, a new dynamical behaviour, namely strange nonchaotic attractor (SNA) is observed. Several routes, including the standard ones by which the appearance of strange nonchaotic attractors takes place, are shown to be realizable in this model over a two-parameter \((f-e)\) domain of the system. In particular, the transition through torus doubling to chaos via SNA, torus breaking to chaos via SNA and period doubling bifurcations of fractal torus are demonstrated with the aid of the two-parameter \((f-e)\) phase diagram. More interestingly, in order to approach the strange nonchaotic attractor, the existence of several new bifurcations on the torus corresponding to the novel phenomenon of torus bubbling are described.

Next in Chapter VI, we consider the simplest second order nonlinear dissipative nonautonomous circuit introduced recently by our group, namely the Murali-Lakshmanan-Chua (MLC) circuit. The dynamics of this circuit subjected to the external quasiperiodic forcing has been investigated. The scenarios and mechanisms for the creation of SNAs are illustrated in detail through numerical and experimental studies of this simple piecewise-linear electronic cir-
cuit. In particular, a new route and the associated mechanism are described for the creation of strange nonchaotic attractor during transition from two-frequency quasiperiodicity to chaotic attractor through torus doubling bifurcation. Strange nonchaotic attractor arises when a torus doubled attractor is interrupted by a subharmonic bifurcation, resulting in the inhibition of torus doubling sequence. This transition is shown to exhibit type III intermittent characteristic scaling behaviour. Then, we concentrate on the chaotic dynamics of a two-frequency parametrically driven Duffing oscillator which is shown to be dominated by the occurrence of a variety of strange nonchaotic attractors (SNAs). Different routes and mechanisms for the creation of such SNAs are studied in Chapter VII. It includes Heagy-Hammel, gradual fractalization and type I and type III intermittencies. Finally, in Chapter VIII, we consider a simple quasiperiodically forced one-dimensional cubic map which is shown to exhibit very many routes to chaos via SNAs in a two-parameter (A-Q) space. In particular, torus collision, fractalization, intermittencies, and crisis phenomenon have been investigated.